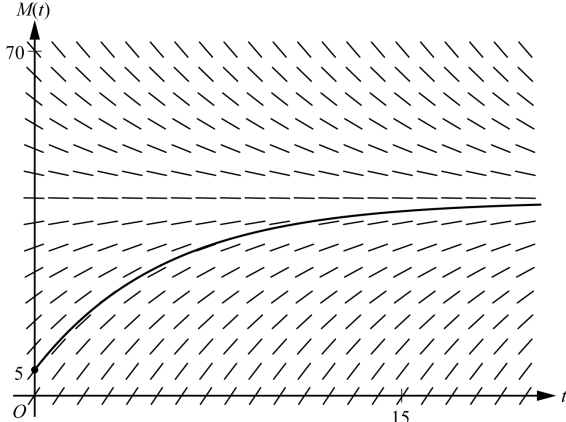


**Part B (AB or BC): Graphing calculator not allowed****Question 3****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function  $M$  models the temperature of the milk at time  $t$ , where  $M(t)$  is measured in degrees Celsius ( $^{\circ}\text{C}$ ) and  $t$  is the number of minutes since the bottle was placed in the pan.  $M$  satisfies the differential equation  $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ . At time  $t = 0$ , the temperature of the milk is  $5^{\circ}\text{C}$ . It can be shown that  $M(t) < 40$  for all values of  $t$ .

| Model Solution                                                                                                                                                                                                                                                        | Scoring                              |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------|
| <p>(a) A slope field for the differential equation <math>\frac{dM}{dt} = \frac{1}{4}(40 - M)</math> is shown. Sketch the solution curve through the point <math>(0, 5)</math>.</p>  | <p>Solution curve <b>1 point</b></p> |

**Scoring notes:**

- The solution curve must pass through the point  $(0, 5)$ , extend reasonably close to the left and right edges of the rectangle, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.
- The solution curve must lie entirely below the horizontal line segments at  $M = 40$ .

- (b) Use the line tangent to the graph of  $M$  at  $t = 0$  to approximate  $M(2)$ , the temperature of the milk at time  $t = 2$  minutes.

|                                                                                                                                                                                                                                                                          |                                                     |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|
| $\left. \frac{dM}{dt} \right _{t=0} = \frac{1}{4}(40 - 5) = \frac{35}{4}$                                                                                                                                                                                                | $\left. \frac{dM}{dt} \right _{t=0}$ <b>1 point</b> |
| <p>The tangent line equation is <math>y = 5 + \frac{35}{4}(t - 0)</math>.</p> <p><math>M(2) \approx 5 + \frac{35}{4} \cdot 2 = 22.5</math></p> <p>The temperature of the milk at time <math>t = 2</math> minutes is approximately <math>22.5^{\circ}</math> Celsius.</p> | Approximation <b>1 point</b>                        |

**Scoring notes:**

- The value of the slope may appear in a tangent line equation or approximation.
- A response of  $5 + \frac{35}{4} \cdot 2$  is the minimal response to earn both points.
- A response of  $\frac{1}{4}(40 - 5)$  earns the first point. If there are any subsequent errors in simplification, the response does not earn the second point.
- In order to earn the second point the response must present an approximation found by using a tangent line that:
  - passes through the point  $(0, 5)$  and
  - has slope  $\frac{35}{4}$  or a nonzero slope that is declared to be the value of  $\frac{dM}{dt}$ .
- An unsupported approximation does not earn the second point.
- The approximation need not be simplified, but the response does not earn the second point if the approximation is simplified incorrectly.

**Total for part (b) 2 points**

- (c) Write an expression for  $\frac{d^2M}{dt^2}$  in terms of  $M$ . Use  $\frac{d^2M}{dt^2}$  to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of  $M(2)$ . Give a reason for your answer.

|                                                                                                                                                              |                                         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------|
| $\frac{d^2M}{dt^2} = -\frac{1}{4} \frac{dM}{dt} = -\frac{1}{4} \left( \frac{1}{4}(40 - M) \right) = -\frac{1}{16}(40 - M)$                                   | $\frac{d^2M}{dt^2}$ <b>1 point</b>      |
| Because $M(t) < 40$ , $\frac{d^2M}{dt^2} < 0$ , so the graph of $M$ is concave down. Therefore, the tangent line approximation of $M(2)$ is an overestimate. | Overestimate with reason <b>1 point</b> |

