

Part B (AB): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Two particles, H and J , are moving along the x -axis. For $0 \leq t \leq 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2-4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2 - 1)^3$.

	Model Solution	Scoring
A	Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.	
	$x_H'(t) = v_H(t) = (2t - 4)e^{t^2-4t}$	Considers x_H' Point 1 (P1)
	$x_H'(1) = v_H(1) = -2e^{-3}$	Answer Point 2 (P2)
Scoring Notes for Part A		
P1 can be earned by presenting $x_H'(t)$, $x_H'(1)$, $x'(t)$, $x'(1)$, $(2t - 4)e^{t^2-4t}$, or $(2 \cdot 1 - 4)e^{1^2-4 \cdot 1}$. - An unsupported answer of $-2e^{-3}$ earns P2 but not P1.		

- B** During what open intervals of time t , for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

From part A, $x_H'(t) = v_H(t) = (2t - 4)e^{t^2-4t}$.

$$x_H'(t) = (2t - 4)e^{t^2-4t} = 0 \Rightarrow t = 2$$

$$x_H'(t) < 0 \text{ for } 0 < t < 2, \text{ and } x_H'(t) > 0 \text{ for } 2 < t < 5.$$

Thus, particle H is moving to the left for $0 < t < 2$ and moving to the right for $2 < t < 5$.

$$v_J(t) = 2t(t^2 - 1)^3 = 0 \text{ for } 0 < t < 5 \Rightarrow t = 1$$

$$v_J(t) < 0 \text{ for } 0 < t < 1, \text{ and } v_J(t) > 0 \text{ for } 1 < t < 5.$$

Thus, particle J is moving to the left for $0 < t < 1$ and moving to the right for $1 < t < 5$.

Therefore, particles H and J are moving in opposite directions for $1 < t < 2$.

Scoring Notes for Part B

- To earn **P3**, a response can do one of the following:
 - Set $x_H'(t) = 0$, $v_H(t) = 0$, or $(2t - 4)e^{t^2-4t} = 0$
 - Set $v_J(t) = 0$ or $2t(t^2 - 1)^3 = 0$
 - Identify $t = 2$ for particle H and no other values in the interval $0 < t < 5$
 - Identify $t = 1$ for particle J and no other values in the interval $0 < t < 5$
 - Identify the interval $1 < t < 2$
- To earn **P4**, a response can provide an analysis of signs of velocity or direction of motion on the interval $0 < t < 5$ for either particle H or particle J .
- To be eligible for **P5**, a response must provide correct analyses of signs of velocity or direction of motion on the interval $0 < t < 5$ for both particles.
- Only analysis within the interval $0 < t < 5$ will be considered in scoring.

C

It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

$$v_J(2) > 0 \text{ and } v_J'(2) > 0.$$

Answer with reason **Point 6 (P6)**

Because $v_J(2)$ and $v_J'(2)$ have the same sign, the speed of particle J is increasing at $t = 2$.

Scoring Notes for Part C

- An evaluation of $v_J(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J(2) = 108$.
- An evaluation of $v_J'(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J'(2) = 486$.
- A response can either import the analysis for the sign of $v_J(2)$ from part B or restart.
- A response that stated “ $v_J(t) > 0$ for $1 < t < 5$ ” in part B does not need to restate $v_J(2) > 0$ and earns **P6** for “ $v_J(2)$ and $v_J'(2)$ have the same sign, so the speed is increasing.”

D

Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

$$\begin{aligned} x_J(2) &= x_J(0) + \int_0^2 v_J(t) dt = 7 + \int_0^2 2t(t^2 - 1)^3 dt \\ &= 7 + \left[\frac{1}{4}(t^2 - 1)^4 \right]_0^2 \\ &= 7 + \frac{1}{4}((3)^4 - (-1)^4) = 7 + \frac{1}{4}(80) = 27 \end{aligned}$$

Integrand **Point 7 (P7)**

Antiderivative **Point 8 (P8)**

Answer **Point 9 (P9)**

Scoring Notes for Part D

- To earn **P7**, a response must present an indefinite or definite integral with an integrand of $v_J(t)$ or $2t(t^2 - 1)^3$. (See below for notes on how to handle a missing differential dt .)
- P8** is earned for an antiderivative of the form $k(t^2 - 1)^4$ or equivalent, for $k > 0$. If $k \neq \frac{1}{4}$, then the response is not eligible to earn **P9**.
- A response of $7 + \frac{1}{4}((3)^4 - (-1)^4)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

Note: An ambiguous response, such as $7 + \frac{1}{4}((3)^4 - (-1)^4)$, does not bank **P9** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., $7 + \frac{1}{4}(80)$ or 27) to earn **P9**.

- If the differential dt is missing:

- Writing $\int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
- Writing $7 + \int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
- Writing $\int_0^2 v_J(t) + 7$ introduces an ambiguity for the intended integrand.

$$\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}(t^2 - 1)^4 \right]_0^2 + 7 \text{ resolves the ambiguity.}$$

Therefore, this earns **P7** and **P8** and is eligible for **P9**.

$$\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}(t^2 - 1)^4 + 7t \right]_0^2 \text{ confirms that an incorrect integrand was used.}$$

Therefore, this does not earn **P7**, earns **P8**, and is not eligible for **P9**.

- If the ambiguity is not resolved, this does not earn **P7**, **P8**, or **P9**.

- Alternate solution using u -substitution:

$$\text{Let } u = t^2 - 1, \text{ then } du = 2t dt.$$

$$t = 0 \Rightarrow u = -1$$

$$t = 2 \Rightarrow u = 3$$

$$x_J(2) = x_J(0) + \int_0^2 v_J(t) dt = 7 + \int_0^2 2t(t^2 - 1)^3 dt$$

$$= 7 + \int_{-1}^3 u^3 du = 7 + \left[\frac{1}{4}u^4 \right]_{-1}^3$$

$$= 7 + \frac{1}{4}((3)^4 - (-1)^4) = 7 + \frac{1}{4}(80) = 27$$

- Alternate solution using indefinite integral:

$$\int 2t(t^2 - 1)^3 dt = \frac{1}{4}(t^2 - 1)^4 + C$$

$$x_J(0) = 7 = \frac{1}{4}(0^2 - 1)^4 + C \Rightarrow C = \frac{27}{4}$$

$$x_J(t) = \frac{1}{4}(t^2 - 1)^4 + \frac{27}{4}$$

$$x_J(2) = \frac{1}{4}(2^2 - 1)^4 + \frac{27}{4} = \frac{108}{4} = 27$$

Part A (AB): Graphing calculator required**Question 2****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A particle moves along the x -axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln(t^2 - 4t + 5) - 0.2t$.

Model Solution	Scoring
(a) There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R. For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.	
$v(t) = 0 \Rightarrow t = 1.425610$	t_R 1 point
Therefore, the particle is at rest (not moving) at $t_R = 1.426$ (or 1.425).	
For $0 < t < t_R$, $v(t) > 0$. Therefore, the particle is moving to the right on that interval.	Direction with explanation 1 point

Scoring notes:

- The first point is earned for considering $v(t) = 0$ and reporting the value $t = 1.426$ (or $t = 1.425$).
- A response that finds no value or an incorrect value of t_R is not eligible to earn the second point.
- A response need not demonstrate an evaluation of $v(t)$ at any value of t , $0 < t < 1.426$. The evaluations do not need to be presented with three correct decimal places, but they must be correct for the number of digits shown for one up to three digits after the decimal.

