

Part B (AB or BC): Graphing calculator not allowed**Question 3****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R , where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

	Model Solution	Scoring	
A	Approximate $R'(1)$ using the average rate of change of R over the interval $0 \leq t \leq 2$. Show the work that leads to your answer. Indicate units of measure.		
	$R'(1) \approx \frac{R(2) - R(0)}{2 - 0}$ $= \frac{100 - 90}{2} = \frac{10}{2} = 5 \text{ words per minute per minute}$	Answer with setup	Point 1 (P1)
		Units	Point 2 (P2)

Scoring Notes for Part A

- To earn **P1**, a response must present the answer along with the supporting work of a difference and a quotient using values from the table.
 - $\frac{100 - 90}{2 - 0}$, $\frac{10}{2 - 0}$, $\frac{100 - 90}{2}$, or $\frac{R(2) - R(0)}{2 - 0} = 5$ is sufficient to earn **P1**.
 - $\frac{R(2) - R(0)}{2 - 0}$ by itself is not sufficient to earn **P1**.
- P2** is earned for correct units, whether or not they are attached to a numerical value for the average rate of change.
- P2** is also earned for the units “words / minute² .”

B Must there be a value c , for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

R is differentiable implies R is continuous.

Differentiable
implies continuous

Point 3 (P3)

$$R(0) = 90 < 155 < R(10) = 162$$

Answer with
justification

Point 4 (P4)

Therefore, by the Intermediate Value Theorem, there must be a value c , with $0 < c < 10$, such that $R(c) = 155$.

Scoring Notes for Part B

- To earn **P3**, a response must state that R is continuous because R is differentiable (or equivalent). A response that simply states “ R is continuous” without justification does not earn **P3**.
- A response does not need to earn **P3** to be eligible for **P4**.
- To earn **P4**, a response must indicate that $R(0) < 155$ (or $R(2) < 155$ or $R(8) < 155$) and $R(10) > 155$, state that “ R is continuous,” and answer “yes” in some way.
- To earn **P4**, a response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

- C Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t) \, dt$. Show the work that leads to your answer.

$\int_0^{10} R(t) \, dt$ $\approx \frac{R(0) + R(2)}{2}(2 - 0) + \frac{R(2) + R(8)}{2}(8 - 2)$ $+ \frac{R(8) + R(10)}{2}(10 - 8)$	Form of trapezoidal sum	Point 5 (P5)
$= \frac{90 + 100}{2}(2 - 0) + \frac{100 + 150}{2}(8 - 2) + \frac{150 + 162}{2}(10 - 8)$ $= \frac{190}{2}(2) + \frac{250}{2}(6) + \frac{312}{2}(2) = 190 + 750 + 312 = 1252$	Answer with supporting work	Point 6 (P6)

Scoring Notes for Part C

- Read “=” as “ $\approx$ ” for **P5**.
- The form of a trapezoidal sum includes three terms, each of which includes a product of two factors, where one of the factors incorporates the $\frac{1}{2}$ as part of the product. To earn **P5**, at least five of the six factors must be correct. If any of the six factors is incorrect, the response does not earn **P6**. Consider the following examples:
 - $\frac{90 + 100}{2}(2 - 0) + \frac{100 + 150}{2}(8 - 2) + \frac{150 + 162}{2}(10 - 8)$ earns **P5** and is sufficient to earn **P6**.
 - $\frac{190}{2}(2) + \frac{250}{2}(6) + \frac{312}{2}(2)$ earns **P5** and is sufficient to earn **P6**.
 - $\frac{1}{2}((R(0) + R(2))(2) + (R(2) + R(8))(6) + (R(8) + R(10))(2))$ earns **P5** and is eligible for **P6**.
 - $\frac{90 + 100}{2}(2) + \frac{100 + 150}{2}(2) + \frac{150 + 162}{2}(2)$ earns **P5** but is not eligible for **P6**.
(Note that the factor of 2 in the second term of this expression is incorrect.)
- Special case:** A response of $(90 + 100) + (100 + 150)3 + (150 + 162)$ earns both **P5** and **P6**.
- To be eligible for **P6**, a response must have earned **P5**.
Special case: A response of $95 \cdot 2 + 125 \cdot 6 + 156 \cdot 2$ earns **P6** but does not earn **P5**.
- A response of $\frac{90 + 100}{2}(2 - 0) + \frac{100 + 150}{2}(8 - 2) + \frac{150 + 162}{2}(10 - 8)$ or equivalent earns **P6** (i.e., subsequent errors in simplification will not be considered in scoring for **P6**).
- A response of $\frac{(90 \cdot 2 + 100 \cdot 6 + 150 \cdot 2) + (100 \cdot 2 + 150 \cdot 6 + 162 \cdot 2)}{2}$ or equivalent earns both **P5** and **P6**. (Note that the average of the left Riemann sum and right Riemann sum is equivalent to the trapezoidal sum.)
- A completely correct left Riemann sum (e.g., $90 \cdot 2 + 100 \cdot 6 + 150 \cdot 2 = 1080$) or a completely correct right Riemann sum (e.g., $100 \cdot 2 + 150 \cdot 6 + 162 \cdot 2 = 1424$) earns **P5** but does not earn **P6**.

