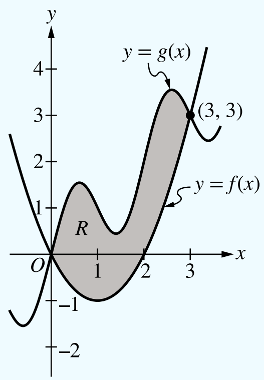
Part A (AB): Graphing calculator required
Question 2 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.



(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring	
A	Find the area of R. Show the setup for your calculations.		
	$\int_0^3 (g(x) - f(x))\,dx$	Form of integrand	**Point 1 (P1)**
	$= 5.136620$ The area is 5.137 (or 5.136).	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned for a response that presents an integrand of $g(x) - f(x)$, $|g(x) - f(x)|$, $f(x) - g(x)$, or $|f(x) - g(x)|$ in a definite integral, with or without the differential dx.
- **P1** could also be earned for a difference of definite integrals with integrands $g(x)$ and $f(x)$.
- **P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect communication between the integral and the correct answer is treated as scratch work and is not considered in scoring.
  - $\int_0^3 (f(x) - g(x))\,dx = -5.137$ so the area is 5.137.

    Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer.
  - $\int_0^3 (f(x) - g(x))\,dx = 5.137$

    Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- The exact answer is $\dfrac{4 + 9\pi}{2\pi}$.

B	Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.	
$\int_0^3 x(g(x) - f(x))\,dx$	Form of integrand	**Point 3 (P3)**
$= 7.704930$ The volume of the solid is 7.705 (or 7.704).	Answer	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned for a definite integral with an integrand presented as a product of two nonconstant factors, with one of the factors equal to x, $g(x) - f(x)$, or $f(x) - g(x)$.
- The presence or absence of the differential dx will not be considered in scoring **P3** or **P4**.
- **P4** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- Incorrect or unclear communication between the integral and the correct answer is treated as scratch work and is not considered in scoring. For example:
  - $\int_0^3 x(f(x) - g(x))\,dx = -7.705$ so the volume is 7.705.

    Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer.
  - $\int_0^3 x(f(x) - g(x))\,dx = 7.705$

    Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- The exact answer is $\dfrac{12 + 27\pi}{4\pi}$.

AP Calculus AB: **2025**

C	Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.	
Volume $= \pi\int_0^3 \left((g(x) - (-2))^2 - (f(x) - (-2))^2\right) dx$	Form of integrand	**Point 5 (P5)**
	Integrand	**Point 6 (P6)**
	Limits, constant, and differential	**Point 7 (P7)**

Scoring Notes for Part C

- **P5** is earned for a definite integral with an integrand of $R^2 - r^2$ or $\left|R^2 - r^2\right|$, where one of $\{R, r\}$ is correct or a difference between g and a nonzero constant, and the other is correct or a difference between f and a nonzero constant.
- **P6** is earned for the integral $\int_0^3 \left((g(x) + 2)^2 - (f(x) + 2)^2\right) dx$, $\int_0^3 \left((f(x) + 2)^2 - (g(x) + 2)^2\right) dx$, or a mathematically equivalent expression.
- Note **P5** and **P6** could be earned for a difference of definite integrals.
- A response that presents an integral expression that does not include the constant π is eligible for **P5** and **P6** but does not earn **P7**.
- To be eligible for **P7**, a response must have earned **P5**.
- A response that reverses the difference of squares must resolve the reversal with either the constant or the limits of integration AND include the differential to earn **P7**. For example:
  - A response of $-\pi\int_0^3 \left((f(x) + 2)^2 - (g(x) + 2)^2\right) dx$ or $\pi\int_3^0 \left((f(x) + 2)^2 - (g(x) + 2)^2\right) dx$ earns **P5**, **P6**, and **P7**.
  - A response of $\pi\int_0^3 \left((f(x) + 2)^2 - (g(x) + 2)^2\right) dx$ earns **P5** and **P6** but does not earn **P7**.
- A response of only $\pi\int_0^3 \left((g(x) - (-2))^2 - (f(x) - (-2))^2\right) dx$ earns **P5**, **P6**, and **P7**.

