

Part B (AB or BC): Graphing calculator not allowed**Question 3****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R , where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

Model Solution**Scoring**

- A** Approximate $R'(1)$ using the average rate of change of R over the interval $0 \leq t \leq 2$. Show the work that leads to your answer. Indicate units of measure.

$$R'(1) \approx \frac{R(2) - R(0)}{2 - 0}$$

$$= \frac{100 - 90}{2} = \frac{10}{2} = 5 \text{ words per minute per minute}$$

Answer with setup **Point 1 (P1)**Units **Point 2 (P2)****Scoring Notes for Part A**

- To earn **P1**, a response must present the answer along with the supporting work of a difference and a quotient using values from the table.
 - $\frac{100 - 90}{2 - 0}$, $\frac{10}{2 - 0}$, $\frac{100 - 90}{2}$, or $\frac{R(2) - R(0)}{2 - 0} = 5$ is sufficient to earn **P1**.
 - $\frac{R(2) - R(0)}{2 - 0}$ by itself is not sufficient to earn **P1**.
- P2** is earned for correct units, whether or not they are attached to a numerical value for the average rate of change.
- P2** is also earned for the units “words / minute² .”

B Must there be a value c , for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

R is differentiable implies R is continuous.

Differentiable
implies continuous

Point 3 (P3)

$$R(0) = 90 < 155 < R(10) = 162$$

Answer with
justification

Point 4 (P4)

Therefore, by the Intermediate Value Theorem, there must be a value c , with $0 < c < 10$, such that $R(c) = 155$.

Scoring Notes for Part B

- To earn **P3**, a response must state that R is continuous because R is differentiable (or equivalent). A response that simply states “ R is continuous” without justification does not earn **P3**.
- A response does not need to earn **P3** to be eligible for **P4**.
- To earn **P4**, a response must indicate that $R(0) < 155$ (or $R(2) < 155$ or $R(8) < 155$) and $R(10) > 155$, state that “ R is continuous,” and answer “yes” in some way.
- To earn **P4**, a response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

- C Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t) \, dt$. Show the work that leads to your answer.

$\int_0^{10} R(t) \, dt$ $\approx \frac{R(0) + R(2)}{2}(2 - 0) + \frac{R(2) + R(8)}{2}(8 - 2)$ $+ \frac{R(8) + R(10)}{2}(10 - 8)$	Form of trapezoidal sum	Point 5 (P5)
$= \frac{90 + 100}{2}(2 - 0) + \frac{100 + 150}{2}(8 - 2) + \frac{150 + 162}{2}(10 - 8)$ $= \frac{190}{2}(2) + \frac{250}{2}(6) + \frac{312}{2}(2) = 190 + 750 + 312 = 1252$	Answer with supporting work	Point 6 (P6)

Scoring Notes for Part C

- Read “=” as “ $\approx$ ” for **P5**.
- The form of a trapezoidal sum includes three terms, each of which includes a product of two factors, where one of the factors incorporates the $\frac{1}{2}$ as part of the product. To earn **P5**, at least five of the six factors must be correct. If any of the six factors is incorrect, the response does not earn **P6**. Consider the following examples:
 - $\frac{90 + 100}{2}(2 - 0) + \frac{100 + 150}{2}(8 - 2) + \frac{150 + 162}{2}(10 - 8)$ earns **P5** and is sufficient to earn **P6**.
 - $\frac{190}{2}(2) + \frac{250}{2}(6) + \frac{312}{2}(2)$ earns **P5** and is sufficient to earn **P6**.
 - $\frac{1}{2}((R(0) + R(2))(2) + (R(2) + R(8))(6) + (R(8) + R(10))(2))$ earns **P5** and is eligible for **P6**.
 - $\frac{90 + 100}{2}(2) + \frac{100 + 150}{2}(2) + \frac{150 + 162}{2}(2)$ earns **P5** but is not eligible for **P6**.
(Note that the factor of 2 in the second term of this expression is incorrect.)
- Special case:** A response of $(90 + 100) + (100 + 150)3 + (150 + 162)$ earns both **P5** and **P6**.
- To be eligible for **P6**, a response must have earned **P5**.
Special case: A response of $95 \cdot 2 + 125 \cdot 6 + 156 \cdot 2$ earns **P6** but does not earn **P5**.
- A response of $\frac{90 + 100}{2}(2 - 0) + \frac{100 + 150}{2}(8 - 2) + \frac{150 + 162}{2}(10 - 8)$ or equivalent earns **P6** (i.e., subsequent errors in simplification will not be considered in scoring for **P6**).
- A response of $\frac{(90 \cdot 2 + 100 \cdot 6 + 150 \cdot 2) + (100 \cdot 2 + 150 \cdot 6 + 162 \cdot 2)}{2}$ or equivalent earns both **P5** and **P6**. (Note that the average of the left Riemann sum and right Riemann sum is equivalent to the trapezoidal sum.)
- A completely correct left Riemann sum (e.g., $90 \cdot 2 + 100 \cdot 6 + 150 \cdot 2 = 1080$) or a completely correct right Riemann sum (e.g., $100 \cdot 2 + 150 \cdot 6 + 162 \cdot 2 = 1424$) earns **P5** but does not earn **P6**.