Total for part (a) 2 points

- (b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.

$a(1.5) = v'(1.5) = -1$	Acceleration with setup 1 point
The acceleration of the particle at time $t = 1.5$ is -1 (or -0.999).	
$v(1.5) = -0.076856 < 0$	Answer with explanation 1 point
Because $a(1.5)$ and $v(1.5)$ have the same sign, the speed is increasing at time $t = 1.5$.	

Scoring notes:

- A response must demonstrate the relationship $v' = a$ in order to earn the first point. This relationship could be shown by " $v'(1.5) = -1$ (or -0.999)" or " $v' = a$ and $a(1.5) = -1$ (or -0.999).". A response of just " $a(1.5) = -1$ (or -0.999)" is not sufficient to earn the first point.
- A response must declare a value for $a(1.5)$ to be eligible for the second point.
- The second point can only be earned for a response that is consistent with a negative velocity at time $t = 1.5$ and the presented value of $a(1.5)$.
- Any presented value of $v(1.5)$ must be correct for the number of digits presented, from one up to three decimal places in order to earn the second point.
- A response does not need to report a value for $v(1.5)$; an implied sign is sufficient:
 - Any statement equivalent to "The speed of the particle is increasing because $a(1.5)$ and $v(1.5)$ have the same sign" will earn the second point, provided the presented value of $a(1.5)$ is negative.
- A response that presents or references an incorrect value or sign of $v(1.5)$ does not earn the second point.
- Alternate solution for the second point:

Speed = $|v(t)|$ and its derivative is positive when $t = 1.5$, therefore the speed of the particle is increasing.

Total for part (b) 2 points

- (c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.

$x(4) = x(1) + \int_1^4 v(t) dt$	Integral	1 point
$= -3 + 0.197117 = -2.802883$	Uses initial condition	1 point
The position of the particle at time $t = 4$ is -2.803 (or -2.802).	Answer	1 point

Scoring notes:

- The first point is earned for a definite integral with integrand $v(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A response with a linkage error such as $x(4) = \int_1^4 v(t) dt = -3 + 0.197$ or $\int_1^4 v(t) dt = 0.197 = -2.803$ earns the first 2 points but does not earn the third point.
- Missing differential (dt):
 - Unambiguous responses of $x(1) + \int_1^4 v(t)$ and $-3 + \int_1^4 v(t)$ both earn the first 2 points and are eligible for the third point.
 - Ambiguous responses of $\int_1^4 v(t) + x(1)$ and $\int_1^4 v(t) - 3$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, neither of these responses earn the third point.
- The second point is earned for either symbolically or numerically adding $x(1)$ or -3 to a definite integral with a lower limit of 1.
- The third point is earned for an answer of $-3 + 0.197$ or $0.197 - 3$ with no additional simplification, provided there is some supporting work for these values.
- An answer of just -2.803 (or -2.802) with no supporting work does not earn any points.

Total for part (c) 3 points

- (d) Find the total distance traveled by the particle over the interval $1 \leq t \leq 4$. Show the setup for your calculations.

$\int_1^4 v(t) dt$	Integral	1 point
$= 0.9581$	Answer	1 point
The total distance traveled by the particle over the interval $1 \leq t \leq 4$ is 0.958.		

Scoring notes:

- The first point is earned for any one of the following:
 - $\int_1^4 |v(t)| dt$
 - $\int_1^{1.425} v(t) dt - \int_{1.425}^{2.883} v(t) dt + \int_{2.883}^4 v(t) dt$
 - $\int_1^{1.426} v(t) dt - \int_{1.426}^{2.883} v(t) dt + \int_{2.883}^4 v(t) dt$
- Due to variations in numerical integration techniques on some calculators, responses of 0.958, 0.959, or 0.96 earn the second point.

Total for part (d) 2 points**Total for question 2 9 points**

Part A (AB): Graphing calculator required**Question 2****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t} \sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.

Model Solution		Scoring
(a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.		
For $0 < t < 90$, $v(t) = 0 \Rightarrow t = 56$.	Considers sign of $v(t)$	1 point
Stephen changes direction when his velocity changes sign. This occurs at $t = 56$ seconds.	Answer with reason	1 point
Scoring notes:		
- A response that considers $v(t) = 0$ earns the first point. - A response of “Stephen changes direction when his velocity changes sign” earns the first point for considering the sign of $v(t)$ but must include the answer of $t = 56$ in order to earn the second point. - A response of $t = 56$ with no supporting work does not earn either point. - Any presented values of t outside the interval $0 < t < 90$ will not affect scoring.		
Total for part (a)		2 points
(b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.		
$v'(60) = a(60) = -0.0360162$	$a(60)$ with setup	1 point
Stephen's acceleration at time $t = 60$ seconds is -0.036 meter per second per second.	Acceleration units	1 point

$v(60) = -0.1595124 < 0$	Speeding up with reason	1 point
Stephen is speeding up at time $t = 60$ seconds because Stephen's velocity and acceleration are both negative at that time.		
Scoring notes:		
- The minimum work needed to earn the first point is $v'(60) = -0.036$. $a(60) = -0.0360162$ is not sufficient to earn the first point. The connection $v'(t) = a(t)$ or $v'(60) = a(60)$ must be explicitly shown. - A response must declare a value for $a(60)$ to be eligible for the second point. - In order to earn the third point the presented conclusion must be consistent with a negative velocity at time $t = 60$ and the presented value of $a(60)$. - A response does not need to find the value of $v(60)$; an implied sign is sufficient. - The statement “Stephen is speeding up because $a(60)$ and $v(60)$ have the same sign” (or equivalent) earns the third point, provided a negative value is presented for $a(60)$. - A response that reports an incorrect sign or value of $v(60)$ does not earn the third point. Any presented value of $v(60)$ must be correct for the number of digits presented, from one up to three decimal places in order to earn the third point. - Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). - In degree mode, there are two possible values for $v'(60)$. A response that declares $v'(60) = -0.000141$ does not earn the first point but would earn the third point in the presence of $v(60) = 0.042089$ or $v(60) > 0$ and the conclusion that Stephen is slowing down. - Similarly, a response that declares $v'(60) = 0.039304$ does not earn the first point but would earn the third point in the presence of $v(60) = 0.042089$ or $v(60) > 0$ and the conclusion that Stephen is speeding up.		
Total for part (b)		3 points
(c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.		
$\int_{20}^{80} v(t) dt$	Integral	1 point
$= 23.383997$	Answer	1 point
The distance between Stephen's positions at $t = 20$ seconds and $t = 80$ seconds is 23.384 (or 23.383) meters.		

Scoring notes:

- The first point is earned only for $\int_{20}^{80} v(t) \, dt$ (or the equivalent) with or without the differential.
- The second point is earned only for an answer of 23.384 (or 23.383) regardless of whether the first point was earned.
- Degree mode: In degree mode, $\int_{20}^{80} v(t) \, dt = 2.407982$.

Total for part (c) 2 points

- (d) Find the total distance Stephen swims over the time interval $0 \leq t \leq 90$ seconds. Show the setup for your calculations.

$\int_0^{90} v(t) \, dt$	Integral	1 point
$= 62.164216$	Answer	1 point
The total distance Stephen swims over the time interval $0 \leq t \leq 90$ seconds is 62.164 meters.		

