

1. Male birds of a certain species arrive at a nesting area over a thirty-day period. The rate at which the male birds arrive at the nesting area at time t days is modeled by a differentiable function M , where $M(t)$ is measured in number of birds per day. Selected values of $M(t)$ are shown in the table.

t (days)	0	5	10	15	20	25	30
$M(t)$ (birds per day)	2	7	16	6	5	2	0

(Note: Your calculator should be in radian mode.)

Part A

Approximate $M'(7.5)$ using the average rate of change of M over the interval $5 \leq t \leq 10$. Show the work that leads to your answer, and indicate units of measure.

Part B

- Use a midpoint Riemann sum with the three subintervals $[0, 10]$, $[10, 20]$, and $[20, 30]$ to approximate $\int_0^{30} M(t) dt$. Show the work that leads to your answer.
- Interpret the meaning of $\int_0^{30} M(t) dt$ in the context of the problem.

Part C

The rate at which female birds of the same species arrive at the same nesting area, in birds per day, is modeled by the function F defined as follows.

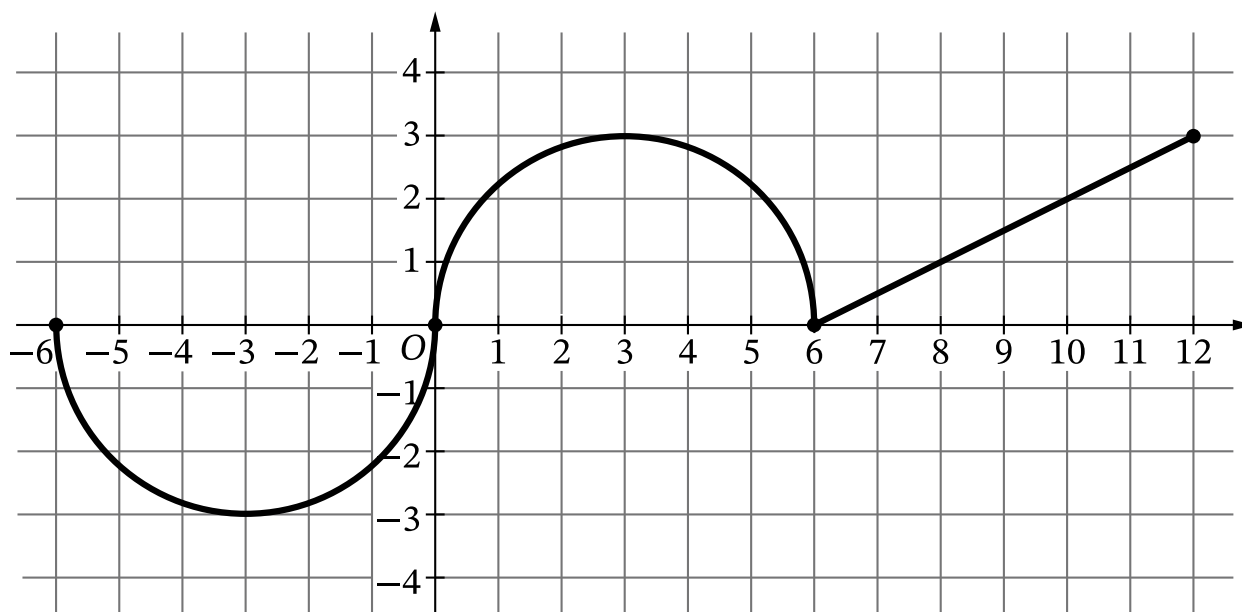
$$F(t) = \begin{cases} 0 & \text{for } 0 \leq t < 15 \\ 18 + 16 \sin\left(\frac{\pi}{20}(t + 15)\right) & \text{for } 15 \leq t \leq 45 \end{cases}$$

How many female birds of this species arrive at the nesting area from $t = 15$ to $t = 45$? Show the setup for your calculations, and round your answer to the nearest integer.

Part D

On the interval $15 < t < 30$, the difference in the rates at which male and female birds of this species arrive at the nesting area can be modeled by the differentiable function $D(t) = M(t) - F(t)$, where F is the function defined in part C. Is there a time t in the interval $15 < t < 20$ when $D(t) = 0$? Justify your answer.

4. The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f , consisting of two semicircles and one line segment, is shown in the figure.

Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t) dt$.

- A. Find $g'(8)$. Give a reason for your answer.
- B. Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.
- C. Find $g(12)$ and $g(0)$. Label your answers.
- D. Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f , where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

- (a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t) dt$ in the context of the problem. Use a right

Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of

$$\int_{60}^{135} f(t) dt.$$

- (b) Must there exist a value of c , for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

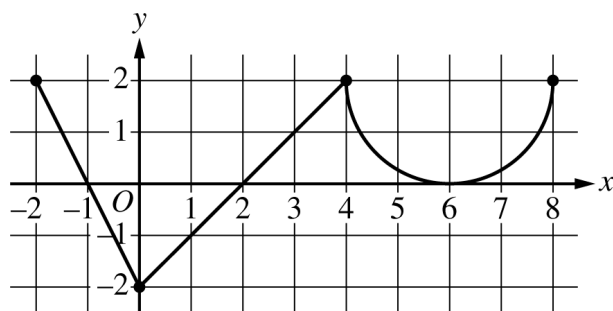
- (c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right) \cos\left(\left(\frac{t}{120}\right)^2\right)$ for

$0 \leq t \leq 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \leq t \leq 150$.

