

1. Male birds of a certain species arrive at a nesting area over a thirty-day period. The rate at which the male birds arrive at the nesting area at time t days is modeled by a differentiable function M , where $M(t)$ is measured in number of birds per day. Selected values of $M(t)$ are shown in the table.

t (days)	0	5	10	15	20	25	30
$M(t)$ (birds per day)	2	7	16	6	5	2	0

(Note: Your calculator should be in radian mode.)

Part A

Approximate $M'(7.5)$ using the average rate of change of M over the interval $5 \leq t \leq 10$. Show the work that leads to your answer, and indicate units of measure.

Part B

- i. Use a midpoint Riemann sum with the three subintervals $[0, 10]$, $[10, 20]$, and $[20, 30]$ to approximate $\int_0^{30} M(t) dt$. Show the work that leads to your answer.
- ii. Interpret the meaning of $\int_0^{30} M(t) dt$ in the context of the problem.

Part C

The rate at which female birds of the same species arrive at the same nesting area, in birds per day, is modeled by the function F defined as follows.

$$F(t) = \begin{cases} 0 & \text{for } 0 \leq t < 15 \\ 18 + 16 \sin\left(\frac{\pi}{20}(t + 15)\right) & \text{for } 15 \leq t \leq 45 \end{cases}$$

How many female birds of this species arrive at the nesting area from $t = 15$ to $t = 45$? Show the setup for your calculations, and round your answer to the nearest integer.

Part D

On the interval $15 < t < 30$, the difference in the rates at which male and female birds of this species arrive at the nesting area can be modeled by the differentiable function $D(t) = M(t) - F(t)$, where F is the function defined in part C. Is there a time t in the interval $15 < t < 20$ when $D(t) = 0$? Justify your answer.

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R , where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

- A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \leq t \leq 2$. Show the work that leads to your answer. Indicate units of measure.
- B. Must there be a value c , for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.
- C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t) dt$. Show the work that leads to your answer.
- D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

- 1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.
 - (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).
 - (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).
 - (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.
 - (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t , for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400) dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

1. The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f , where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

(a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression

$$2\pi \int_0^4 r f(r) \, dr.$$

Approximate the value of $2\pi \int_0^4 r f(r) \, dr$ using a right Riemann sum with the four

subintervals indicated by the data in the table.

(c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

(d) The density of bacteria in the petri dish, for $1 \leq r \leq 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k , $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \leq r \leq 4$?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \leq t \leq 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

- (a) How many people enter the line for the escalator during the time interval $0 \leq t \leq 300$?
- (b) During the time interval $0 \leq t \leq 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?
- (c) For $t > 300$, what is the first time t that there are no people in line for the escalator?
- (d) For $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

CALCULUS AB
SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A , where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

- (a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.
- (b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.
- (c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \frac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.
- (d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \leq 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln(t^2 + 2t) \text{ for } 3 < t \leq 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

- (a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?
- (b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.
- (c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.
- (d) How many pounds of bananas are on the display table at time $t = 8$?

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.
- (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.
- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
- (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
- (c) At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.