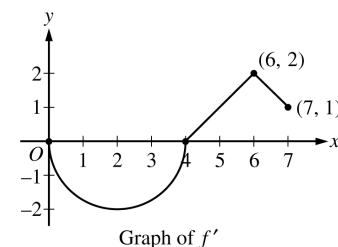
Scoring notes:

- The first point is earned only for $\int_0^{90} |v(t)| \, dt$ or $\int_0^{56} v(t) \, dt - \int_{56}^{90} v(t) \, dt$ (or the equivalent), with or without the differential(s).
- The second point is earned only for an answer of 62.164 regardless of whether the first point was earned.
- Degree mode: In degree mode, $\int_0^{90} |v(t)| \, dt = 3.127892$.

Total for part (d) 2 points**Total for question 2 9 points****Part B (AB or BC): Graphing calculator not allowed****Question 3****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f' , the derivative of f , consists of a semicircle and two line segments, as shown in the figure above.

Model Solution	Scoring
(a) Find $f(0)$ and $f(5)$.	
$f(0) = f(4) + \int_4^0 f'(x) \, dx = 3 - \int_0^4 f'(x) \, dx = 3 + 2\pi$	Area of either region 1 point – OR – $\int_0^4 f'(x) \, dx$
$f(5) = f(4) + \int_4^5 f'(x) \, dx = 3 + \frac{1}{2} = \frac{7}{2}$	– OR – $\int_4^5 f'(x) \, dx$
	$f(0)$ 1 point
	$f(5)$ 1 point

Scoring notes:

- A response with answers of only $f(0) = \pm 2\pi$, or only $f(5) = \frac{1}{2}$, or both earns 1 of the 3 points.
- A response displaying $f(5) = \frac{7}{2}$ and a missing or incorrect value for $f(0)$ earns 2 of the 3 points.
- The second and third points can be earned in either order.
- Read unlabeled values from left to right and from top to bottom as $f(0)$ and $f(5)$. A single value must be labeled as either $f(0)$ or $f(5)$ in order to earn any points.

Total for part (a) 3 points

- (b) Find the x -coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

The graph of f has a point of inflection at each of $x = 2$ and $x = 6$, because $f'(x)$ changes from decreasing to increasing at $x = 2$ and from increasing to decreasing at $x = 6$.	Answer	1 point
	Justification	1 point

Scoring notes:

- A response that gives only one of $x = 2$ or $x = 6$, along with a correct justification, earns 1 of the 2 points.
- A response that claims that there is a point of inflection at any value other than $x = 2$ or $x = 6$ earns neither point.
- To earn the second point a response must use correct reasoning based on the graph of f' . Examples of correct reasoning include:
 - Correctly discussing the signs of the slopes of the graph of f'
 - Citing $x = 2$ and $x = 6$ as the locations of local extrema on the graph of f'
- Examples of reasoning not (sufficiently) connected to the graph of f' include:
 - Reasoning based on sign changes in f'' unless the connection is made between the sign of f'' and the slopes of the graph of f'
 - Reasoning based only on the concavity of the graph of f
- The second point cannot be earned by use of vague or undefined terms such as “it” or “the function” or “the derivative.”
- Responses that report inflection points as ordered pairs must report the points $(2, 3 + \pi)$ and $(6, 5)$ in order to earn the first point. If the y -coordinates are reported incorrectly, the response remains eligible for the second point.

Total for part (b) 2 points

- (c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.

$g'(x) = f'(x) - 1$	$g'(x) = f'(x) - 1$	1 point
$f'(x) - 1 \leq 0 \Rightarrow f'(x) \leq 1$	Interval with reason	1 point
The graph of g is decreasing on the interval $0 \leq x \leq 5$ because $g'(x) \leq 0$ on this interval.		

Scoring notes:

- The first point can be earned for $f'(x) \leq 1$ or the equivalent, in words or symbols.
- Endpoints do not need to be included in the interval to be eligible for the second point.

Total for part (c) 2 points

- (d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

g is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$.	Considers $g'(x) = 0$	1 point
	Answer with justification	1 point

Therefore, the absolute minimum occurs at $x = 5$, and

$g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$ is the absolute minimum value of g .

Scoring notes:

- A justification that uses a local argument, such as “ g' changes from negative to positive (or g changes from decreasing to increasing) at $x = 5$ ” must also state that $x = 5$ is the only critical point.
- If $g'(x) = 0$ (or equivalent) is not declared explicitly, a response that isolates $x = 5$ as the only critical number belonging to $(0, 7)$ earns the first point.
- A response that imports $g'(x) = f'(x)$ from part (c) is eligible for the first point but not the second.
 - In this case, consideration of $x = 4$ as the only critical number on $(0, 7)$ earns the first point.
- Solution using Candidates Test:

$$g'(x) = f'(x) - 1 = 0 \Rightarrow x = 5, x = 7$$

x	$g(x)$
0	$3 + 2\pi$
5	$-\frac{3}{2}$
7	$-\frac{1}{2}$

The absolute minimum value of g on the interval $0 \leq x \leq 7$ is $-\frac{3}{2}$.

- When using a Candidates Test, a response may import an incorrect value of $f(0) = g(0) > -\frac{3}{2}$ from part (a). The second point can only be earned for an answer of $-\frac{3}{2}$.

Total for part (d) 2 points

Total for question 3 9 points

Part B (AB): Graphing calculator not allowed**Question 6****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Particle P moves along the x -axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y -axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At

time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

Model Solution**Scoring**

- (a) Find $v_P(t)$, the velocity of particle P at time t .

$$v_P(t) = x_P'(t) = 4e^{-t}$$

Answer

1 point**Scoring notes:**

- A response that equates $x_P(t)$ with $v_P(t)$ does not earn the point.
- An unlabeled response earns the point.

Total for part (a)**1 point**

- (b) Find $a_Q(t)$, the acceleration of particle Q at time t . Find all times t , for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

$$a_Q(t) = v_Q'(t) = \frac{-2}{t^3}$$

 $a_Q(t)$ **1 point**

For $t > 0$, $a_Q(t) < 0$ and $v_Q(t) > 0$.

Considers signs of
 $a_Q(t)$ and $v_Q(t)$

1 point

Because the velocity and acceleration have opposite signs, the speed of particle Q is decreasing for all $t > 0$.

Answer with
justification

1 point**Scoring notes:**

- Earning the first point is not necessary for a response to be eligible to earn the second or third points; however, the response must present an expression for $a_Q(t)$ to be eligible for third point.
- A response earns the second point with either of the following statements: “ $v_Q(t)$ and $a_Q(t)$ have opposite signs” or “ $v_Q(t)$ and $a_Q(t)$ have the same sign.” This statement, however, must be consistent with $v_Q(t)$ and the presented expression for $a_Q(t)$.
- A response must earn the second point to be eligible for the third point. The answer must be consistent with the presented justification. Furthermore, responses for which $a_Q(t) > 0$ for $t > 0$ must conclude that there is no time at which the speed of the particle is decreasing.
- A response that indicates $v_Q(t) < 0$ does not earn the third point, even if the answer and justification are consistent with a reported sign of $a_Q(t)$.

Total for part (b)**3 points**

- (c) Find
- $y_Q(t)$
- , the position of particle
- Q
- at time
- t
- .