- D** A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

$$\int_0^{10} W(t) \, dt = \int_0^{10} \left(-\frac{3}{10}t^2 + 8t + 100 \right) dt$$

Integrand **Point 7 (P7)**

$$= \left(-\frac{1}{10}t^3 + 4t^2 + 100t \right) \Big|_0^{10}$$

Antiderivative **Point 8 (P8)**

$$= \left(-\frac{1}{10} \cdot 1000 + 4 \cdot 100 + 100 \cdot 10 \right) - \left(-\frac{1}{10} \cdot 0 + 4 \cdot 0 + 100 \cdot 0 \right) \\ = 1300$$

Answer **Point 9 (P9)**

Based on the model, the teacher has read 1300 words by the end of the 10 minutes.

Scoring Notes for Part D

- **P7** is earned for an indefinite or definite integral with integrand $W(t)$, with or without the differential dt .
- **P8** is earned for the correct antiderivative, with or without the constant of integration.
- To be eligible for **P9**, a response must have earned **P8**.
- A response of $\left(-\frac{1}{10} \cdot 1000 + 4 \cdot 100 + 100 \cdot 10 \right) - \left(-\frac{1}{10} \cdot 0 + 4 \cdot 0 + 100 \cdot 0 \right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by

$A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

Model Solution	Scoring
(a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).	
The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by $\int_1^5 A(t) dt$.	Answer 1 point
Scoring notes: - The response must be a definite integral with correct lower and upper limits to earn this point. - Because $A(t) = A(t)$ for $1 \leq t \leq 5$, a response of $\int_1^5 450\sqrt{\sin(0.62t)} dt$ or $\int_1^5 A(t) dt$ earns the point. - A response missing dt or using dx is eligible to earn the point. - A response with a copy error in the expression for $A(t)$ will earn the point only in the presence of $\int_1^5 A(t) dt$.	
Total for part (a) 1 point	

- (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

$$\text{Average} = \frac{1}{5-1} \int_1^5 A(t) dt = 375.536966$$

Uses average value formula: **1 point**

$$\frac{1}{b-a} \int_a^b A(t) dt$$

The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is 375.537 (or 375.536) vehicles per hour.

Answer **1 point**

Scoring notes:

- The use of the average value formula, indicating that $a = 1$ and $b = 5$, can be presented in single or multiple steps to earn the first point. For example, the following response earns both points:

$$\int_1^5 A(t) dt = 1502.147865, \text{ so the average value is } 375.536966.$$

- A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points: $\int_1^5 A(t) dt = 1502.147865 = 375.536966$.

- The answer must be correct to three decimal places. For example,

$$\frac{1}{5-1} \int_1^5 A(t) dt = 375.536966 \approx 376 \text{ earns only the first point.}$$

- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{4} \int_1^5 A(t) dt = 79.416068$.

- Special case: $\frac{1}{5} \int_1^5 A(t) dt = 300.429573$ earns 1 out of 2 points.

Total for part (b) 2 points

- (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

$A'(1) = 148.947272$	Considers $A'(1)$	1 point
Because $A'(1) > 0$, the rate at which the vehicles arrive at the toll plaza is increasing.	Answer with reason	1 point

Scoring notes:

- The response need not present the value of $A'(1)$. The second line of the model solution earns both points.
- An incorrect value assigned to $A'(1)$ earns the first point (but will not earn the second point).
- Without a reference to $t = 1$, the first point is earned by any of the following:
 - 148.947 accurate to the number of decimals presented, with zero up to three decimal places (i.e., 149, 148, 148.9, 148.95, or 148.94)
 - $A'(t) = 148.947$ by itself
- To be eligible for the second point, the first point must be earned.
- To earn the second point, there must be a reference to $t = 1$.
- Degree mode: $A'(1) = 23.404311$

Total for part (c) 2 points

- (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t , for $a \leq t \leq 4$, is given by

$N(t) = \int_a^t (A(x) - 400) dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

$N'(t) = A(t) - 400 = 0$ $\Rightarrow A(t) = 400 \Rightarrow t = 1.469372, t = 3.597713$ $a = 1.469372$ $b = 3.597713$	Considers $N'(t) = 0$	1 point								
	$t = a$ and $t = b$	1 point								
<table><tr><td>t</td><td>$N(t) = \int_a^t (A(x) - 400) dx$</td></tr><tr><td>$a$</td><td>0</td></tr><tr><td>b</td><td>71.254129</td></tr><tr><td>4</td><td>62.338346</td></tr></table> The greatest number of vehicles in line is 71.	t	$N(t) = \int_a^t (A(x) - 400) dx$	a	0	b	71.254129	4	62.338346	Answer	1 point
	t	$N(t) = \int_a^t (A(x) - 400) dx$								
	a	0								
	b	71.254129								
4	62.338346									
Justification	1 point									