D	It can be shown that $g'(x) = 1 + \pi\cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.	
$f'(x) = g'(x) \Rightarrow 2x - 2 = 1 + \pi\cos(\pi x)$	$f'(x) = g'(x)$	**Point 8 (P8)**
$\Rightarrow x = 0.675819$ The lines tangent to the graphs of f and g are parallel at $x = 0.676$ (or 0.675).	Answer	**Point 9 (P9)**

Scoring Notes for Part D

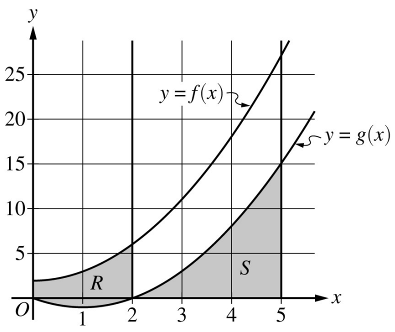
- **P8** is earned for a general statement, such as $f'(x) = g'(x)$, or for any correct equation formed by substituting $2x - 2$ for $f'(x)$, $1 + \pi\cos(\pi x)$ for $g'(x)$, or both.
- **P9** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



The functions f and g are defined by $f(x) = x^2 + 2$ and $g(x) = x^2 - 2x$, as shown in the graph.

Model Solution	Scoring

(a) Let R be the region bounded by the graphs of f and g, from $x = 0$ to $x = 2$, as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region R.

Area $= \int_0^2 (f(x) - g(x))\,dx$	Integrand	**1 point**
	Answer	**1 point**

Scoring notes:

- The first point is earned for a response that presents an integrand of $f(x) - g(x)$, $|f(x) - g(x)|$, $g(x) - f(x)$, or $|g(x) - f(x)|$ in one or more definite integrals.
- The first point could also be earned for a difference of definite integrals with integrands $f(x)$ and $g(x)$.
- The second point is earned only for one or more integrals equivalent to $\int_0^2 (f(x) - g(x))\,dx$, such as $\int_0^2 f(x)\,dx - \int_0^2 g(x)\,dx$, $\int_0^2 |f(x) - g(x)|\,dx$, $-\int_0^2 (g(x) - f(x))\,dx$, or $\int_0^2 |g(x) - f(x)|\,dx$.
  - Note: $\int_0^2 f(x)\,dx + \left|\int_0^2 g(x)\,dx\right|$ would earn both points.

Total for part (a) 2 points

(b) Let S be the region bounded by the graph of g and the x-axis, from $x = 2$ to $x = 5$, as shown in the graph. Region S is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height equal to half its base in region S. Find the volume of the solid. Show the work that leads to your answer.

Volume $= \int_2^5 \frac{1}{2}(g(x))^2\,dx = \int_2^5 \frac{1}{2}\left(x^2 - 2x\right)^2 dx$	Integrand	**1 point**
	Limits	**1 point**
$= \frac{1}{2}\int_2^5 \left(x^4 - 4x^3 + 4x^2\right) dx$ $= \frac{1}{2}\left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3}\right]_2^5$	Antiderivative	**1 point**
$= \frac{1}{2}\left[\left(\frac{5^5}{5} - 5^4 + \frac{500}{3}\right) - \left(\frac{32}{5} - 16 + \frac{32}{3}\right)\right]$ $= \frac{1}{2}\left(\frac{500}{3} - \frac{16}{15}\right) = \frac{414}{5}$	Answer	**1 point**

Scoring notes:

- The first point is earned only by a response with an integrand of the form $k(g(x))^2$ in a definite integral, where k is any nonzero constant.
- The second point is earned for limits of $x = 2$ and $x = 5$ in a definite integral with an integrand of the form $a(x) \cdot g(x)$ for any nonzero factor $a(x)$.
- To earn the third point a response must provide a correct antiderivative of $k\left(x^2 - 2x\right)^n$ for some integer $n \ge 2$.
- The fourth point is earned only for a numeric answer equivalent to $\frac{1}{2}\left(\frac{500}{3} - \frac{16}{15}\right)$.
- Special case: A response of Volume $= \int_2^5 \frac{1}{2}(f(x))^2\,dx$ or Volume $= \int_2^5 \frac{1}{2}\left(x^2 + 2\right)^2 dx$ earns the first 2 points and is not eligible for the last 2 points.

Total for part (b) 4 points

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region S, as described in part (b), is rotated about the horizontal line $y = 20$.