**Scoring notes:**

- The first point is earned for either  $\frac{d^2M}{dt^2} = -\frac{1}{4}\left(\frac{1}{4}(40 - M)\right)$  or  $\frac{d^2M}{dt^2} = -\frac{1}{16}(40 - M)$  (or equivalent). A response that presents any subsequent simplification error does not earn the second point.
- A response that presents an expression for  $\frac{d^2M}{dt^2}$  in terms of  $\frac{dM}{dt}$  but fails to continue to an expression in terms of  $M$  (i.e.,  $\frac{d^2M}{dt^2} = -\frac{1}{4}\frac{dM}{dt}$ ) does not earn the first point but is eligible for the second point.
- If the response presents an expression for  $\frac{d^2M}{dt^2}$  that is incorrect, the response is eligible for the second point only if the expression is a nonconstant linear function that is negative for  $5 < M < 40$ .
  - Special case: A response that presents  $\frac{d^2M}{dt^2} = \frac{1}{16}(40 - M)$  does not earn the first point but is eligible to earn the second point for a consistent answer and reason.
- To earn the second point a response must include  $\frac{d^2M}{dt^2} < 0$ , or  $\frac{dM}{dt}$  is decreasing, or the graph of  $M$  is concave down, as well as the conclusion that the approximation is an overestimate.
- A response that presents an argument based on  $\frac{d^2M}{dt^2}$  or concavity at a single point does not earn the second point.

**Total for part (c) 2 points**

- (d) Use separation of variables to find an expression for  $M(t)$ , the particular solution to the differential equation  $\frac{dM}{dt} = \frac{1}{4}(40 - M)$  with initial condition  $M(0) = 5$ .

|                                                                                                                                                                                                        |                                                    |                |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------|----------------|
| $\frac{dM}{40 - M} = \frac{1}{4}dt$<br>$\int \frac{dM}{40 - M} = \int \frac{1}{4}dt$                                                                                                                   | Separates variables                                | <b>1 point</b> |
| $-\ln 40 - M  = \frac{1}{4}t + C$                                                                                                                                                                      | Finds antiderivatives                              | <b>1 point</b> |
| $-\ln 40 - 5  = 0 + C \Rightarrow C = -\ln 35$<br>$M(t) < 40 \Rightarrow 40 - M > 0 \Rightarrow  40 - M  = 40 - M$<br>$-\ln(40 - M) = \frac{1}{4}t - \ln 35$<br>$\ln(40 - M) = -\frac{1}{4}t + \ln 35$ | Constant of integration and uses initial condition | <b>1 point</b> |

|                                                |                |                |
|------------------------------------------------|----------------|----------------|
| $40 - M = 35e^{-t/4}$<br>$M = 40 - 35e^{-t/4}$ | Solves for $M$ | <b>1 point</b> |
|------------------------------------------------|----------------|----------------|

**Scoring notes:**

- A response with no separation of variables earns 0 out of 4 points.
- A response that presents an antiderivative of  $-\ln(40 - M)$  without absolute value symbols is eligible for all 4 points.
- A response with no constant of integration can earn at most the first 2 points.
- A response is eligible for the third point only if it has earned the first 2 points.
  - Special Case: A response that presents  $+\ln(40 - M) = \frac{t}{4} + C$  (or equivalent) does not earn the second point, is eligible for the third point, but not eligible for the fourth.
- An eligible response earns the third point by correctly including the constant of integration in an equation and substituting 0 for  $t$  and 5 for  $M$ .
- A response is eligible for the fourth point only if it has earned the first 3 points.
- A response earns the fourth point only for an answer of  $M = 40 - 35e^{-t/4}$  or equivalent.

**Total for part (d) 4 points****Total for question 3 9 points**

**Part B (AB): Graphing calculator not allowed****Question 5****9 points****General Scoring Notes**

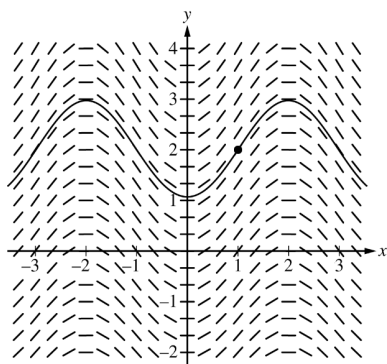
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the differential equation  $\frac{dy}{dx} = \frac{1}{2} \sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ . Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(1) = 2$ . The function  $f$  is defined for all real numbers.