Scoring notes:

- It is not necessary to indicate that $A(t) = 400$ to earn the first point, although this statement alone would earn the first point.
- A response of “ $A(t) \geq 400$ when $1.469372 \leq t \leq 3.597713$ ” will earn the first 2 points. A response of “ $A(t) \geq 400$ ” along with the presence of exactly one of the two numbers above will earn the first point, but not the second. A response of “ $A(t) \geq 400$ ” by itself will not earn either of the first 2 points.
- To earn the second point the values for a and b must be accurate to the number of decimals presented, with at least one and up to three decimal places. These may appear only in a candidates table, as limits of integration, or on a number line.
- A response with incorrect notation involving t or x is eligible to earn all 4 points.
- A response that does not earn the first point is still eligible for the remaining 3 points.
- To earn the third point, a response must present the greatest number of vehicles. This point is earned for answers of either 71 or 71.254*** only.
- A correct justification earns the fourth point, even if the third point is not earned because of a decimal presentation error.
- When using a Candidates Test, the response must include the values for $N(a)$, $N(b)$, and $N(4)$ to earn the fourth point. These values must be correct to the number of decimals presented, with up to three decimal places. (Correctly rounded integer values are acceptable.)
- Alternate solution for the third and fourth points:
 For $a \leq t \leq b$, $A(t) \geq 400$. For $b \leq t \leq 4$, $A(t) \leq 400$.
 Thus, $N(t) = \int_a^t (A(x) - 400) dx$ is greatest at $t = b$.
 $N(b) = 71.254129$, and the greatest number of vehicles in line is 71.
- Degree mode: The response is only eligible to earn the first point because in degree mode $A(t) < 400$.

Total for part (d) 4 points

Total for question 1 9 points

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f , where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

Model Solution**Scoring**

- (a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{0.5} = 8$$

Estimate **1 point**

At a distance of $r = 2.25$ centimeters from the center of the petri dish, the density of the bacteria population is increasing at a rate of 8 milligrams per square centimeter per centimeter.

Interpretation with units **1 point**

Scoring notes:

- To earn the first point the response must provide both a difference and a quotient and must explicitly use values of f from the table.
- Simplification of the numerical value is not required to earn the first point. If the numerical value is simplified, it must be correct.
- The interpretation requires all of the following: distance $r = 2.25$, density of bacteria (population) is increasing or changing, at a rate of 8, and units of milligrams per square centimeter per centimeter.
- The second point (interpretation) cannot be earned without a nonzero presented value for $f'(2.25)$.
- To earn the second point the interpretation must be consistent with the presented nonzero value for $f'(2.25)$. In particular, if the presented value for $f'(2.25)$ is negative, the interpretation must include “decreasing at a rate of $|f'(2.25)|$ ” or “changing at a rate of $f'(2.25)$.” The second point cannot be earned for an incorrect statement such as “the bacteria density is decreasing at a rate of $-8 \dots$ ” even for a presented $f'(2.25) = -8$.
- The units ($\text{mg}/\text{cm}^2/\text{cm}$) may be attached to the estimate of $f'(2.25)$ and, if so, do not need to be repeated in the interpretation.
- If units attached to the estimate do not agree with units in the interpretation, read the units in the interpretation.

Total for part (a) 2 points

- (b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression

$2\pi \int_0^4 r f(r) dr$. Approximate the value of $2\pi \int_0^4 r f(r) dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

$2\pi \int_0^4 r f(r) dr \approx 2\pi(1 \cdot f(1) \cdot (1 - 0) + 2 \cdot f(2) \cdot (2 - 1) + 2.5 \cdot f(2.5) \cdot (2.5 - 2) + 4 \cdot f(4) \cdot (4 - 2.5))$	Right Riemann sum setup	1 point
$= 2\pi(1 \cdot 2 \cdot 1 + 2 \cdot 6 \cdot 1 + 2.5 \cdot 10 \cdot 0.5 + 4 \cdot 18 \cdot 1.5)$ $= 269\pi = 845.088$	Approximation	1 point

Scoring notes:

- The presence or absence of 2π has no bearing on earning the first point.
- The first point is earned for a sum of four products with at most one error in any single value among the four products. Multiplication by 1 in any term does not need to be shown, but all other products must be explicitly shown.
- A response of $1 \cdot f(1) \cdot (1 - 0) + 2 \cdot f(2) \cdot (2 - 1) + 2.5 \cdot f(2.5) \cdot (2.5 - 2) + 4 \cdot f(4) \cdot (4 - 2.5)$ earns the first point but not the second.
- A response with any error in the Riemann sum is not eligible for the second point.
- A response that provides a completely correct left Riemann sum for $2\pi \int_0^4 r f(r) dr$ and approximation (91π) earns one of the two points. A response that has any error in a left Riemann sum or evaluation for $2\pi \int_0^4 r f(r) dr$ earns no points.
- A response that provides a completely correct right Riemann sum for $2\pi \int_0^4 r f(r) dr$ and approximation (80π) earns one of the two points. A response that has any error in a right Riemann sum or evaluation for $2\pi \int_0^4 r f(r) dr$ earns no points.
- Simplification of the numerical value is not required to earn the second point. If a numerical value is given, it must be correct to three decimal places.

Total for part (b) 2 points

- (c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

$\frac{d}{dr}(rf(r)) = f(r) + rf'(r)$	Product rule expression for $\frac{d}{dr}(rf(r))$	1 point
Because f is nonnegative and increasing, $\frac{d}{dr}(rf(r)) > 0$ on the interval $0 \leq r \leq 4$. Thus, the integrand $rf(r)$ is strictly increasing. Therefore, the right Riemann sum approximation of $2\pi \int_0^4 rf(r) dr$ is an overestimate.	Answer with explanation	1 point

Scoring notes:

- To earn the second point a response must explain that $rf(r)$ is increasing and, therefore, the right Riemann sum is an overestimate. The second point can be earned without having earned the first point.
- A response that attempts to explain based on a left Riemann sum for $2\pi \int_0^4 rf(r) dr$ from part (b) earns no points.
- A response that attempts to explain based on a right Riemann sum for $2\pi \int_0^4 f(r) dr$ from part (b) earns no points.

Total for part (c) 2 points

- (d) The density of bacteria in the petri dish, for $1 \leq r \leq 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k , $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \leq r \leq 4$?

Average value = $g_{\text{avg}} = \frac{1}{4-1} \int_1^4 g(r) dr$	Definite integral	1 point
$\frac{1}{4-1} \int_1^4 g(r) dr = 9.875795$	Average value	1 point
$g(k) = g_{\text{avg}} \Rightarrow k = 2.497$	Answer	1 point

Scoring notes:

- The first point is earned for a definite integral, with or without $\frac{1}{4-1}$ or $\frac{1}{3}$.
- A response that presents a definite integral with incorrect limits but a correct integrand earns the first point.
- Presentation of the numerical value 9.875795 is not required to earn the second point. This point can be earned by the average value setup: $\frac{1}{3} \int_1^4 g(r) dr$.
- Once earned for the average value setup, the second point cannot be lost. Subsequent errors will result in not earning the third point.
- The third point is earned only for the value $k = 2.497$.
- The third point cannot be earned without the second.
- Special case: A response that does not provide the average value setup but presents an average value of -13.955 is using degree mode on their calculator. This response would not earn the second point but could earn the third point for an answer of $k = 2.5$ (or 2.499).

Total for part (d) 3 points

Total for question 1 9 points

2018

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Scoring Guidelines

2018 SCORING GUIDELINES

Question 1

(a) $\int_0^{300} r(t) dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $20 + \int_0^{300} (r(t) - 0.7) dt = 20 + \int_0^{300} r(t) dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

2 : $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or 414.285) seconds.}$$

1 : answer

(d) The total number of people in line at time t , $0 \leq t \leq 300$, is modeled by

$$20 + \int_0^t r(x) dx - 0.7t.$$

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

4 : $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

2018 SCORING GUIDELINES

Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118 .

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760 .

3 : $\begin{cases} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{3.5} v(t) dt = 2.844$ (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral $\int_0^{3.5} v(t) dt$ is the displacement of the particle over the time interval $0 \leq t \leq 3.5$.

The integral $\int_0^{3.5} |v(t)| dt$ is the total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

3 : $\begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{cases}$

(d) $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

2 : $\begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$

2018 SCORING GUIDELINES

Question 3

$$\begin{aligned} \text{(a)} \quad f(-5) &= f(1) + \int_1^{-5} g(x) \, dx = f(1) - \int_{-5}^1 g(x) \, dx \\ &= 3 - \left(-9 - \frac{3}{2} + 1 \right) = 3 - \left(-\frac{19}{2} \right) = \frac{25}{2} \end{aligned}$$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b)} \quad \int_1^6 g(x) \, dx &= \int_1^3 g(x) \, dx + \int_3^6 g(x) \, dx \\ &= \int_1^3 2 \, dx + \int_3^6 2(x-4)^2 \, dx \\ &= 4 + \left[\frac{2}{3}(x-4)^3 \right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3} \right) = 10 \end{aligned}$$

3 : $\begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f''(x) = g'(x)$ is increasing on those intervals.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

2018 SCORING GUIDELINES

Question 4

$$(a) \quad H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$$(b) \quad \frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$$

Because H is differentiable on $3 \leq t \leq 5$, H is continuous on $3 \leq t \leq 5$.