Volume $= \pi\int_2^5 \left[\left(20^2\right) - (20 - g(x))^2\right] dx$	Form of integrand	**1 point**
$= \pi\int_2^5 \left[400 - (20 - g(x))^2\right] dx$	Integrand	**1 point**
$= \pi\int_2^5 \left[400 - \left(20 - \left(x^2 - 2x\right)\right)^2\right] dx$	Limits and constant	**1 point**

Scoring notes:

- The first point is earned for a response that presents an integrand of $20^2 - (20 - g(x))^2$, $20^2 - (g(x) - 20)^2$, $(20 - g(x))^2 - 20^2$, $(g(x) - 20)^2 - 20^2$, or any mathematically equivalent expression, in one or more definite integrals.
- The second point is earned only for an integrand mathematically equivalent to $20^2 - (20 - g(x))^2$ or $20^2 - (g(x) - 20)^2$ in a definite integral. Note that $\int_a^b \left|(20 - g(x))^2 - 400\right| dx$ or $\left|\int_a^b \left((20 - g(x))^2 - 400\right) dx\right|$ earns the first 2 points.
- The integral may be split into two integrals.
  - For example, Volume $= \pi\int_2^5 \left(20^2\right) dx - \pi\int_2^5 (20 - g(x))^2\, dx$ or Volume $= \pi \cdot 20^2 \cdot 3 - \pi\int_2^5 (20 - g(x))^2\, dx$.
- A response that presents an allowable integrand involving $g(x)$, but continues and makes an error in using the expression for $g(x)$, does not earn the second point.
  - For example, $\pi\int_2^5 \left[20^2 - (20 - g(x))^2\right] dx = \pi\int_2^5 \left[20^2 - \left(20 - x^2 - 2x\right)^2\right] dx$ earns the first point but does not earn the second point.
- To be eligible for the third point a response must have earned at least 1 of the first 2 points or must have presented an integrand involving $g(x)$ of the form $R^2 - r^2$ in a definite integral.
- The third point is earned only by a definite integral including the constant π and limits $x = 2$ to $x = 5$. A response that presents any other constant or limits (including $x = 5$ to $x = 2$, except in the note below) does not earn the third point.
  - Note: $\pi\int_5^2 \left[(20 - g(x))^2 - 400\right] dx$ would earn all three points.
- Special case: A response of $\pi\int_2^5 \left[400 - (20 - f(x))^2\right] dx$ or equivalent earns 2 of the 3 points.

Total for part (c)	**3 points**
Total for question 6	**9 points**

2019

AP® CollegeBoard

AP® Calculus AB
Scoring Guidelines

2019 SCORING GUIDELINES

Question 1

(a) $\int_0^5 E(t)\,dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{1}{5-0}\int_0^5 L(t)\,dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$E(t) - L(t) = 0 \Rightarrow t = 6.20356$

$E(t) - L(t) > 0$ for $0 \le t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \le 8$. Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$A(t) = \int_0^t (E(s) - L(s))\,ds$
$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

$3 : \begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$

2019 SCORING GUIDELINES

Question 2

(a) v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$

By the Mean Value Theorem, there is a value c, $0.3 < c < 2.8$, such that $v_P'(c) = 0$.

$2 : \begin{cases} 1 : v_P(2.8) - v_P(0.3) = 0 \\ 1 : \text{justification, using Mean Value Theorem} \end{cases}$

— OR —

v_P is differentiable $\Rightarrow$ v_P is continuous on $0.3 \le t \le 2.8$.

By the Extreme Value Theorem, v_p has a minimum on $[0.3, 2.8]$.

$v_P(0.3) = 55 > -29 = v_P(1.7)$ and $v_P(1.7) = -29 < 55 = v_P(2.8)$.
Thus v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P'(t)$ must equal 0 at this minimum.