**Model Solution****Scoring**

- (a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point  $(1, 2)$ .



Solution curve

**1 point****Scoring notes:**

- The solution curve must pass through the point  $(1, 2)$ , extend reasonably close to the left and right edges of the square and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.
- The solution curve must indicate  $f(x) > 0$  for all points on the curve.
- All local maximum/minimum points on the solution curve must occur at horizontal line segments in the slope field.

**Total for part (a) 1 point**

- (b) Write an equation for the line tangent to the solution curve in part (a) at the point  $(1, 2)$ . Use the equation to approximate  $f(0.8)$ .

$$\left. \frac{dy}{dx} \right|_{(x,y)=(1,2)} = \frac{1}{2} \cdot 3 \cdot \sin\left(\frac{\pi}{2}\right) = \frac{3}{2}$$

Tangent line equation **1 point**

$$\text{An equation for the tangent line is } y = 2 + \frac{3}{2}(x - 1).$$

$$f(0.8) \approx 2 + \frac{3}{2}(0.8 - 1) = 1.7$$

Approximation **1 point****Scoring notes:**

- The tangent line equation can be presented in any equivalent form.
- An incorrect tangent line equation with a slope of  $\frac{3}{2}$  is eligible to earn the second point for a consistent answer.
- A response of only  $2 + \frac{3}{2}(0.8 - 1)$  earns the second point but not the first.

**Total for part (b) 2 points**

- (c) It is known that  $f''(x) > 0$  for  $-1 \leq x \leq 1$ . Is the approximation found in part (b) an overestimate or an underestimate for  $f(0.8)$ ? Give a reason for your answer.

Because  $f''(x) > 0$ ,  $f$  is concave up on  $-1 \leq x \leq 1$ , the tangent line lies below the graph of  $y = f(x)$  at  $x = 0.8$ , and the approximation for  $f(0.8)$  is an underestimate.

Answer with reason **1 point****Scoring notes:**

- The reason must include  $f''(x) > 0$ ,  $f'(x)$  is increasing, or  $f(x)$  is concave up.

**Total for part (c) 1 point**

- (d) Use separation of variables to find  $y = f(x)$ , the particular solution to the differential equation

$$\frac{dy}{dx} = \frac{1}{2} \sin\left(\frac{\pi}{2}x\right) \sqrt{y+7} \text{ with the initial condition } f(1) = 2.$$

|                                                                                                                                                                                                                                                                  |                                                    |         |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------|---------|
| $\int \frac{dy}{\sqrt{y+7}} = \int \frac{1}{2} \sin\left(\frac{\pi}{2}x\right) dx$                                                                                                                                                                               | Separation of variables                            | 1 point |
| $2\sqrt{y+7} = -\frac{1}{\pi} \cos\left(\frac{\pi}{2}x\right) + C$                                                                                                                                                                                               | One correct antiderivative                         | 1 point |
|                                                                                                                                                                                                                                                                  | The other correct antiderivative                   | 1 point |
| $f(1) = 2 \Rightarrow 2\sqrt{2+7} = -\frac{1}{\pi} \cos\left(\frac{\pi}{2} \cdot 1\right) + C$<br>$\Rightarrow 6 = -\frac{1}{\pi} \cos\left(\frac{\pi}{2}\right) + C \Rightarrow C = 6$<br><br>$\sqrt{y+7} = 3 - \frac{1}{2\pi} \cos\left(\frac{\pi}{2}x\right)$ | Constant of integration and uses initial condition | 1 point |
| $y = \left(3 - \frac{1}{2\pi} \cos\left(\frac{\pi}{2}x\right)\right)^2 - 7$                                                                                                                                                                                      | Solves for $y$                                     | 1 point |

**Scoring notes:**

- A response with no separation of variables earns 0 out of 5 points.
- A response with no constant of integration can earn at most the first 3 points.
- A response is eligible for the fourth point only if it has earned the first point and at least 1 of the 2 antiderivative points.
  - Special case: The incorrect separation of  $\sqrt{y+7} \, dy = \frac{1}{2} \sin\left(\frac{\pi}{2}x\right) dx$  does not earn the first point, is only eligible for the antiderivative point for  $-\frac{1}{\pi} \cos\left(\frac{\pi}{2}x\right)$ , and is eligible for the fourth point.
- An eligible response earns the fourth point by correctly including the constant of integration in an equation and substituting 1 for  $x$  and 2 for  $y$ .
- A response is eligible for the fifth point only if it has earned the first 4 points.