By the Mean Value Theorem, there exists a value c , $3 < c < 5$, such that $H'(c) = 2$.

$$(c) \quad \text{The average height of the tree over the time interval } 2 \leq t \leq 10 \text{ is given by } \frac{1}{10 - 2} \int_2^{10} H(t) \, dt.$$

$$\begin{aligned} \frac{1}{8} \int_2^{10} H(t) \, dt &\approx \frac{1}{8} \left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3 \right) \\ &= \frac{1}{8} (65.75) = \frac{263}{32} \end{aligned}$$

The average height of the tree over the time interval $2 \leq t \leq 10$ is $\frac{263}{32}$ meters.

$$(d) \quad G(x) = 50 \Rightarrow x = 1$$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$$

$$\left. \frac{d}{dt}(G(x)) \right|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\frac{3}{4}$ meter per year.

$$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$$

$$2 : \begin{cases} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$$

$$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$$

$$3 : \begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$$

Note: max 1/3 [1-0] if no chain rule

2018 SCORING GUIDELINES

Question 5

- (a) The average rate of change of
- f
- on the interval
- $0 \leq x \leq \pi$
- is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

1 : answer

- (b)
- $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

2 : $\begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

- (c)
- $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

3 : $\begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$

The absolute minimum value of f on $0 \leq x \leq 2\pi$ is $-\frac{1}{\sqrt{2}}e^{5\pi/4}$.

- (d)
- $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

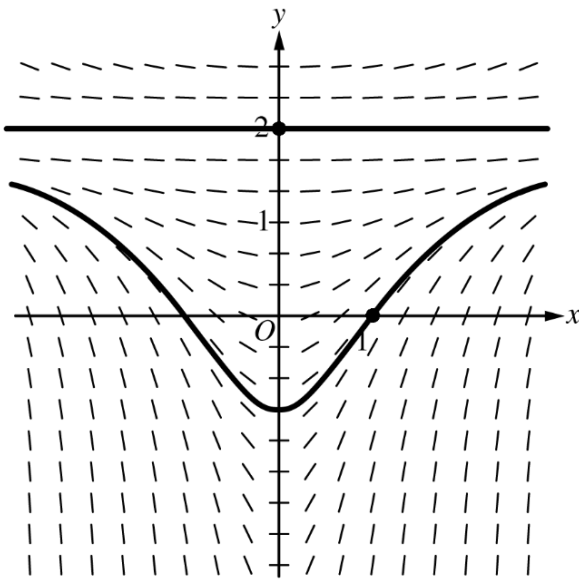
3 : $\begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \text{and limits equal 0} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

2018 SCORING GUIDELINES

Question 6

(a)



$$2 : \begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

$$(b) \left. \frac{dy}{dx} \right|_{(x, y)=(1, 0)} = \frac{4}{3}$$

An equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ is $y = \frac{4}{3}(x - 1)$.

$$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$$

$$2 : \begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$$

$$(c) \frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$$

$$\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$$

$$y = 2 - \frac{6}{x^2 + 2}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: this solution is valid for $-\infty < x < \infty$.

AP[®] Calculus AB

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2016 SCORING GUIDELINES

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

- (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

$$(a) \quad R'(2) \approx \frac{R(3) - R(1)}{3 - 1} = \frac{950 - 1190}{3 - 1} = -120 \text{ liters/hr}^2$$

$$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$$

$$(b) \quad \text{The total amount of water removed is given by } \int_0^8 R(t) \, dt.$$

$$\begin{aligned} \int_0^8 R(t) \, dt &\approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters} \end{aligned}$$

This is an overestimate since R is a decreasing function.

$$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$$

$$\begin{aligned} (c) \quad \text{Total} &\approx 50000 + \int_0^8 W(t) \, dt - 8050 \\ &= 50000 + 7836.195325 - 8050 \approx 49786 \text{ liters} \end{aligned}$$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$$

$$(d) \quad W(0) - R(0) > 0, \quad W(8) - R(8) < 0, \quad \text{and } W(t) - R(t) \text{ is continuous.}$$

$$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$$

Therefore, the Intermediate Value Theorem guarantees at least one time t , $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

For this value of t , the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

2016 SCORING GUIDELINES

Question 2

For $t \geq 0$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

- At time $t = 4$, is the particle speeding up or slowing down?
- Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.
- Find the position of the particle at time $t = 0$.
- Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

(a) $v(4) = 2.978716 > 0$
 $v'(4) = -1.164000 < 0$

The particle is slowing down since the velocity and acceleration have different signs.