$y_Q(t) = y_Q(1) + \int_1^t \frac{1}{s^2} ds$	Integral	1 point
	Uses initial condition	1 point
$= 2 - \left(\frac{1}{s} \right) \Big _1^t = 2 - \frac{1}{t} + 1 = 3 - \frac{1}{t}$	Answer	1 point

Scoring notes:

- A response that presents $\int_1^t \frac{1}{s^2} dt$ (using the same variable as a limit and integrand function) does not earn the first point unless it is followed by an attempt at integration.
- A response that presents either $\int \frac{1}{t^2} dt$ or $-\frac{1}{t}$ (with no integral) earns the first point. If the response continues and presents $2 = -1 + C$, then the response earns the second point.
- A response that presents only $y_Q(t) = -\frac{1}{t} + 3$ will earn all 3 points. Note that the right side of this equation suffices to earn all points. A response of $y_Q(t) = -\frac{1}{t} + C$, where $C \neq 3$, (with no additional supporting work) earns only the first point.

Total for part (c) 3 points

- (d) As
- $t \rightarrow \infty$
- , which particle will eventually be farther from the origin? Give a reason for your answer.

For particle P , $\lim_{t \rightarrow \infty} (6 - 4e^{-t}) = 6$.	One correct limit	1 point
For particle Q , $\lim_{t \rightarrow \infty} \left(3 - \frac{1}{t}\right) = 3$.		
Because $6 > 3$, particle P will eventually be farther from the origin.	Answer with reason	1 point

Scoring notes:

- A response with an incorrect $y_Q(t)$ from part (c) is eligible for both points in part (d) provided $y_Q(t)$ is a non-constant function. The second point is earned for a consistent answer with reason, and limits correct for particle P and the presented $y_Q(t)$.
- Responses that present statements such as “ $6 - 4e^{-t}$ approaches 6” or “ Q goes to 3” earn the first point and are eligible for the second point.
- A response that treats ∞ as an input for $x_P(t)$ or $y_Q(t)$, such as “ $6 - 4e^{-\infty}$ ” or “ $3 - \frac{1}{\infty}$ ” is not eligible for the second point.

Total for part (d) 2 points**Total for question 6 9 points****Part A (AB): Graphing calculator required****Question 2****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

A particle, P , is moving along the x -axis. The velocity of particle P at time t is given by $v_P(t) = \sin(t^{1.5})$ for $0 \leq t \leq \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q , also moves along the x -axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \leq t \leq \pi$. At time $t = 0$, particle Q is at position $x = 10$.

Model Solution**Scoring**

- (a) Find the positions of particles
- P
- and
- Q
- at time
- $t = 1$
- .

$x_P(1) = 5 + \int_0^1 v_P(t) dt = 5.370660$	One definite integral	1 point
At time $t = 1$, the position of particle P is $x = 5.371$ (or 5.370).	One position	1 point
$x_Q(1) = 10 + \int_0^1 v_Q(t) dt = 8.564355$	The other position	1 point
At time $t = 1$, the position of particle Q is $x = 8.564$.		

Scoring notes:

- The first point is earned for the explicit presentation of at least one definite integral, either $\int_0^1 v_P(t) dt$ or $\int_0^1 v_Q(t) dt$.
- The first point must be earned to be eligible for the second and third points.
- The second point is earned for adding the initial condition to at least one of the definite integrals and finding the correct position.
- Writing $\int_0^1 v_P(t) + 5 = 5.370660$ does not earn a position point, because the missing dt makes this statement unclear or false. However, $5 + \int_0^1 v_P(t) = 5.370660$ does earn the position point because it is not ambiguous. Similarly, for the position of Q .
- Read unlabeled answers presented left to right, or top to bottom, as $x_P(1)$ and $x_Q(1)$, respectively.
- Special case 1: A response of $x_P(1) = 5 + \int_0^a v_P(t) dt = 5.370660$ AND $x_Q(1) = 10 + \int_0^a v_Q(t) dt = 8.564355$ for $a \neq 1$ earns one point.
- Special case 2: A response of $x_P(1) = 5 + \int v_P(t) dt = 5.370660$ AND $x_Q(1) = 10 + \int v_Q(t) dt = 8.564355$ or the equivalent, never providing the definite integrals, earns one point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $x_P(1)$ is 5.007 (or 5.006).

Total for part (a) 3 points

- (b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

$v_P(1) = \sin(1^{1.5}) = 0.841471 > 0$ At time $t = 1$, particle P is moving to the right.	Direction of motion for one particle	1 point
$v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$ At time $t = 1$, particle Q is moving to the left. At time $t = 1$, $x_P(1) < x_Q(1)$, so particle P is to the left of particle Q . Thus, at time $t = 1$, particles P and Q are moving toward each other.	Answer with explanation	1 point

Scoring notes:

- The first point is earned for using the sign of $v_P(1)$ or $v_Q(1)$ to determine the direction of motion for one of the particles. This point cannot be earned without reference to the sign of $v_P(1)$ or $v_Q(1)$.
- It is not necessary to present an explicit value for $v_P(1)$, or $v_Q(1)$, but if a value is presented, it must be correct as far as reported, up to three places after the decimal.
- Read with imported incorrect position values from part (a).
- If one or both position values were not found in part (a), but are found in part (b), the points for part (a) are not earned retroactively.
- To earn the second point the explanation must be based on the signs of $v_P(1)$ and $v_Q(1)$ and the relative positions of particle P and particle Q at $t = 1$. References to other values of time, such as $t = 0$, are not sufficient.
- Degree mode: $v_P(1) = 0.017$. (See degree mode statement in part (a).)

Total for part (b) 2 points

- (c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

$a_Q(1) = v'_Q(1) = 1.026856$ The acceleration of particle Q is 1.027 (or 1.026) at time $t = 1$.	Setup and acceleration	1 point
$v_Q(1) = -1 < 0$ and $a_Q(1) > 0$ The speed of particle Q is decreasing at time $t = 1$ because the velocity and acceleration have opposite signs.	Speed decreasing with reason	1 point

Scoring notes:

- To earn the first point the acceleration must be explicitly connected to v'_Q (e.g., $v'_Q(1) = 1.026856$).
- The first point is not earned for an unsupported value of 1.027 (or 1.026). The setup, $v'_Q(1)$, must be shown. Presenting only $a_Q(1) = 1.027$ (or 1.026) without indication that $v'_Q = a_Q$ is not enough to earn the first point.
- A response does not need to present a value for $v_Q(1)$; the sign is sufficient.
- To earn the second point a response must compare the signs of a_Q and v_Q at $t = 1$. Considering only one sign is not sufficient.
- After the first point has been earned, a response declaring only “velocity and acceleration are of opposite signs at $t = 1$ so the particle is slowing down” (or equivalent) earns the second point.
- The second point may be earned without the first, as long as the response does not present an incorrect value or sign for $v_Q(1)$ and concludes the particle is slowing down because velocity and acceleration have opposite signs at $t = 1$.

Total for part (c) 2 points

- (d) Find the total distance traveled by particle P over the time interval $0 \leq t \leq \pi$.