— OR —

$2 : \begin{cases} 1 : v_P(0.3) > v_P(1.7) \text{ and } v_P(1.7) < v_P(2.8) \\ 1 : \text{justification, using Extreme Value Theorem} \end{cases}$

(b)
$$\begin{aligned}\int_0^{2.8} v_P(t)\,dt &\approx 0.3\left(\frac{v_P(0) + v_P(0.3)}{2}\right) + 1.4\left(\frac{v_P(0.3) + v_P(1.7)}{2}\right) + 1.1\left(\frac{v_P(1.7) + v_P(2.8)}{2}\right) \\ &= 0.3\left(\frac{0 + 55}{2}\right) + 1.4\left(\frac{55 + (-29)}{2}\right) + 1.1\left(\frac{-29 + 55}{2}\right) \\ &= 40.75\end{aligned}$$

1 : answer, using trapezoidal sum

(c) $v_Q(t) = 60 \Rightarrow t = A = 1.866181$ or $t = B = 3.519174$
$v_Q(t) \ge 60$ for $A \le t \le B$

$\int_A^B |v_Q(t)|\,dt = 106.108754$

The distance traveled by particle Q during the interval $A \le t \le B$ is 106.109 (or 106.108) meters.

$3 : \begin{cases} 1 : \text{interval} \\ 1 : \text{definite integral} \\ 1 : \text{distance} \end{cases}$

(d) From part (b), the position of particle P at time $t = 2.8$ is

$x_P(2.8) = \int_0^{2.8} v_P(t)\,dt \approx 40.75.$

$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t)\,dt = -90 + 135.937653 = 45.937653$

Therefore at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

$3 : \begin{cases} 1 : \int_0^{2.8} v_Q(t)\,dt \\ 1 : \text{position of particle } Q \\ 1 : \text{answer} \end{cases}$

2019 SCORING GUIDELINES

Question 3

(a) $\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$

$\Rightarrow 7 = \int_{-6}^{-2} f(x)\,dx + 2 + \left(9 - \frac{9\pi}{4}\right)$

$\Rightarrow \int_{-6}^{-2} f(x)\,dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4$

3 :
- 1 : $\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$
- 1 : $\int_{-2}^{5} f(x)\,dx$
- 1 : answer

(b) $\int_3^5 (2f'(x) + 4)\,dx = 2\int_3^5 f'(x)\,dx + \int_3^5 4\,dx$

$= 2(f(5) - f(3)) + 4(5-3)$

$= 2\left(0 - (3 - \sqrt{5})\right) + 8$

$= 2(-3 + \sqrt{5}) + 8 = 2 + 2\sqrt{5}$

— OR —

$\int_3^5 (2f'(x) + 4)\,dx = [2f(x) + 4x]_{x=3}^{x=5}$

$= (2f(5) + 20) - (2f(3) + 12)$

$= (2\cdot 0 + 20) - \left(2(3 - \sqrt{5}) + 12\right)$

$= 2 + 2\sqrt{5}$

2 :
- 1 : Fundamental Theorem of Calculus
- 1 : answer

(c) $g'(x) = f(x) = 0 \Rightarrow x = -1,\ x = \frac{1}{2},\ x = 5$

x	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval $-2 \le x \le 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$.

3 :
- 1 : $g'(x) = f(x)$
- 1 : identifies $x = -1$ as a candidate
- 1 : answer with justification

(d) $\lim_{x\to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} = \frac{10^1 - 3f'(1)}{f(1) - \arctan 1}$

$= \frac{10 - 3\cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}}$

1 : answer

2019 SCORING GUIDELINES

Question 4

(a) $V = \pi r^2 h = \pi(1)^2 h = \pi h$

$\left.\frac{dV}{dt}\right|_{h=4} = \pi \left.\frac{dh}{dt}\right|_{h=4} = \pi\left(-\frac{1}{10}\sqrt{4}\right) = -\frac{\pi}{5}$ cubic feet per second

2 :
- 1 : $\frac{dV}{dt} = \pi\frac{dh}{dt}$
- 1 : answer with units

(b) $\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}}\cdot\frac{dh}{dt} = -\frac{1}{20\sqrt{h}}\cdot\left(-\frac{1}{10}\sqrt{h}\right) = \frac{1}{200}$

Because $\frac{d^2h}{dt^2} = \frac{1}{200} > 0$ for $h > 0$, the rate of change of the height is increasing when the height of the water is 3 feet.