(b) $v(t) = 0 \Rightarrow t = 2.707468$

$v(t)$ changes from positive to negative at $t = 2.707$.
 Therefore, the particle changes direction at this time.

(c) $x(0) = x(4) + \int_4^0 v(t) dt$
 $= 2 + (-5.815027) = -3.815$

(d) Distance $= \int_0^3 |v(t)| dt = 5.301$

2 : conclusion with reason

2 : $\begin{cases} 1 : t = 2.707 \\ 1 : \text{justification} \end{cases}$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

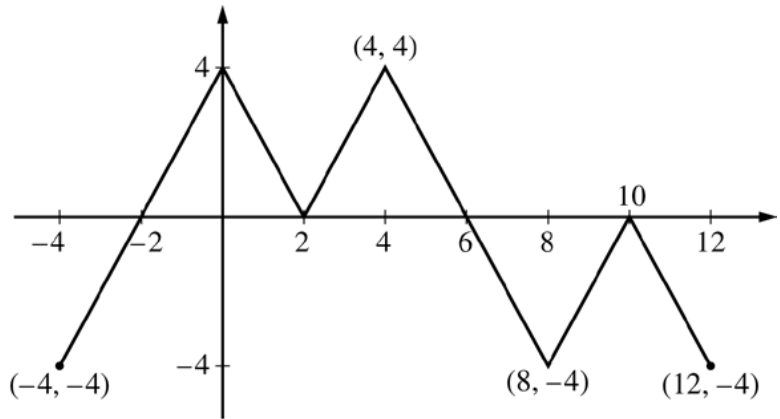
2016 SCORING GUIDELINES

Question 3

The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by

$$g(x) = \int_2^x f(t) \, dt.$$

- (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
- (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
- (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.
- (d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

Graph of f

- (a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \leq 0$ for $8 \leq x \leq 12$.
- (b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \leq x \leq 4$ and decreasing for $4 \leq x \leq 8$.
- (c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
-4	-4
-2	-8
6	8
12	-4

On the interval $-4 \leq x \leq 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

- (d) $g(x) \leq 0$ for $-4 \leq x \leq 2$ and $10 \leq x \leq 12$.

1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

1 : answer with justification

1 : answer with justification

4 : $\left\{ \begin{array}{l} 1 : \text{considers } x = -2 \text{ and } x = 6 \\ \quad \text{as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{array} \right.$

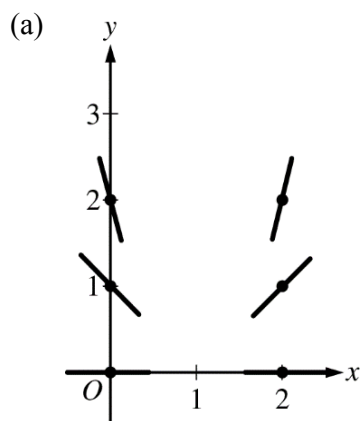
2 : intervals

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Question 4

Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$.
Use your equation to approximate $f(2.1)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.



2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = \frac{3^2}{2-1} = 9$

An equation for the tangent line is $y = 9(x - 2) + 3$.

$$f(2.1) \approx 9(2.1 - 2) + 3 = 3.9$$

2 : $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{1}{y^2} dy = \frac{1}{x-1} dx$
 $\int \frac{1}{y^2} dy = \int \frac{1}{x-1} dx$
 $-\frac{1}{y} = \ln|x-1| + C$
 $-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$
 $-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$
 $y = \frac{1}{\frac{1}{3} - \ln(x-1)}$

5 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and} \\ \quad \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

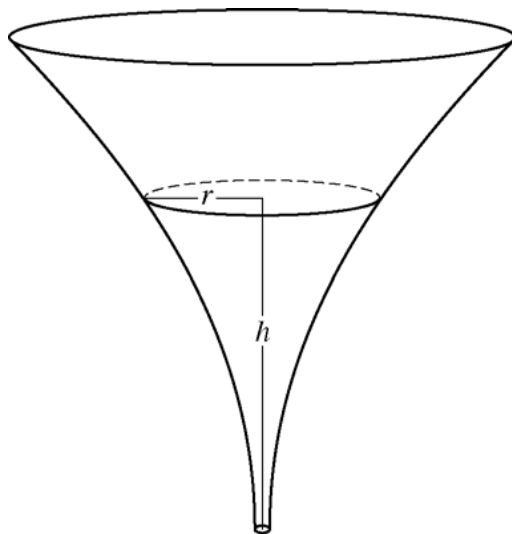
Note: max 3/5 [1-2-0-0] if no constant of integration