**Total for part (d) 5 points**

**Total for question 5 9 points**

## AP<sup>®</sup> Calculus AB 2015 Scoring Guidelines

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## 2015 SCORING GUIDELINES

## Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function  $R$ , where  $R(t) = 20\sin\left(\frac{t^2}{35}\right)$  cubic feet per hour,  $t$  is measured in hours, and  $0 \leq t \leq 8$ . The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by  $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$  cubic feet per hour, for  $0 \leq t \leq 8$ . There are 30 cubic feet of water in the pipe at time  $t = 0$ .

- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval  $0 \leq t \leq 8$ ?
- (b) Is the amount of water in the pipe increasing or decreasing at time  $t = 3$  hours? Give a reason for your answer.
- (c) At what time  $t$ ,  $0 \leq t \leq 8$ , is the amount of water in the pipe at a minimum? Justify your answer.
- (d) The pipe can hold 50 cubic feet of water before overflowing. For  $t > 8$ , water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time  $w$  when the pipe will begin to overflow.

(a)  $\int_0^8 R(t) dt = 76.570$

2 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b)  $R(3) - D(3) = -0.313632 < 0$   
 Since  $R(3) < D(3)$ , the amount of water in the pipe is decreasing at time  $t = 3$  hours.

2 :  $\begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time  $t$ ,  $0 \leq t \leq 8$ , is  $30 + \int_0^t [R(x) - D(x)] dx$ .

3 :  $\begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

$$R(t) - D(t) = 0 \Rightarrow t = 0, 3.271658$$

| $t$      | Amount of water in the pipe |
|----------|-----------------------------|
| 0        | 30                          |
| 3.271658 | 27.964561                   |
| 8        | 48.543686                   |

The amount of water in the pipe is a minimum at time  $t = 3.272$  (or 3.271) hours.

(d)  $30 + \int_0^w [R(t) - D(t)] dt = 50$

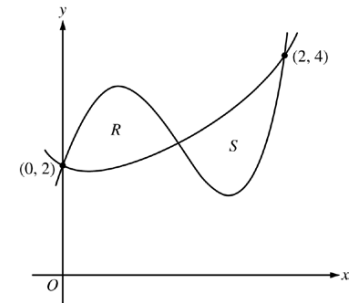
2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

## 2015 SCORING GUIDELINES

## Question 2

Let  $f$  and  $g$  be the functions defined by  $f(x) = 1 + x + e^{x^2-2x}$  and  $g(x) = x^4 - 6.5x^2 + 6x + 2$ . Let  $R$  and  $S$  be the two regions enclosed by the graphs of  $f$  and  $g$  shown in the figure above.

- (a) Find the sum of the areas of regions  $R$  and  $S$ .
- (b) Region  $S$  is the base of a solid whose cross sections perpendicular to the  $x$ -axis are squares. Find the volume of the solid.
- (c) Let  $h$  be the vertical distance between the graphs of  $f$  and  $g$  in region  $S$ . Find the rate at which  $h$  changes with respect to  $x$  when  $x = 1.8$ .



- (a) The graphs of  $y = f(x)$  and  $y = g(x)$  intersect in the first quadrant at the points  $(0, 2)$ ,  $(2, 4)$ , and  $(A, B) = (1.032832, 2.401108)$ .

$$\begin{aligned} \text{Area} &= \int_0^A [g(x) - f(x)] dx + \int_A^2 [f(x) - g(x)] dx \\ &= 0.997427 + 1.006919 = 2.004 \end{aligned}$$

4 :  $\begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$

(b)  $\text{Volume} = \int_A^2 [f(x) - g(x)]^2 dx = 1.283$

3 :  $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c)  $h(x) = f(x) - g(x)$   
 $h'(x) = f'(x) - g'(x)$   
 $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$  (or  $-3.811$ )