3 :
- 1 : $\frac{d}{dh}\left(-\frac{1}{10}\sqrt{h}\right) = -\frac{1}{20\sqrt{h}}$
- 1 : $\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}}\cdot\frac{dh}{dt}$
- 1 : answer with explanation

(c) $\frac{dh}{\sqrt{h}} = -\frac{1}{10}\,dt$

$\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10}\,dt$

$2\sqrt{h} = -\frac{1}{10}t + C$

$2\sqrt{5} = -\frac{1}{10}\cdot 0 + C \Rightarrow C = 2\sqrt{5}$

$2\sqrt{h} = -\frac{1}{10}t + 2\sqrt{5}$

$h(t) = \left(-\frac{1}{20}t + \sqrt{5}\right)^2$

4 :
- 1 : separation of variables
- 1 : antiderivatives
- 1 : constant of integration and uses initial condition
- 1 : $h(t)$

Note: $0/4$ if no separation of variables

Note: max $2/4$ [1-1-0-0] if no constant of integration

2019 SCORING GUIDELINES

Question 5

(a) $\int_0^2 (h(x) - g(x))\,dx = \int_0^2 \left(\left(6 - 2(x-1)^2\right) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right)\right)\right) dx$

$$= \left[\left(6x - \frac{2}{3}(x-1)^3\right) - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right)\right)\right]_{x=0}^{x=2}$$

$$= \left(\left(12 - \frac{2}{3}\right) - (-4 + 0)\right) - \left(\left(0 + \frac{2}{3}\right) - (0 + 0)\right)$$

$$= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}$$

The area of R is $\frac{44}{3}$.

4 : { 1 : integrand
1 : antiderivative of $3\cos\left(\frac{\pi}{2}x\right)$
1 : antiderivative of remaining terms
1 : answer

(b) $\int_0^2 A(x)\,dx = \int_0^2 \frac{1}{x+3}\,dx$

$$= \left[\ln(x+3)\right]_{x=0}^{x=2} = \ln 5 - \ln 3$$

The volume of the solid is $\ln 5 - \ln 3$.

2 : { 1 : integral
1 : answer

(c) $\pi\int_0^2 \left((6 - g(x))^2 - (6 - h(x))^2\right) dx$

3 : { 1 : limits and constant
1 : form of integrand
1 : integrand

2019 SCORING GUIDELINES

Question 6

(a) $h'(2) = \frac{2}{3}$

1 : answer

(b) $a'(x) = 9x^2h(x) + 3x^3h'(x)$

$a'(2) = 9\cdot 2^2 h(2) + 3\cdot 2^3 h'(2) = 36\cdot 4 + 24\cdot\frac{2}{3} = 160$

3 : { 1 : form of product rule
1 : $a'(x)$
1 : $a'(2)$

(c) Because h is differentiable, h is continuous, so $\lim_{x\to 2} h(x) = h(2) = 4$.

Also, $\lim_{x\to 2} h(x) = \lim_{x\to 2} \frac{x^2 - 4}{1 - (f(x))^3}$, so $\lim_{x\to 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$.

Because $\lim_{x\to 2}\left(x^2 - 4\right) = 0$, we must also have $\lim_{x\to 2}\left(1 - (f(x))^3\right) = 0$.

Thus $\lim_{x\to 2} f(x) = 1$.

Because f is differentiable, f is continuous, so $f(2) = \lim_{x\to 2} f(x) = 1$.

Also, because f is twice differentiable, f' is continuous, so $\lim_{x\to 2} f'(x) = f'(2)$ exists.

Using L'Hospital's Rule,

$$\lim_{x\to 2} \frac{x^2 - 4}{1 - (f(x))^3} = \lim_{x\to 2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{4}{-3(1)^2\cdot f'(2)} = 4.$$

Thus $f'(2) = -\frac{1}{3}$.

4 : { 1 : $\lim_{x\to 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$
1 : $f(2)$
1 : L'Hospital's Rule
1 : $f'(2)$

(d) Because g and h are differentiable, g and h are continuous, so $\lim_{x\to 2} g(x) = g(2) = 4$ and $\lim_{x\to 2} h(x) = h(2) = 4$.

Because $g(x) \le k(x) \le h(x)$ for $1 < x < 3$, it follows from the squeeze theorem that $\lim_{x\to 2} k(x) = 4$.

Also, $4 = g(2) \le k(2) \le h(2) = 4$, so $k(2) = 4$.

Thus k is continuous at $x = 2$.

1 : continuous with justification