$\int_0^\pi v_P(t) dt = 1.93148$	Definite integral	1 point
Over the time interval $0 \leq t \leq \pi$, the total distance traveled by particle P is 1.931.	Answer	1 point

Scoring notes:

- The first point is earned for $\int_0^\pi |v_P(t)| dt$.
- The first point can also be earned for a sum (or difference) of definite integrals, such as $\int_0^{2.145029} v_P(t) dt - \int_{2.145029}^\pi v_P(t) dt$, provided the response has indicated $v_P(2.145029) = 0$.
- The second point can only be earned for the correct answer.
- The unsupported value 1.931 earns no points.
- A response reporting the distance traveled by particle Q as $\int_0^\pi |v_Q(t)| dt = 3.506$ earns the first point and is not eligible for the second point.
- In degree mode, the total distance traveled is 0.122. (See degree mode statement in part (a).) In the degree mode case, the response must present $\int_0^\pi |v_P(t)| dt$ in order to earn the first point because $\int_0^\pi |v_P(t)| dt = \int_0^\pi v_P(t) dt$.

Total for part (d) 2 points**Total for question 2 9 points**

2019

AP[®] CollegeBoard

AP[®] Calculus AB

Scoring Guidelines

2019 SCORING GUIDELINES

Question 1

(a) $\int_0^5 E(t) dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b) $\frac{1}{5-0} \int_0^5 L(t) dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

- (c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$$E(t) - L(t) = 0 \Rightarrow t = 6.20356$$

$E(t) - L(t) > 0$ for $0 \leq t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$. Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$$A(t) = \int_0^t (E(s) - L(s)) ds$$

$$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

$$2 : \begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$$

2019 SCORING GUIDELINES

Question 2

- (a) v_P is differentiable $\Rightarrow v_P$ is continuous on $0.3 \leq t \leq 2.8$.

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value c , $0.3 < c < 2.8$, such that $v_P'(c) = 0$.

— OR —

v_P is differentiable $\Rightarrow v_P$ is continuous on $0.3 \leq t \leq 2.8$.

By the Extreme Value Theorem, v_P has a minimum on $[0.3, 2.8]$.
 $v_P(0.3) = 55 > -29 = v_P(1.7)$ and $v_P(1.7) = -29 < 55 = v_P(2.8)$.
 Thus v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P'(t)$ must equal 0 at this minimum.

$$2 : \begin{cases} 1 : v_P(2.8) - v_P(0.3) = 0 \\ 1 : \text{justification, using Mean Value Theorem} \end{cases}$$

— OR —

$$2 : \begin{cases} 1 : v_P(0.3) > v_P(1.7) \\ \text{and } v_P(1.7) < v_P(2.8) \\ 1 : \text{justification, using Extreme Value Theorem} \end{cases}$$

$$\begin{aligned} \text{(b)} \quad \int_0^{2.8} v_P(t) dt &\approx 0.3 \left(\frac{v_P(0) + v_P(0.3)}{2} \right) + 1.4 \left(\frac{v_P(0.3) + v_P(1.7)}{2} \right) \\ &\quad + 1.1 \left(\frac{v_P(1.7) + v_P(2.8)}{2} \right) \\ &= 0.3 \left(\frac{0 + 55}{2} \right) + 1.4 \left(\frac{55 + (-29)}{2} \right) + 1.1 \left(\frac{-29 + 55}{2} \right) \\ &= 40.75 \end{aligned}$$

1 : answer, using trapezoidal sum

- (c) $v_Q(t) = 60 \Rightarrow t = A = 1.866181$ or $t = B = 3.519174$

$$v_Q(t) \geq 60 \text{ for } A \leq t \leq B$$

$$\int_A^B |v_Q(t)| dt = 106.108754$$

The distance traveled by particle Q during the interval $A \leq t \leq B$ is 106.109 (or 106.108) meters.

$$3 : \begin{cases} 1 : \text{interval} \\ 1 : \text{definite integral} \\ 1 : \text{distance} \end{cases}$$

- (d) From part (b), the position of particle P at time $t = 2.8$ is

$$x_P(2.8) = \int_0^{2.8} v_P(t) dt \approx 40.75.$$

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t) dt = -90 + 135.937653 = 45.937653$$

Therefore at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

$$3 : \begin{cases} 1 : \int_0^{2.8} v_Q(t) dt \\ 1 : \text{position of particle } Q \\ 1 : \text{answer} \end{cases}$$

2019 SCORING GUIDELINES

Question 3

$$\begin{aligned} \text{(a)} \quad \int_{-6}^5 f(x) \, dx &= \int_{-6}^{-2} f(x) \, dx + \int_{-2}^5 f(x) \, dx \\ &\Rightarrow 7 = \int_{-6}^{-2} f(x) \, dx + 2 + \left(9 - \frac{9\pi}{4}\right) \\ &\Rightarrow \int_{-6}^{-2} f(x) \, dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \int_3^5 (2f'(x) + 4) \, dx &= 2 \int_3^5 f'(x) \, dx + \int_3^5 4 \, dx \\ &= 2(f(5) - f(3)) + 4(5 - 3) \\ &= 2(0 - (3 - \sqrt{5})) + 8 \\ &= 2(-3 + \sqrt{5}) + 8 = 2 + 2\sqrt{5} \end{aligned}$$

— OR —

$$\begin{aligned} \int_3^5 (2f'(x) + 4) \, dx &= [2f(x) + 4x]_{x=3}^{x=5} \\ &= (2f(5) + 20) - (2f(3) + 12) \\ &= (2 \cdot 0 + 20) - (2(3 - \sqrt{5}) + 12) \\ &= 2 + 2\sqrt{5} \end{aligned}$$

$$\text{(c)} \quad g'(x) = f(x) = 0 \Rightarrow x = -1, x = \frac{1}{2}, x = 5$$

x	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval $-2 \leq x \leq 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$.

$$\begin{aligned} \text{(d)} \quad \lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} &= \frac{10^1 - 3f'(1)}{f(1) - \arctan 1} \\ &= \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}} \end{aligned}$$

$$3 : \begin{cases} 1 : \int_{-6}^5 f(x) \, dx = \int_{-6}^{-2} f(x) \, dx + \int_{-2}^5 f(x) \, dx \\ 1 : \int_{-2}^5 f(x) \, dx \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -1 \text{ as a candidate} \\ 1 : \text{answer with justification} \end{cases}$$

1 : answer

2019 SCORING GUIDELINES

Question 4

$$\begin{aligned} \text{(a)} \quad V &= \pi r^2 h = \pi(1)^2 h = \pi h \\ \frac{dV}{dt} \Big|_{h=4} &= \pi \frac{dh}{dt} \Big|_{h=4} = \pi \left(-\frac{1}{10} \sqrt{4} \right) = -\frac{\pi}{5} \text{ cubic feet per second} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{d^2 h}{dt^2} &= -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10} \sqrt{h} \right) = \frac{1}{200} \\ \text{Because } \frac{d^2 h}{dt^2} &= \frac{1}{200} > 0 \text{ for } h > 0, \text{ the rate of change of the} \\ &\text{height is increasing when the height of the water is 3 feet.} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \frac{dh}{\sqrt{h}} &= -\frac{1}{10} dt \\ \int \frac{dh}{\sqrt{h}} &= \int -\frac{1}{10} dt \\ 2\sqrt{h} &= -\frac{1}{10} t + C \\ 2\sqrt{5} &= -\frac{1}{10} \cdot 0 + C \Rightarrow C = 2\sqrt{5} \\ 2\sqrt{h} &= -\frac{1}{10} t + 2\sqrt{5} \\ h(t) &= \left(-\frac{1}{20} t + \sqrt{5} \right)^2 \end{aligned}$$

