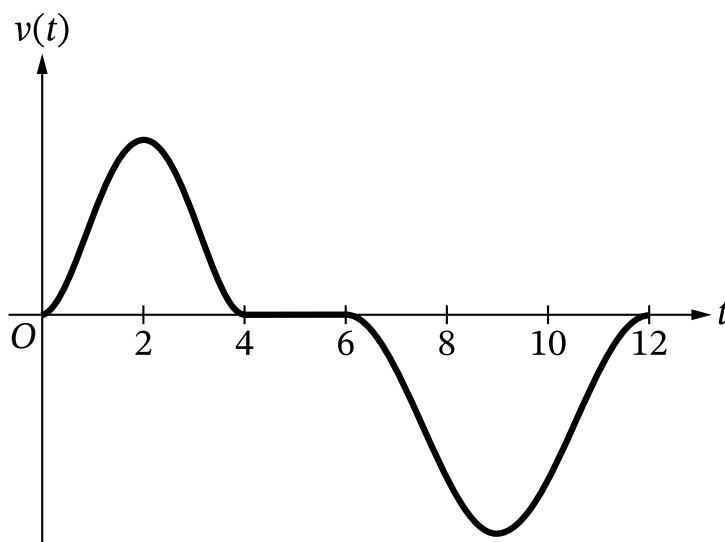


5. A remote-controlled toy car moves back and forth along a straight path so that its velocity at time t is given by the function v , where $v(t)$ is measured in feet per second and t is measured in seconds.

$$v(t) = \begin{cases} t^4 - 8t^3 + 16t^2 & \text{for } 0 \leq t \leq 4 \\ 0 & \text{for } 4 < t < 6 \\ 10 \cos\left(\frac{\pi}{3}t\right) - 10 & \text{for } 6 \leq t \leq 12 \end{cases}$$

The graph of $v(t)$ is shown.



Part A

Find the acceleration of the car at time $t = 1$ second. Show the work that leads to your answer.

Part B

Is the car speeding up or slowing down at time $t = 1$ second? Give a reason for your answer.

Part C

Find the distance, in feet, that the car traveled over the time interval $0 \leq t \leq 4$ seconds. Show the work that leads to your answer.

Part D

Find the average velocity of the car over the time interval $6 \leq t \leq 12$ seconds. Show the work that leads to your answer.

5. Two particles, H and J , are moving along the x -axis. For $0 \leq t \leq 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2 - 4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2 - 1)^3$.
- A. Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.
- B. During what open intervals of time t , for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.
- C. It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.
- D. Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

2. A particle moves along the x -axis so that its velocity at time $t \geq 0$ is given by $v(t) = \ln(t^2 - 4t + 5) - 0.2t$.
- (a) There is one time, $t = t_R$, in the interval $0 < t < 2$ when the particle is at rest (not moving). Find t_R . For $0 < t < t_R$, is the particle moving to the right or to the left? Give a reason for your answer.
- (b) Find the acceleration of the particle at time $t = 1.5$. Show the setup for your calculations. Is the speed of the particle increasing or decreasing at time $t = 1.5$? Explain your reasoning.
- (c) The position of the particle at time t is $x(t)$, and its position at time $t = 1$ is $x(1) = -3$. Find the position of the particle at time $t = 4$. Show the setup for your calculations.
- (d) Find the total distance traveled by the particle over the interval $1 \leq t \leq 4$. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Name:

AP Calculus AB: **2024**

END OF PART A

CALCULUS AB

SECTION II, Part B

Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

2. Stephen swims back and forth along a straight path in a 50-meter-long pool for 90 seconds. Stephen's velocity is modeled by $v(t) = 2.38e^{-0.02t}\sin\left(\frac{\pi}{56}t\right)$, where t is measured in seconds and $v(t)$ is measured in meters per second.
- (a) Find all times t in the interval $0 < t < 90$ at which Stephen changes direction. Give a reason for your answer.
- (b) Find Stephen's acceleration at time $t = 60$ seconds. Show the setup for your calculations, and indicate units of measure. Is Stephen speeding up or slowing down at time $t = 60$ seconds? Give a reason for your answer.
- (c) Find the distance between Stephen's position at time $t = 20$ seconds and his position at time $t = 80$ seconds. Show the setup for your calculations.
- (d) Find the total distance Stephen swims over the time interval $0 \leq t \leq 90$ seconds. Show the setup for your calculations.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

Name:

AP Calculus AB: **2023**

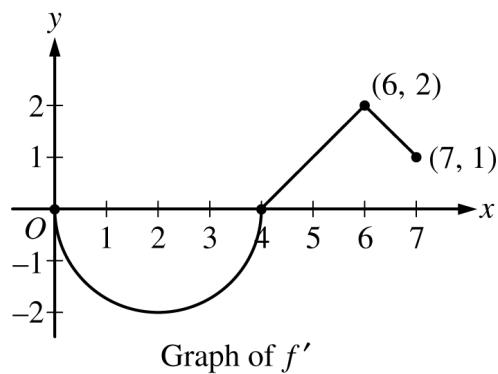
CALCULUS AB

SECTION II, Part B

Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



3. Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f' , the derivative of f , consists of a semicircle and two line segments, as shown in the figure above.
- (a) Find $f(0)$ and $f(5)$.
- (b) Find the x -coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.
- (c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.
- (d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

6. Particle P moves along the x -axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y -axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

- (a) Find $v_P(t)$, the velocity of particle P at time t .
- (b) Find $a_Q(t)$, the acceleration of particle Q at time t . Find all times t , for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.
- (c) Find $y_Q(t)$, the position of particle Q at time t .
- (d) As $t \rightarrow \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

A particle, P , is moving along the x -axis. The velocity of particle P at time t is given by $v_P(t) = \sin(t^{1.5})$ for $0 \leq t \leq \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q , also moves along the x -axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \leq t \leq \pi$. At time $t = 0$, particle Q is at position $x = 10$.

- (a) Find the positions of particles P and Q at time $t = 1$.
- (b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.
- (c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.
- (d) Find the total distance traveled by particle P over the time interval $0 \leq t \leq \pi$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

CALCULUS AB
SECTION II, Part B
Time—1 hour
4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

t (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	-29	55	48

2. The velocity of a particle, P , moving along the x -axis is given by the differentiable function v_P , where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$.
- (a) Justify why there must be at least one time t , for $0.3 \leq t \leq 2.8$, at which $v_P'(t)$, the acceleration of particle P , equals 0 meters per hour per hour.
- (b) Use a trapezoidal sum with the three subintervals $[0, 0.3]$, $[0.3, 1.7]$, and $[1.7, 2.8]$ to approximate the value of $\int_0^{2.8} v_P(t) dt$.
- (c) A second particle, Q , also moves along the x -axis so that its velocity for $0 \leq t \leq 4$ is given by $v_Q(t) = 45\sqrt{t} \cos(0.063t^2)$ meters per hour. Find the time interval during which the velocity of particle Q is at least 60 meters per hour. Find the distance traveled by particle Q during the interval when the velocity of particle Q is at least 60 meters per hour.
- (d) At time $t = 0$, particle Q is at position $x = -90$. Using the result from part (b) and the function v_Q from part (c), approximate the distance between particles P and Q at time $t = 2.8$.
-

END OF PART A OF SECTION II

2. A particle moves along the x -axis with velocity given by $v(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3}$ for time $0 \leq t \leq 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) Evaluate $\int_0^{3.5} v(t) \, dt$, and evaluate $\int_0^{3.5} |v(t)| \, dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x -axis with position given by $x_2(t) = t^2 - t$ for $0 \leq t \leq 3.5$. At what time t are the two particles moving with the same velocity?

END OF PART A OF SECTION II

5. Two particles move along the x -axis. For $0 \leq t \leq 8$, the position of particle P at time t is given by

$$x_P(t) = \ln(t^2 - 2t + 10), \text{ while the velocity of particle } Q \text{ at time } t \text{ is given by } v_Q(t) = t^2 - 8t + 15.$$

Particle Q is at position $x = 5$ at time $t = 0$.

- (a) For $0 \leq t \leq 8$, when is particle P moving to the left?
 - (b) For $0 \leq t \leq 8$, find all times t during which the two particles travel in the same direction.
 - (c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.
 - (d) Find the position of particle Q the first time it changes direction.
-

2. For $t \geq 0$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$$v(t) = 1 + 2 \sin\left(\frac{t^2}{2}\right). \text{ The particle is at position } x = 2 \text{ at time } t = 4.$$

- (a) At time $t = 4$, is the particle speeding up or slowing down?
 - (b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.
 - (c) Find the position of the particle at time $t = 0$.
 - (d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.
-

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B**Time—60 minutes****Number of problems—4****No calculator is allowed for these problems.**

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

3. Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.