Note: 0/5 if no separation of variables

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

2016 SCORING GUIDELINES

Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

- (a) Find the average value of the radius of the funnel.
- (b) Find the volume of the funnel.
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

$$\begin{aligned} \text{(a) Average radius} &= \frac{1}{10} \int_0^{10} \frac{1}{20}(3 + h^2) dh = \frac{1}{200} \left[3h + \frac{h^3}{3} \right]_0^{10} \\ &= \frac{1}{200} \left(\left(30 + \frac{1000}{3} \right) - 0 \right) = \frac{109}{60} \text{ in} \end{aligned}$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^{10} \left(\left(\frac{1}{20} \right) (3 + h^2) \right)^2 dh = \frac{\pi}{400} \int_0^{10} (9 + 6h^2 + h^4) dh \\ &= \frac{\pi}{400} \left[9h + 2h^3 + \frac{h^5}{5} \right]_0^{10} \\ &= \frac{\pi}{400} \left(\left(90 + 2000 + \frac{100000}{5} \right) - 0 \right) = \frac{2209\pi}{40} \text{ in}^3 \end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(c) } \frac{dr}{dt} &= \frac{1}{20}(2h) \frac{dh}{dt} \\ -\frac{1}{5} &= \frac{3}{10} \frac{dh}{dt} \\ \frac{dh}{dt} &= -\frac{1}{5} \cdot \frac{10}{3} = -\frac{2}{3} \text{ in/sec} \end{aligned}$$

3 : $\begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

2016 SCORING GUIDELINES

Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x) dx$.

$$\begin{aligned} \text{(a)} \quad k(3) &= f(g(3)) = f(6) = 4 \\ k'(3) &= f'(g(3)) \cdot g'(3) = f'(6) \cdot 2 = 5 \cdot 2 = 10 \end{aligned}$$

An equation for the tangent line is $y = 10(x - 3) + 4$.

$$\begin{aligned} \text{(b)} \quad h'(1) &= \frac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{(f(1))^2} \\ &= \frac{(-6) \cdot 8 - 2 \cdot 3}{(-6)^2} = \frac{-54}{36} = -\frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \int_1^3 f''(2x) dx &= \frac{1}{2} [f'(2x)]_1^3 = \frac{1}{2} [f'(6) - f'(2)] \\ &= \frac{1}{2} [5 - (-2)] = \frac{7}{2} \end{aligned}$$

3 : $\begin{cases} 2 : \text{slope at } x = 3 \\ 1 : \text{equation for tangent line} \end{cases}$

3 : $\begin{cases} 2 : \text{expression for } h'(1) \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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2015 SCORING GUIDELINES

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
- (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
- (c) At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t) dt = 76.570$

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$
 Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

2 : $\begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t , $0 \leq t \leq 8$, is $30 + \int_0^t [R(x) - D(x)] dx$.

3 : $\begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

$R(t) - D(t) = 0 \Rightarrow t = 0, 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

(d) $30 + \int_0^w [R(t) - D(t)] dt = 50$

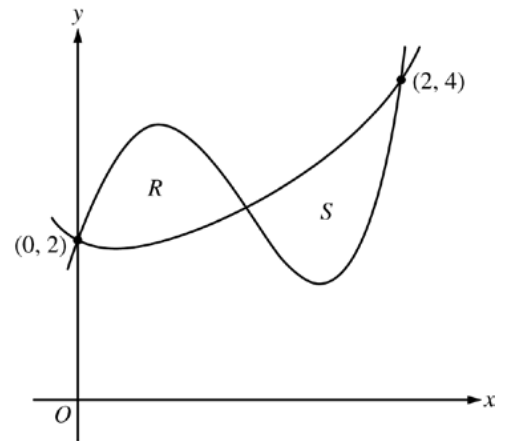
2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

2015 SCORING GUIDELINES

Question 2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

- (a) Find the sum of the areas of regions R and S .
- (b) Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.
- (c) Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.



- (a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$.

$$\begin{aligned}\text{Area} &= \int_0^A [g(x) - f(x)] dx + \int_A^2 [f(x) - g(x)] dx \\ &= 0.997427 + 1.006919 = 2.004\end{aligned}$$

- (b) $\text{Volume} = \int_A^2 [f(x) - g(x)]^2 dx = 1.283$

- (c) $h(x) = f(x) - g(x)$
 $h'(x) = f'(x) - g'(x)$
 $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

$$4 : \begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$$

2015 SCORING GUIDELINES

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

- (a) Use the data in the table to estimate the value of $v'(16)$.
- (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.
- Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.
- (c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.
- (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

(a) $v'(16) \approx \frac{240 - 200}{20 - 12} = 5 \text{ meters/min}^2$

1 : approximation

- (b) $\int_0^{40} |v(t)| dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \leq t \leq 40$ minutes.

