

Calculus free-response points are lost to language more than to algebra: an answer that computes correctly but never says “rate of change of ... with respect to ...”, or states a theorem without checking its hypotheses. Learn these sentences and use them verbatim. 微积分失分多在表述: 必须写清 “关于 的变化率”, 并核对定理成立条件。

Limits and continuity

Limit 极限 — the value a function approaches as the input approaches a point; it can exist where the function is undefined

Continuity at a point 在一点连续 — f is continuous at c when f(c) exists, the limit as $x \rightarrow c$ exists, and the two are equal – all three conditions

Intermediate Value Theorem 介值定理 — if f is continuous on [a, b] and k lies between f(a) and f(b), then $f(x) = k$ for some x in (a, b); continuity must be stated to use it

Removable discontinuity 可去间断点 — a point where the limit exists but does not equal the function value, so redefining one point would make f continuous

Derivatives

Derivative 导数 — the instantaneous rate of change of f with respect to x, defined as the limit of the difference quotient

Differentiability 可导性 — the existence of the derivative at a point; differentiability implies continuity, but continuity does not imply differentiability

Chain rule 链式法则 — the derivative of a composite function is the derivative of the outer function evaluated at the inner, times the derivative of the inner

Implicit differentiation 隐函数求导 — differentiating both sides of a relation with respect to x, treating y as a function of x so that dy/dx appears by the chain rule

Critical point 临界点 — a point in the domain where the derivative is zero or undefined

Mean Value Theorem 中值定理 — if f is continuous on [a, b] and differentiable on (a, b), some c in (a, b) has $f'(c)$ equal to the average rate of change over [a, b]

Point of inflection 拐点 — a point where the concavity changes, which requires the second derivative to CHANGE SIGN, not merely to be zero

Related rates 相关变化率 — using a relation between quantities and the chain rule to find one rate from another, both with respect to time

Integrals

Definite integral 定积分 — the limit of Riemann sums over a partition, equal to the net signed area between the curve and the x-axis

Riemann sum 黎曼和 — an approximation of a definite integral by rectangles; left, right, midpoint and trapezoidal rules differ in the height chosen

Fundamental Theorem of Calculus 微积分基本定理 — differentiation and integration are inverse: the derivative of the accumulation function is the integrand, and a definite integral equals the change in an antiderivative

Average value of a function 函数平均值 — the definite integral over an interval divided by the LENGTH of that interval

Accumulation function 累积函数 — a function defined as an integral from a fixed lower limit to a variable upper limit; its derivative is the integrand

Separable differential equation 可分离变量微分方程 — an equation that can be written so all y terms are on one side and all x terms on the other, then integrated

Slope field 斜率场 — a grid of short segments whose slopes are given by dy/dx at each point, showing the shape of the solution curves

Interpretation in context

Every rate answer needs UNITS and a reference: “the volume is increasing at 3 cm³ per second at t = 4”, not “3”.

A definite integral of a rate gives the NET CHANGE in the quantity, and saying so in context is usually a separate point.

Before invoking the Mean Value Theorem or the Intermediate Value Theorem, state that the hypotheses hold. The justification is the mark, not the conclusion.