$$2 : \begin{cases} 1 : \frac{dV}{dt} = \pi \frac{dh}{dt} \\ 1 : \text{answer with units} \end{cases}$$

$$3 : \begin{cases} 1 : \frac{d}{dh} \left(-\frac{1}{10} \sqrt{h} \right) = -\frac{1}{20\sqrt{h}} \\ 1 : \frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} \\ 1 : \text{answer with explanation} \end{cases}$$

$$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : h(t) \end{cases}$$

Note: 0/4 if no separation of variables

Note: max 2/4 [1-1-0-0] if no constant of integration

2019 SCORING GUIDELINES

Question 5

$$\begin{aligned}
 \text{(a)} \quad \int_0^2 (h(x) - g(x)) \, dx &= \int_0^2 \left((6 - 2(x-1)^2) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right) \right) \right) dx \\
 &= \left[6x - \frac{2}{3}(x-1)^3 - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right) \right) \right]_{x=0}^{x=2} \\
 &= \left(\left(12 - \frac{2}{3} \right) - (-4 + 0) \right) - \left(\left(0 + \frac{2}{3} \right) - (0 + 0) \right) \\
 &= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}
 \end{aligned}$$

The area of R is $\frac{44}{3}$.

$$\begin{aligned}
 \text{(b)} \quad \int_0^2 A(x) \, dx &= \int_0^2 \frac{1}{x+3} \, dx \\
 &= \left[\ln(x+3) \right]_{x=0}^{x=2} = \ln 5 - \ln 3
 \end{aligned}$$

The volume of the solid is $\ln 5 - \ln 3$.

$$\text{(c)} \quad \pi \int_0^2 \left((6 - g(x))^2 - (6 - h(x))^2 \right) dx$$

4 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative of } 3\cos\left(\frac{\pi}{2}x\right) \\ 1 : \text{antiderivative of remaining terms} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : \text{limits and constant} \\ 1 : \text{form of integrand} \\ 1 : \text{integrand} \end{cases}$

2019 SCORING GUIDELINES

Question 6

$$\text{(a)} \quad h'(2) = \frac{2}{3}$$

$$\text{(b)} \quad a'(x) = 9x^2 h(x) + 3x^3 h'(x)$$

$$a'(2) = 9 \cdot 2^2 h(2) + 3 \cdot 2^3 h'(2) = 36 \cdot 4 + 24 \cdot \frac{2}{3} = 160$$

$$\text{(c)} \quad \text{Because } h \text{ is differentiable, } h \text{ is continuous, so } \lim_{x \rightarrow 2} h(x) = h(2) = 4.$$

$$\text{Also, } \lim_{x \rightarrow 2} h(x) = \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3}, \text{ so } \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4.$$

$$\text{Because } \lim_{x \rightarrow 2} (x^2 - 4) = 0, \text{ we must also have } \lim_{x \rightarrow 2} (1 - (f(x))^3) = 0.$$

$$\text{Thus } \lim_{x \rightarrow 2} f(x) = 1.$$

$$\text{Because } f \text{ is differentiable, } f \text{ is continuous, so } f(2) = \lim_{x \rightarrow 2} f(x) = 1.$$

Also, because f is twice differentiable, f' is continuous, so

$$\lim_{x \rightarrow 2} f'(x) = f'(2) \text{ exists.}$$

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = \lim_{x \rightarrow 2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{4}{-3(1)^2 \cdot f'(2)} = 4.$$

$$\text{Thus } f'(2) = -\frac{1}{3}.$$

$$\text{(d)} \quad \text{Because } g \text{ and } h \text{ are differentiable, } g \text{ and } h \text{ are continuous, so}$$

$$\lim_{x \rightarrow 2} g(x) = g(2) = 4 \text{ and } \lim_{x \rightarrow 2} h(x) = h(2) = 4.$$

Because $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$, it follows from the squeeze theorem that $\lim_{x \rightarrow 2} k(x) = 4$.

$$\text{Also, } 4 = g(2) \leq k(2) \leq h(2) = 4, \text{ so } k(2) = 4.$$

Thus k is continuous at $x = 2$.

1 : answer

3 : $\begin{cases} 1 : \text{form of product rule} \\ 1 : a'(x) \\ 1 : a'(2) \end{cases}$

4 : $\begin{cases} 1 : \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4 \\ 1 : f(2) \\ 1 : \text{L'Hospital's Rule} \\ 1 : f'(2) \end{cases}$

1 : continuous with justification

AP Calculus AB

Scoring Guidelines

2018 SCORING GUIDELINES

Question 1

(a) $\int_0^{300} r(t) \, dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$.

(b) $20 + \int_0^{300} (r(t) - 0.7) \, dt = 20 + \int_0^{300} r(t) \, dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or 414.285) seconds.}$$

(d) The total number of people in line at time t , $0 \leq t \leq 300$, is modeled by $20 + \int_0^t r(x) \, dx - 0.7t$.

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

1 : answer

4 : $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

2018 SCORING GUIDELINES

Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118 .

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760 .

3 : $\begin{cases} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{3.5} v(t) dt = 2.844$ (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral $\int_0^{3.5} v(t) dt$ is the displacement of the particle over the time interval $0 \leq t \leq 3.5$.

The integral $\int_0^{3.5} |v(t)| dt$ is the total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

3 : $\begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{cases}$

(d) $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

2 : $\begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$

2018 SCORING GUIDELINES

Question 3

$$\begin{aligned} \text{(a)} \quad f(-5) &= f(1) + \int_1^{-5} g(x) dx = f(1) - \int_{-5}^1 g(x) dx \\ &= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2} \end{aligned}$$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b)} \quad \int_1^6 g(x) dx &= \int_1^3 g(x) dx + \int_3^6 g(x) dx \\ &= \int_1^3 2 dx + \int_3^6 2(x-4)^2 dx \\ &= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10 \end{aligned}$$

3 : $\begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f''(x) = g(x)$ is increasing on those intervals.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ because $f''(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

2018 SCORING GUIDELINES

Question 4

$$(a) \quad H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$$(b) \quad \frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$$

Because H is differentiable on $3 \leq t \leq 5$, H is continuous on $3 \leq t \leq 5$.

By the Mean Value Theorem, there exists a value c , $3 < c < 5$, such that $H'(c) = 2$.

(c) The average height of the tree over the time interval $2 \leq t \leq 10$ is given by $\frac{1}{10 - 2} \int_2^{10} H(t) \, dt$.

$$\begin{aligned} \frac{1}{8} \int_2^{10} H(t) \, dt &\approx \frac{1}{8} \left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3 \right) \\ &= \frac{1}{8} (65.75) = \frac{263}{32} \end{aligned}$$

The average height of the tree over the time interval $2 \leq t \leq 10$ is $\frac{263}{32}$ meters.

$$(d) \quad G(x) = 50 \Rightarrow x = 1$$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$$

$$\left. \frac{d}{dt}(G(x)) \right|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\frac{3}{4}$ meter per year.

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

2 : $\begin{cases} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

2 : $\begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

3 : $\begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0] if no chain rule

2018 SCORING GUIDELINES

Question 5

(a) The average rate of change of f on the interval $0 \leq x \leq \pi$ is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

(b) $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

(c) $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}} e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \leq x \leq 2\pi$ is $-\frac{1}{\sqrt{2}} e^{5\pi/4}$.