- D** A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

$$\int_0^{10} W(t) \, dt = \int_0^{10} \left(-\frac{3}{10}t^2 + 8t + 100 \right) dt$$

Integrand **Point 7 (P7)**

$$= \left(-\frac{1}{10}t^3 + 4t^2 + 100t \right) \Big|_0^{10}$$

Antiderivative **Point 8 (P8)**

$$= \left(-\frac{1}{10} \cdot 1000 + 4 \cdot 100 + 100 \cdot 10 \right) - \left(-\frac{1}{10} \cdot 0 + 4 \cdot 0 + 100 \cdot 0 \right) \\ = 1300$$

Answer **Point 9 (P9)**

Based on the model, the teacher has read 1300 words by the end of the 10 minutes.

Scoring Notes for Part D

- **P7** is earned for an indefinite or definite integral with integrand $W(t)$, with or without the differential dt .
- **P8** is earned for the correct antiderivative, with or without the constant of integration.
- To be eligible for **P9**, a response must have earned **P8**.
- A response of $\left(-\frac{1}{10} \cdot 1000 + 4 \cdot 100 + 100 \cdot 10 \right) - \left(-\frac{1}{10} \cdot 0 + 4 \cdot 0 + 100 \cdot 0 \right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C , where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

Model Solution**Scoring**

- (a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \leq t \leq 7$. Show the work that leads to your answer and include units of measure.

$$C'(5) \approx \frac{C(7) - C(3)}{7 - 3} = \frac{69 - 85}{4} = -4 \text{ degrees Celsius per minute}$$

Estimate with supporting work **1 point**

Units **1 point**

Scoring notes:

- To earn the first point a response must include a difference and a quotient as the supporting work.
- $\frac{-16}{7-3}$, $\frac{69-85}{7-3}$, or $\frac{69-85}{4}$ is sufficient to earn the first point.
- A response that presents only units without a numerical approximation for $C'(5)$ does not earn the second point.
- The second point is also earned for “degrees per minute” attached to a numerical value.

Total for part (a) 2 points

- (b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t) dt$. Interpret the meaning of $\frac{1}{12} \int_0^{12} C(t) dt$ in the context of the problem.

$\int_0^{12} C(t) dt \approx (3 - 0) \cdot C(0) + (7 - 3) \cdot C(3) + (12 - 7) \cdot C(7)$ $= 3 \cdot 100 + 4 \cdot 85 + 5 \cdot 69 = 985$	Form of left Riemann sum	1 point
	Estimate	1 point
$\frac{1}{12} \int_0^{12} C(t) dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	1 point

Scoring notes:

- Read “=” as “ $\approx$ ” for the first point.
- To earn the first point at least five of the six factors in the Riemann sum must be correct. If any of the six factors is incorrect, the response does not earn the second point.
- A response of $(3 - 0) \cdot C(0) + (7 - 3) \cdot C(3) + (12 - 7) \cdot C(7)$ earns the first point. Values must be pulled from the table to earn the second point.
- A response of $3 \cdot 100 + 4 \cdot 85 + 5 \cdot 69$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A completely correct right Riemann sum (e.g., $3 \cdot 85 + 4 \cdot 69 + 5 \cdot 55$) earns 1 of the first 2 points. An unsupported answer of 806 does not earn either of the first 2 points.
- Units will not affect scoring for the second point.
- To earn the third point the interpretation must include both “average temperature” and the time interval. The response need not include a reference to units. However, if incorrect units are given in the interpretation, the response does not earn the third point.