2 :  $\begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$

## 2015 SCORING GUIDELINES

## Question 3

|                               |   |     |     |      |     |
|-------------------------------|---|-----|-----|------|-----|
| $t$<br>(minutes)              | 0 | 12  | 20  | 24   | 40  |
| $v(t)$<br>(meters per minute) | 0 | 200 | 240 | -220 | 150 |

Johanna jogs along a straight path. For  $0 \leq t \leq 40$ , Johanna's velocity is given by a differentiable function  $v$ . Selected values of  $v(t)$ , where  $t$  is measured in minutes and  $v(t)$  is measured in meters per minute, are given in the table above.

- (a) Use the data in the table to estimate the value of  $v'(16)$ .
- (b) Using correct units, explain the meaning of the definite integral  $\int_0^{40} |v(t)| dt$  in the context of the problem.

Approximate the value of  $\int_0^{40} |v(t)| dt$  using a right Riemann sum with the four subintervals indicated in the table.

- (c) Bob is riding his bicycle along the same path. For  $0 \leq t \leq 10$ , Bob's velocity is modeled by  $B(t) = t^3 - 6t^2 + 300$ , where  $t$  is measured in minutes and  $B(t)$  is measured in meters per minute.

Find Bob's acceleration at time  $t = 5$ .

- (d) Based on the model  $B$  from part (c), find Bob's average velocity during the interval  $0 \leq t \leq 10$ .

(a)  $v'(16) \approx \frac{240 - 200}{20 - 12} = 5$  meters/min<sup>2</sup>

1 : approximation

- (b)  $\int_0^{40} |v(t)| dt$  is the total distance Johanna jogs, in meters, over the time interval  $0 \leq t \leq 40$  minutes.

3 :  $\begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned} \int_0^{40} |v(t)| dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters} \end{aligned}$$

- (c) Bob's acceleration is  $B'(t) = 3t^2 - 12t$ .

$$B'(5) = 3(25) - 12(5) = 15 \text{ meters/min}^2$$

2 :  $\begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

- (d) Avg vel =  $\frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300) dt$

$$\begin{aligned} &= \frac{1}{10} \left[ \frac{t^4}{4} - 2t^3 + 300t \right]_0^{10} \\ &= \frac{1}{10} \left[ \frac{10000}{4} - 2000 + 3000 \right] = 350 \text{ meters/min} \end{aligned}$$

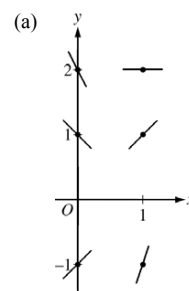
3 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

## 2015 SCORING GUIDELINES

## Question 4

Consider the differential equation  $\frac{dy}{dx} = 2x - y$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- (c) Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(2) = 3$ . Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 2$ ? Justify your answer.
- (d) Find the values of the constants  $m$  and  $b$  for which  $y = mx + b$  is a solution to the differential equation.



2 :  $\begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$

(b)  $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$

In Quadrant II,  $x < 0$  and  $y > 0$ , so  $2 - 2x + y > 0$ .

Therefore, all solution curves are concave up in Quadrant II.

2 :  $\begin{cases} 1 : \frac{d^2y}{dx^2} \\ 1 : \text{concave up with reason} \end{cases}$

(c)  $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = 2(2) - 3 = 1 \neq 0$

Therefore,  $f$  has neither a relative minimum nor a relative maximum at  $x = 2$ .

2 :  $\begin{cases} 1 : \text{considers } \left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} \\ 1 : \text{conclusion with justification} \end{cases}$

(d)  $y = mx + b \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(mx + b) = m$

$$2x - y = m$$

$$2x - (mx + b) = m$$

$$(2 - m)x - (m + b) = 0$$

$$2 - m = 0 \Rightarrow m = 2$$

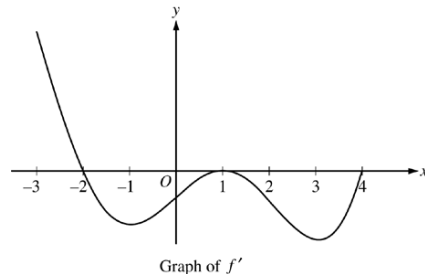
$$b = -m \Rightarrow b = -2$$

3 :  $\begin{cases} 1 : \frac{d}{dx}(mx + b) = m \\ 1 : 2x - y = m \\ 1 : \text{answer} \end{cases}$

