

6. The function f is twice differentiable. The table gives values of f and its derivative f' at selected values of x .

x	0	2	3	6
$f(x)$	-1	3	8	5
$f'(x)$	-5	4	9	-2

Part A

Find $\lim_{x \rightarrow 2} \frac{f(x)}{x}$, or state that the limit does not exist.

Part B

Let $g(x) = f(f(x))$. Find $g'(2)$. Show the work that leads to your answer.

Part C

Let h be a differentiable function such that $h(0) = 10$ and $h'(x) = f'(3x)$. Find $h(2)$. Show the work that leads to your answer.

Part D

Let k be the function defined by $k(x) = \int_0^x t^2 f(t) dt$.

- i. Find $k'(x)$.
- ii. Find $k''(3)$. Show the work that leads to your answer.

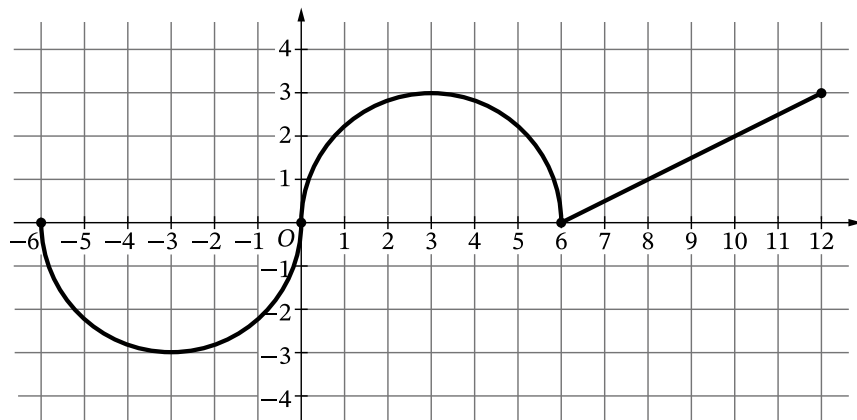
STOP
END OF EXAM

3. A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R , where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

- A. Approximate $R'(1)$ using the average rate of change of R over the interval $0 \leq t \leq 2$. Show the work that leads to your answer. Indicate units of measure.
- B. Must there be a value c , for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.
- C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t) dt$. Show the work that leads to your answer.
- D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

4. The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f , consisting of two semicircles and one line segment, is shown in the figure.

Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t) dt$.

- Find $g'(8)$. Give a reason for your answer.
- Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.
- Find $g(12)$ and $g(0)$. Label your answers.
- Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

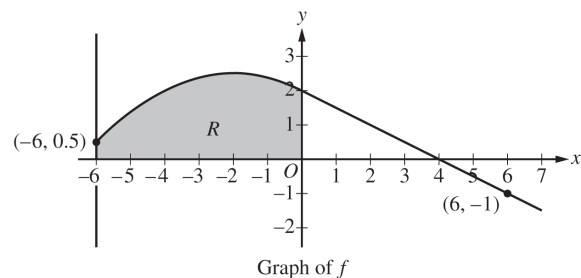
t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

- The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C , where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.
 - Approximate $C'(5)$ using the average rate of change of C over the interval $3 \leq t \leq 7$. Show the work that leads to your answer and include units of measure.
 - Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t) dt$. Interpret the meaning of $\frac{1}{12} \int_0^{12} C(t) dt$ in the context of the problem.
 - For $12 \leq t \leq 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$.

Show the setup for your calculations.
 - For the model defined in part (c), it can be shown that $C''(t) = \frac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate.

Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



4. The graph of the differentiable function f , shown for $-6 \leq x \leq 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \leq x \leq 7$. Let R be the region in the second quadrant bounded by the graph of f , the vertical line $x = -6$, and the x - and y -axes. Region R has area 12.