Total for part (b) 3 points

- (c) For $12 \leq t \leq 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

$C(20) = C(12) + \int_{12}^{20} C'(t) dt$	Integral	1 point
	Uses initial condition	1 point
$= 55 - 14.670812 = 40.329188$	Answer	1 point
The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.		

Scoring notes:

- The first point is earned for a definite integral with integrand $C'(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A linkage error such as $C(20) = \int_{12}^{20} C'(t) dt = 55 - 14.670812$ or $\int_{12}^{20} C'(t) dt = -14.670812 = 40.329188$ earns the first 2 points but does not earn the third point.
- Missing differential (dt):
 - Unambiguous responses of $C(20) = C(12) + \int_{12}^{20} C'(t)$ or $C(20) = 55 + \int_{12}^{20} C'(t)$ earn the first 2 points and are eligible for the third point.
 - Ambiguous responses of $C(20) = \int_{12}^{20} C'(t) + C(12)$ or $C(20) = \int_{12}^{20} C'(t) + 55$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, these responses do not earn the third point.
- The second point is earned for adding $C(12)$ or 55 to a definite integral with a lower limit of 12, either symbolically or numerically.
- The third point is earned for an answer of $55 - 14.671$ or $-14.671 + 55$ with no additional simplification, provided there is some supporting work for these values.
- An answer of just 40.329 with no supporting work does not earn any points.

Total for part (c) 3 points

(d) For the model defined in part (c), it can be shown that $C''(t) = \frac{0.2455e^{0.01t}(100-t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.		
Because $C''(t) > 0$ on the interval $12 < t < 20$, the rate of change in the temperature of the coffee, $C'(t)$, is increasing on this interval. That is, on the interval $12 < t < 20$, the temperature of the coffee is changing at an increasing rate.	Answer with reason	1 point

Scoring notes:

- This point is earned only for a correct answer with a correct reason that references the sign of the second derivative of C .
- A response that provides a reason based on the evaluation of $C''(t)$ at a single point does not earn this point.
- A response that uses ambiguous pronouns (such as “It is positive, so increasing”) does not earn this point.
- A response does not need to reference the interval $12 < t < 20$ to earn the point.

Total for part (d) 1 point**Total for question 1 9 points**

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f , where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

Model Solution**Scoring**

- (a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t) dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t) dt$.

$\int_{60}^{135} f(t) dt$ represents the total number of gallons of gasoline pumped into the gas tank from time $t = 60$ seconds to time $t = 135$ seconds.

Interpretation with units **1 point**

$\int_{60}^{135} f(t) dt$
 $\approx f(90)(90 - 60) + f(120)(120 - 90) + f(135)(135 - 120)$
 $= (0.15)(30) + (0.1)(30) + (0.05)(15) = 8.25$

Form of Riemann sum **1 point**

Answer **1 point**

Scoring notes:

- To earn the first point the response must reference gallons of gasoline added/pumped and the time interval $t = 60$ to $t = 135$.
- To earn the second point at least five of the six factors in the Riemann sum must be correct.
- If there is any error in the Riemann sum, the response does not earn the third point.
- A response of $(0.15)(30) + (0.1)(30) + (0.05)(15)$ earns both the second and third points, unless there is a subsequent error in simplification, in which case the response would earn only the second point.

- A response that presents a correct value with accompanying work that shows the three products in the Riemann sum but does not show all six of the factors and/or the sum process, does not earn the second point but does earn the third point. For example, responses of either $4.5 + 3.0 + 0.75$ or $(0.15)(30)$, $0.1(30)$, $0.05(15) \rightarrow 8.25$ earn the third point but not the second.
- A response of $f(90)(90 - 60) + f(120)(120 - 90) + f(135)(135 - 120) = 8.25$ earns both the second and the third points.
- A response that presents an answer of only 8.25 does not earn either the second or third point.
- A response that provides a completely correct left Riemann sum with accompanying work, $f(60)(30) + f(90)(30) + f(120)(15) = 9$, or $(0.1)(30) + (0.15)(30) + (0.1)(15)$ earns 1 of the last 2 points. A response with any errors or missing factors in a left Riemann sum earns neither of the last 2 points.