Show the setup for your calculations.

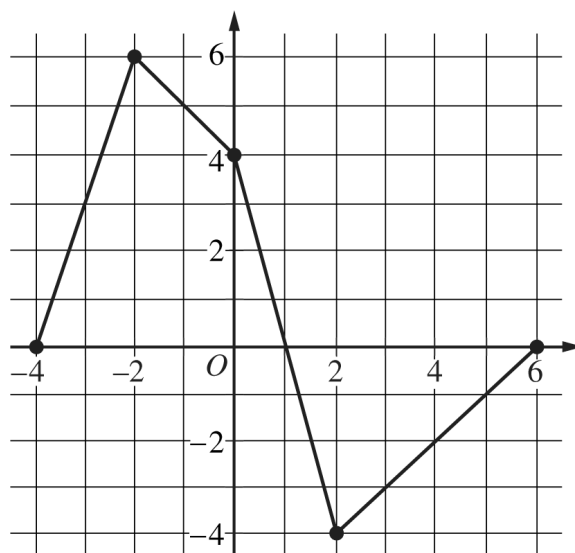
- (d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f' , the derivative of f , consists of two line segments and a semicircle, as shown in the figure.
- (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.
- (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.
- (c) Find the value of $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.
- (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Graph of f

Let f be a continuous function defined on the closed interval $-4 \leq x \leq 6$. The graph of f , consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t) \, dt$.

(a) On what open intervals is the graph of G concave up? Give a reason for your answer.

(b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.

(c) Find $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$.

(d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c , $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

CALCULUS AB
SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \leq t \leq 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

- (a) How many people enter the line for the escalator during the time interval $0 \leq t \leq 300$?
- (b) During the time interval $0 \leq t \leq 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?
- (c) For $t > 300$, what is the first time t that there are no people in line for the escalator?
- (d) For $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.
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2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \leq 12,$$

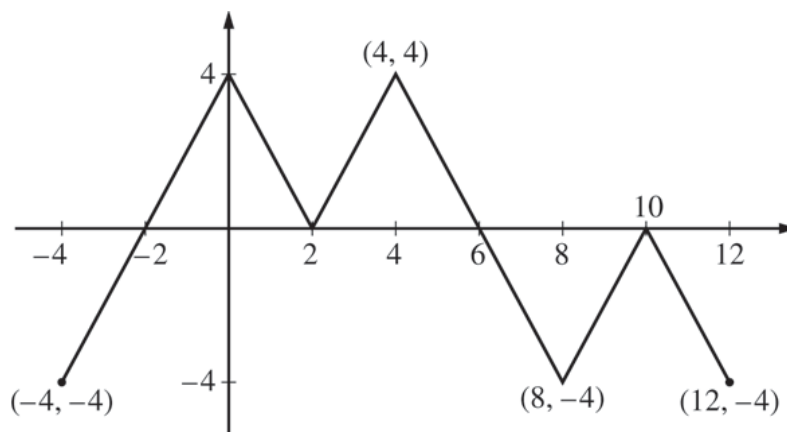
where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln(t^2 + 2t) \text{ for } 3 < t \leq 12,$$

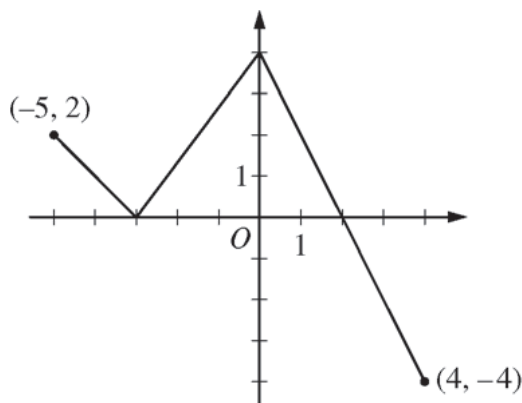
where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

- (a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?
- (b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.
- (c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.
- (d) How many pounds of bananas are on the display table at time $t = 8$?
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END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B**Time—60 minutes****Number of problems—4****No calculator is allowed for these problems.**Graph of f

3. The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_{-4}^x f(t) dt$.
- (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
 - (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
 - (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.
 - (d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.
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CALCULUS AB
SECTION II, Part B**Time—60 minutes****Number of problems—4****No calculator is allowed for these problems.**Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^x f(t) dt$.
- (a) Find $g(3)$.
- (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
- (c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.
- (d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.
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