- (a) The function g is defined by $g(x) = \int_0^x f(t) \, dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.
- (b) For the function g defined in part (a), find all values of x in the interval $0 \leq x \leq 6$ at which the graph of g has a critical point. Give a reason for your answer.
- (c) The function h is defined by $h(x) = \int_{-6}^x f'(t) \, dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

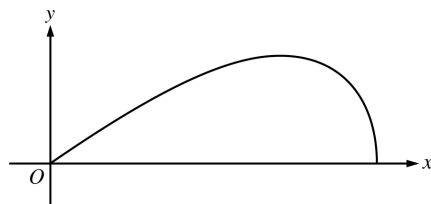
Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x .

- (a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.
- (b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.
- (c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t) \, dt$. Find $m(2)$. Show the work that leads to your answer.
- (d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



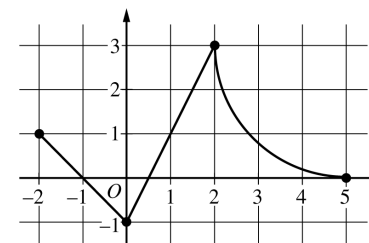
A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c . Each spinning toy is in the shape of the solid generated when such a region is revolved about the x -axis. Both x and y are measured in inches.

- (a) Find the area of the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.
- (b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?
- (c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

CALCULUS AB
SECTION II, Part B
Time—1 hour
Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f

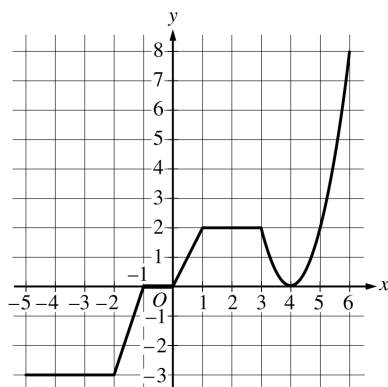
3. The continuous function f is defined on the closed interval $-6 \leq x \leq 5$. The figure above shows a portion of the graph of f , consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f .
- (a) If $\int_{-6}^5 f(x) \, dx = 7$, find the value of $\int_{-6}^{-2} f(x) \, dx$. Show the work that leads to your answer.
- (b) Evaluate $\int_3^5 (2f'(x) + 4) \, dx$.
- (c) The function g is given by $g(x) = \int_{-2}^x f(t) \, dt$. Find the absolute maximum value of g on the interval $-2 \leq x \leq 5$. Justify your answer.
- (d) Find $\lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$.

CALCULUS AB
SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of g

3. The graph of the continuous function g , the derivative of the function f , is shown above. The function g is piecewise linear for $-5 \leq x < 3$, and $g(x) = 2(x - 4)^2$ for $3 \leq x \leq 6$.

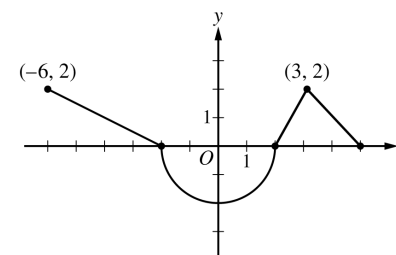
- If $f(1) = 3$, what is the value of $f(-5)$?
- Evaluate $\int_1^6 g(x) \, dx$.
- For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.
- Find the x -coordinate of each point of inflection of the graph of f . Give a reason for your answer.

CALCULUS AB
SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f' , the derivative of f , consists of a semicircle and three line segments, as shown in the figure above.
- Find the values of $f(-6)$ and $f(5)$.
 - On what intervals is f increasing? Justify your answer.
 - Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
 - For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

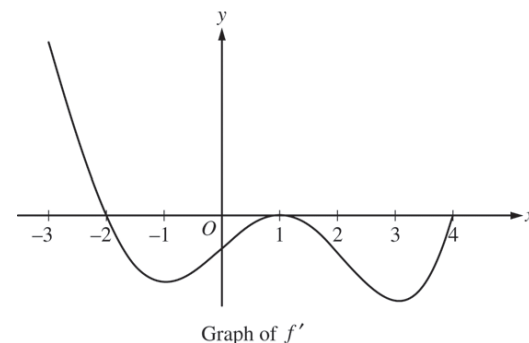
(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x) dx$.

STOP

END OF EXAM



5. The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.
- (a) Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- (c) Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.