(d) $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

1 : answer

2 : $\begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$

3 : $\begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$

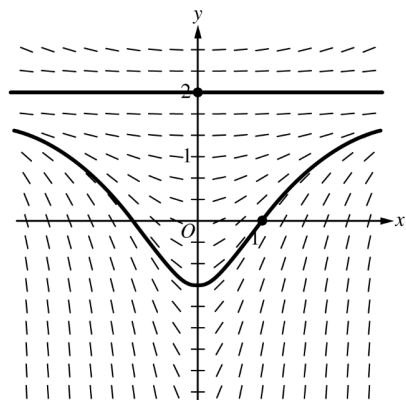
3 : $\begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \text{and limits equal 0} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

2018 SCORING GUIDELINES

Question 6

(a)



- 2 : $\begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b) $\left. \frac{dy}{dx} \right|_{(x, y)=(1, 0)} = \frac{4}{3}$

An equation for the line tangent to the graph of $y = f(x)$ at

$x = 1$ is $y = \frac{4}{3}(x - 1)$.

$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$

(c) $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$

$\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$

$\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$

$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$

$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$

$y = 2 - \frac{6}{x^2 + 2}$

Note: this solution is valid for $-\infty < x < \infty$.

- 2 : $\begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

- 5 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

AP[®] Calculus AB

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2016 SCORING GUIDELINES

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

- (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

(a) $R'(2) \approx \frac{R(3) - R(1)}{3 - 1} = \frac{950 - 1190}{3 - 1} = -120$ liters/hr²

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The total amount of water removed is given by $\int_0^8 R(t) dt$.

$$\begin{aligned} \int_0^8 R(t) dt &\approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters} \end{aligned}$$

This is an overestimate since R is a decreasing function.

3 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$

(c) Total $\approx 50000 + \int_0^8 W(t) dt - 8050$
 $= 50000 + 7836.195325 - 8050 \approx 49786$ liters

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$

(d) $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

2 : $\begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$

Therefore, the Intermediate Value Theorem guarantees at least one time t , $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

For this value of t , the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

2016 SCORING GUIDELINES

Question 2

For $t \geq 0$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$$v(t) = 1 + 2 \sin\left(\frac{t^2}{2}\right).$$

The particle is at position $x = 2$ at time $t = 4$.

- (a) At time $t = 4$, is the particle speeding up or slowing down?
- (b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.
- (c) Find the position of the particle at time $t = 0$.
- (d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

(a) $v(4) = 2.978716 > 0$
 $v'(4) = -1.164000 < 0$

The particle is slowing down since the velocity and acceleration have different signs.

2 : conclusion with reason

(b) $v(t) = 0 \Rightarrow t = 2.707468$

$v(t)$ changes from positive to negative at $t = 2.707$.
 Therefore, the particle changes direction at this time.

2 : $\begin{cases} 1 : t = 2.707 \\ 1 : \text{justification} \end{cases}$

(c) $x(0) = x(4) + \int_4^0 v(t) dt$
 $= 2 + (-5.815027) = -3.815$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(d) Distance $= \int_0^3 |v(t)| dt = 5.301$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

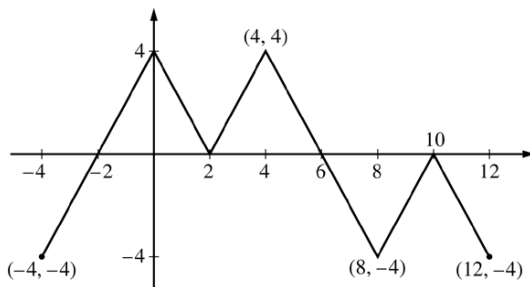
2016 SCORING GUIDELINES

Question 3

The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by

$$g(x) = \int_2^x f(t) dt.$$

- (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
- (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
- (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.
- (d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

Graph of f

- (a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \leq 0$ for $8 \leq x \leq 12$.
- (b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \leq x \leq 4$ and decreasing for $4 \leq x \leq 8$.
- (c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
-4	-4
-2	-8
6	8
12	-4

On the interval $-4 \leq x \leq 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

- (d) $g(x) \leq 0$ for $-4 \leq x \leq 2$ and $10 \leq x \leq 12$.

1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

1 : answer with justification

1 : answer with justification

4 : $\begin{cases} 1 : \text{considers } x = -2 \text{ and } x = 6 \\ \text{as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{cases}$

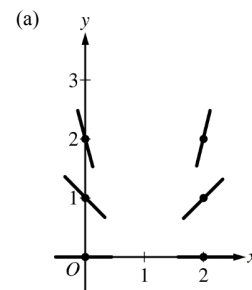
2 : intervals

2016 SCORING GUIDELINES

Question 4

Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.



(b) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = \frac{3^2}{2-1} = 9$

An equation for the tangent line is $y = 9(x - 2) + 3$.

$$f(2.1) \approx 9(2.1 - 2) + 3 = 3.9$$

(c) $\frac{1}{y^2} dy = \frac{1}{x-1} dx$

$$\int \frac{1}{y^2} dy = \int \frac{1}{x-1} dx$$

$$-\frac{1}{y} = \ln|x-1| + C$$

$$-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$$

$$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$$

$$y = \frac{1}{\frac{1}{3} - \ln(x-1)}$$

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

2 : $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

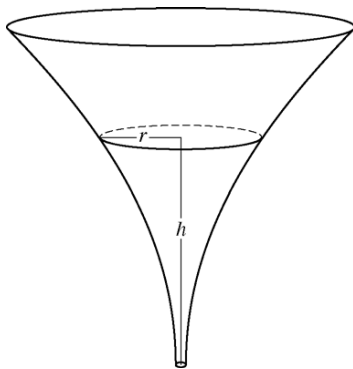
5 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and} \\ \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: 0/5 if no separation of variables

2016 SCORING GUIDELINES

Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

- (a) Find the average value of the radius of the funnel.
- (b) Find the volume of the funnel.
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

$$\begin{aligned} \text{(a) Average radius} &= \frac{1}{10} \int_0^{10} \frac{1}{20}(3 + h^2) dh = \frac{1}{200} \left[3h + \frac{h^3}{3} \right]_0^{10} \\ &= \frac{1}{200} \left(\left(30 + \frac{1000}{3} \right) - 0 \right) = \frac{109}{60} \text{ in} \end{aligned}$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^{10} \left(\frac{1}{20}(3 + h^2) \right)^2 dh = \frac{\pi}{400} \int_0^{10} (9 + 6h^2 + h^4) dh \\ &= \frac{\pi}{400} \left[9h + 2h^3 + \frac{h^5}{5} \right]_0^{10} \\ &= \frac{\pi}{400} \left(\left(90 + 2000 + \frac{100000}{5} \right) - 0 \right) = \frac{2209\pi}{40} \text{ in}^3 \end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(c) } \frac{dr}{dt} &= \frac{1}{20}(2h) \frac{dh}{dt} \\ -\frac{1}{5} &= \frac{3}{10} \frac{dh}{dt} \\ \frac{dh}{dt} &= -\frac{1}{5} \cdot \frac{10}{3} = -\frac{2}{3} \text{ in/sec} \end{aligned}$$

3 : $\begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

2016 SCORING GUIDELINES

Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

- (a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.
- (b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.
- (c) Evaluate $\int_1^3 f''(2x) dx$.

$$\begin{aligned} \text{(a) } k(3) &= f(g(3)) = f(6) = 4 \\ k'(3) &= f'(g(3)) \cdot g'(3) = f'(6) \cdot 2 = 5 \cdot 2 = 10 \end{aligned}$$

An equation for the tangent line is $y = 10(x - 3) + 4$.