## 2015 SCORING GUIDELINES

## Question 5

The figure above shows the graph of  $f'$ , the derivative of a twice-differentiable function  $f$ , on the interval  $[-3, 4]$ . The graph of  $f'$  has horizontal tangents at  $x = -1$ ,  $x = 1$ , and  $x = 3$ . The areas of the regions bounded by the  $x$ -axis and the graph of  $f'$  on the intervals  $[-2, 1]$  and  $[1, 4]$  are 9 and 12, respectively.



- (a) Find all  $x$ -coordinates at which  $f$  has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in  $-3 < x < 4$  is the graph of  $f$  both concave down and decreasing? Give a reason for your answer.
- (c) Find the  $x$ -coordinates of all points of inflection for the graph of  $f$ . Give a reason for your answer.
- (d) Given that  $f(1) = 3$ , write an expression for  $f(x)$  that involves an integral. Find  $f(4)$  and  $f(-2)$ .

- (a)  $f'(x) = 0$  at  $x = -2$ ,  $x = 1$ , and  $x = 4$ .  
 $f'(x)$  changes from positive to negative at  $x = -2$ .  
 Therefore,  $f$  has a relative maximum at  $x = -2$ .

2 :  $\begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$

- (b) The graph of  $f$  is concave down and decreasing on the intervals  $-2 < x < -1$  and  $1 < x < 3$  because  $f'$  is decreasing and negative on these intervals.

2 :  $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

- (c) The graph of  $f$  has a point of inflection at  $x = -1$  and  $x = 3$  because  $f'$  changes from decreasing to increasing at these points.

2 :  $\begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$

The graph of  $f$  has a point of inflection at  $x = 1$  because  $f'$  changes from increasing to decreasing at this point.

- (d)  $f(x) = 3 + \int_1^x f'(t) dt$

$$f(4) = 3 + \int_1^4 f'(t) dt = 3 + (-12) = -9$$

$$f(-2) = 3 + \int_1^{-2} f'(t) dt = 3 - \int_{-2}^1 f'(t) dt = 3 - (-9) = 12$$

3 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$

## 2015 SCORING GUIDELINES

## Question 6

Consider the curve given by the equation  $y^3 - xy = 2$ . It can be shown that  $\frac{dy}{dx} = \frac{y}{3y^2 - x}$ .

- (a) Write an equation for the line tangent to the curve at the point  $(-1, 1)$ .
- (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.
- (c) Evaluate  $\frac{d^2y}{dx^2}$  at the point on the curve where  $x = -1$  and  $y = 1$ .

(a)  $\frac{dy}{dx}\bigg|_{(x,y)=(-1,1)} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$

An equation for the tangent line is  $y = \frac{1}{4}(x + 1) + 1$ .

2 :  $\begin{cases} 1 : \text{slope} \\ 1 : \text{equation for tangent line} \end{cases}$

(b)  $3y^2 - x = 0 \Rightarrow x = 3y^2$

So,  $y^3 - xy = 2 \Rightarrow y^3 - (3y^2)(y) = 2 \Rightarrow y = -1$

$(-1)^3 - x(-1) = 2 \Rightarrow x = 3$

The tangent line to the curve is vertical at the point  $(3, -1)$ .

3 :  $\begin{cases} 1 : \text{sets } 3y^2 - x = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{coordinates} \end{cases}$

(c)  $\frac{d^2y}{dx^2} = \frac{(3y^2 - x)\frac{dy}{dx} - y\left(6y\frac{dy}{dx} - 1\right)}{(3y^2 - x)^2}$

$$\frac{d^2y}{dx^2}\bigg|_{(x,y)=(-1,1)} = \frac{(3 \cdot 1^2 - (-1)) \cdot \frac{1}{4} - 1 \cdot \left(6 \cdot 1 \cdot \frac{1}{4} - 1\right)}{(3 \cdot 1^2 - (-1))^2} = \frac{1 - \frac{1}{2}}{16} = \frac{1}{32}$$

4 :  $\begin{cases} 2 : \text{implicit differentiation} \\ 1 : \text{substitution for } \frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$