3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned} \int_0^{40} |v(t)| dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters} \end{aligned}$$

- (c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.
 $B'(5) = 3(25) - 12(5) = 15 \text{ meters/min}^2$

2 : $\begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

(d) Avg vel $= \frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300) dt$

$$\begin{aligned} &= \frac{1}{10} \left[\frac{t^4}{4} - 2t^3 + 300t \right]_0^{10} \\ &= \frac{1}{10} \left[\frac{10000}{4} - 2000 + 3000 \right] = 350 \text{ meters/min} \end{aligned}$$

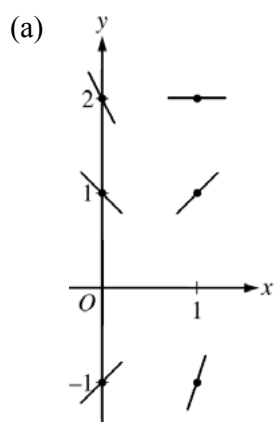
3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.
- (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.



$$2 : \begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$$

(b) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$

In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$.
Therefore, all solution curves are concave up in Quadrant II.

$$2 : \begin{cases} 1 : \frac{d^2y}{dx^2} \\ 1 : \text{concave up with reason} \end{cases}$$

(c) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = 2(2) - 3 = 1 \neq 0$

Therefore, f has neither a relative minimum nor a relative maximum at $x = 2$.

$$2 : \begin{cases} 1 : \text{considers } \left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} \\ 1 : \text{conclusion with justification} \end{cases}$$

(d) $y = mx + b \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(mx + b) = m$

$$2x - y = m$$

$$2x - (mx + b) = m$$

$$(2 - m)x - (m + b) = 0$$

$$2 - m = 0 \Rightarrow m = 2$$

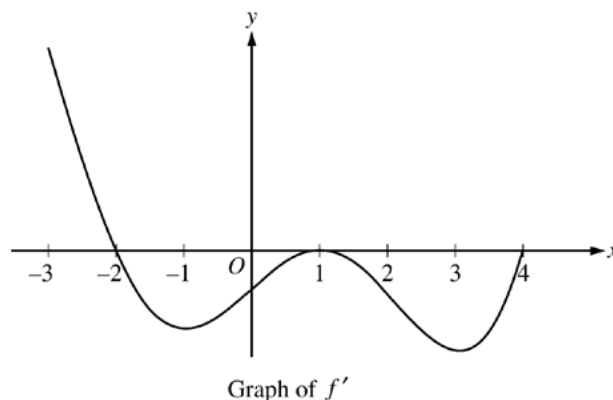
$$b = -m \Rightarrow b = -2$$

$$3 : \begin{cases} 1 : \frac{d}{dx}(mx + b) = m \\ 1 : 2x - y = m \\ 1 : \text{answer} \end{cases}$$

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Question 5

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



- (a) Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- (c) Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

- (a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$.
 $f'(x)$ changes from positive to negative at $x = -2$.
 Therefore, f has a relative maximum at $x = -2$.

2 : $\begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$

- (b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

- (c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points.

2 : $\begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$

The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.

- (d) $f(x) = 3 + \int_1^x f'(t) dt$

$$f(4) = 3 + \int_1^4 f'(t) dt = 3 + (-12) = -9$$

$$\begin{aligned} f(-2) &= 3 + \int_1^{-2} f'(t) dt = 3 - \int_{-2}^1 f'(t) dt \\ &= 3 - (-9) = 12 \end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$

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Question 6

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

- (a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.
- (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.
- (c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

$$(a) \left. \frac{dy}{dx} \right|_{(x,y)=(-1,1)} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$$

An equation for the tangent line is $y = \frac{1}{4}(x + 1) + 1$.

$$(b) 3y^2 - x = 0 \Rightarrow x = 3y^2$$

$$\text{So, } y^3 - xy = 2 \Rightarrow y^3 - (3y^2)(y) = 2 \Rightarrow y = -1$$

$$(-1)^3 - x(-1) = 2 \Rightarrow x = 3$$

The tangent line to the curve is vertical at the point $(3, -1)$.

$$(c) \frac{d^2y}{dx^2} = \frac{(3y^2 - x)\frac{dy}{dx} - y\left(6y\frac{dy}{dx} - 1\right)}{(3y^2 - x)^2}$$

$$\left. \frac{d^2y}{dx^2} \right|_{(x,y)=(-1,1)} = \frac{(3 \cdot 1^2 - (-1)) \cdot \frac{1}{4} - 1 \cdot \left(6 \cdot 1 \cdot \frac{1}{4} - 1\right)}{(3 \cdot 1^2 - (-1))^2}$$

$$= \frac{1 - \frac{1}{2}}{16} = \frac{1}{32}$$

2 : $\begin{cases} 1 : \text{slope} \\ 1 : \text{equation for tangent line} \end{cases}$

3 : $\begin{cases} 1 : \text{sets } 3y^2 - x = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{coordinates} \end{cases}$

4 : $\begin{cases} 2 : \text{implicit differentiation} \\ 1 : \text{substitution for } \frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$