3 : $\begin{cases} 2 : \text{slope at } x = 3 \\ 1 : \text{equation for tangent line} \end{cases}$

$$\begin{aligned} \text{(b) } h'(1) &= \frac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{(f(1))^2} \\ &= \frac{(-6) \cdot 8 - 2 \cdot 3}{(-6)^2} = \frac{-54}{36} = -\frac{3}{2} \end{aligned}$$

3 : $\begin{cases} 2 : \text{expression for } h'(1) \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(c) } \int_1^3 f''(2x) dx &= \frac{1}{2} [f'(2x)]_1^3 = \frac{1}{2} [f'(6) - f'(2)] \\ &= \frac{1}{2} [5 - (-2)] = \frac{7}{2} \end{aligned}$$

3 : $\begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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2015 SCORING GUIDELINES

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

- How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
- Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
- At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t) dt = 76.570$

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$

Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

2 : $\begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

- (c) The amount of water in the pipe at time t , $0 \leq t \leq 8$, is $30 + \int_0^t [R(x) - D(x)] dx$.

$$R(t) - D(t) = 0 \Rightarrow t = 0, 3.271658$$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

3 : $\begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $30 + \int_0^w [R(t) - D(t)] dt = 50$

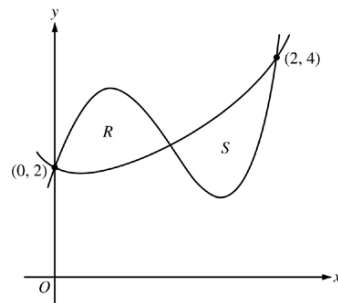
2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

2015 SCORING GUIDELINES

Question 2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

- (a) Find the sum of the areas of regions R and S .
- (b) Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.
- (c) Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.



- (a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$.

$$\begin{aligned}\text{Area} &= \int_0^A [g(x) - f(x)] dx + \int_A^2 [f(x) - g(x)] dx \\ &= 0.997427 + 1.006919 = 2.004\end{aligned}$$

(b) Volume $= \int_A^2 [f(x) - g(x)]^2 dx = 1.283$

(c) $h(x) = f(x) - g(x)$
 $h'(x) = f'(x) - g'(x)$
 $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

4 : $\begin{cases} 1 : \text{limits} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

- (a) Use the data in the table to estimate the value of $v'(16)$.
- (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.

- (c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

- (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

(a) $v'(16) \approx \frac{240 - 200}{20 - 12} = 5$ meters/min²

1 : approximation

- (b) $\int_0^{40} |v(t)| dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \leq t \leq 40$ minutes.

3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned}\int_0^{40} |v(t)| dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters}\end{aligned}$$

- (c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.
 $B'(5) = 3(25) - 12(5) = 15$ meters/min²

2 : $\begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned}\text{Avg vel} &= \frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300) dt \\ &= \frac{1}{10} \left[\frac{t^4}{4} - 2t^3 + 300t \right]_0^{10} \\ &= \frac{1}{10} \left[\frac{10000}{4} - 2000 + 3000 \right] = 350 \text{ meters/min}\end{aligned}$$

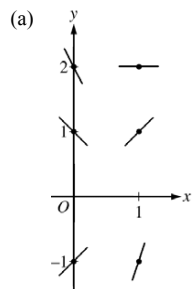
3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.
- (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.



- 2 : $\begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$

(b) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$

In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$.
Therefore, all solution curves are concave up in Quadrant II.

- 2 : $\begin{cases} 1 : \frac{d^2y}{dx^2} \\ 1 : \text{concave up with reason} \end{cases}$

(c) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = 2(2) - 3 = 1 \neq 0$

Therefore, f has neither a relative minimum nor a relative maximum at $x = 2$.

- 2 : $\begin{cases} 1 : \text{considers } \left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} \\ 1 : \text{conclusion with justification} \end{cases}$

(d) $y = mx + b \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(mx + b) = m$

$$2x - y = m$$

$$2x - (mx + b) = m$$

$$(2 - m)x - (m + b) = 0$$

$$2 - m = 0 \Rightarrow m = 2$$

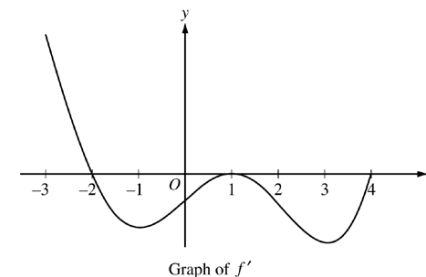
$$b = -m \Rightarrow b = -2$$

- 3 : $\begin{cases} 1 : \frac{d}{dx}(mx + b) = m \\ 1 : 2x - y = m \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 5

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



- (a) Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- (c) Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

- (a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$.
 $f'(x)$ changes from positive to negative at $x = -2$.
Therefore, f has a relative maximum at $x = -2$.

- 2 : $\begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$

- (b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.

- 2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

- (c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points.

- 2 : $\begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$

The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.

(d) $f(x) = 3 + \int_1^x f'(t) dt$

$$f(4) = 3 + \int_1^4 f'(t) dt = 3 + (-12) = -9$$

$$f(-2) = 3 + \int_1^{-2} f'(t) dt = 3 - \int_{-2}^1 f'(t) dt = 3 - (-9) = 12$$

- 3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$

2015 SCORING GUIDELINES

Question 6

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

- (a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.
- (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.
- (c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

$$(a) \left. \frac{dy}{dx} \right|_{(x,y)=(-1,1)} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$$

An equation for the tangent line is $y = \frac{1}{4}(x + 1) + 1$.

$$(b) 3y^2 - x = 0 \Rightarrow x = 3y^2$$

$$\text{So, } y^3 - xy = 2 \Rightarrow y^3 - (3y^2)(y) = 2 \Rightarrow y = -1$$

$$(-1)^3 - x(-1) = 2 \Rightarrow x = 3$$

The tangent line to the curve is vertical at the point $(3, -1)$.

$$(c) \frac{d^2y}{dx^2} = \frac{(3y^2 - x)\frac{dy}{dx} - y\left(6y\frac{dy}{dx} - 1\right)}{(3y^2 - x)^2}$$

$$\left. \frac{d^2y}{dx^2} \right|_{(x,y)=(-1,1)} = \frac{(3 \cdot 1^2 - (-1)) \cdot \frac{1}{4} - 1 \cdot \left(6 \cdot 1 \cdot \frac{1}{4} - 1\right)}{(3 \cdot 1^2 - (-1))^2}$$

$$= \frac{1 - \frac{1}{2}}{16} = \frac{1}{32}$$

$$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{equation for tangent line} \end{cases}$$

$$3 : \begin{cases} 1 : \text{sets } 3y^2 - x = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{coordinates} \end{cases}$$

$$4 : \begin{cases} 2 : \text{implicit differentiation} \\ 1 : \text{substitution for } \frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$$