Total for part (a) 3 points**(b)** Must there exist a value of c , for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer. f is differentiable. $\Rightarrow f$ is continuous on $[60, 120]$. $f(120) - f(60) = 0$ **1 point**

$$\frac{f(120) - f(60)}{120 - 60} = \frac{0.1 - 0.1}{60} = 0$$

Answer with justification **1 point**

By the Mean Value Theorem, there must exist a c , for $60 < c < 120$, such that $f'(c) = 0$.

Scoring notes:

- To earn the first point a response must present either $f(120) - f(60) = 0$, $0.1 - 0.1 = 0$ (perhaps as the numerator of a quotient), or $f(60) = f(120)$.
- To earn the second point a response must:
 - have earned the first point,
 - state that f is continuous because f is differentiable (or equivalent), and
 - answer “yes” in some way.
- A response may reference either the Mean Value Theorem or Rolle’s Theorem.
- A response that references the Intermediate Value Theorem cannot earn the second point.

Total for part (b) 2 points**(c)** The rate of flow of gasoline, in gallons per second, can also be modeled by

$$g(t) = \left(\frac{t}{500}\right) \cos\left(\left(\frac{t}{120}\right)^2\right) \text{ for } 0 \leq t \leq 150. \text{ Using this model, find the average rate of flow of}$$

gasoline over the time interval $0 \leq t \leq 150$. Show the setup for your calculations.

$$\frac{1}{150 - 0} \int_0^{150} g(t) dt$$

Average value formula **1 point**

$$= 0.0959967$$

Answer **1 point**

The average rate of flow of gasoline, in gallons per second, is 0.096 (or 0.095).

Scoring notes:

- The exact value of $\frac{1}{150} \int_0^{150} g(t) dt$ is $\frac{12}{125} \sin\left(\frac{25}{16}\right)$.
- A response may present the average value formula in single or multiple steps. For example, the following response earns both points: $\int_0^{150} g(t) dt = 14.399504$ so the average rate is 0.0959967.
- A response that presents the average value formula in multiple steps but provides incorrect or incomplete communication (e.g., $\int_0^{150} g(t) dt = \frac{14.399504}{150} = 0.0959967$) earns 1 out of 2 points.
- A response of $\int_0^{150} g(t) dt = 0.0959967$ does not earn either point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{150} \int_0^{150} g(t) dt = 0.149981$ or 0.002618.

Total for part (c) 2 points

- (d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

$g'(140) \approx -0.004908$ $g'(140) = -0.005$ (or -0.004)	$g'(140)$ 1 point
The rate at which gasoline is flowing into the tank is decreasing at a rate of 0.005 (or 0.004) gallon per second per second at time $t = 140$ seconds.	Interpretation 1 point

Scoring notes:

- The exact value of $g'(140)$ is $\frac{1}{500} \cos\left(\frac{49}{36}\right) - \frac{49}{9000} \sin\left(\frac{49}{36}\right)$.
- The value of $g'(140)$ may appear only in the interpretation.
- To be eligible for the second point a response must present some numerical value for $g'(140)$.
- To earn the second point the interpretation must include “the rate of flow of gasoline is changing at a rate of [the declared value of $g'(140)$]” and “at $t = 140$ ” (or equivalent).
- An interpretation of “decreasing at a rate of -0.005 ” or “increasing at a rate of 0.005” does not earn the second point.
- Degree mode: In degree mode, $g'(140) = 0.001997$ or 0.00187.

Total for part (d) 2 points**Total for question 1 9 points**

Part B (AB or BC): Graphing calculator not allowed**Question 4****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r , where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \leq t \leq 12$.

Model Solution**Scoring**

- (a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \leq t \leq 10$. Show the computations that lead to your answer, and indicate units of measure.

$$r''(8.5) \approx \frac{r'(10) - r'(7)}{10 - 7} = \frac{-3.8 - (-4.4)}{10 - 7}$$

$r''(8.5)$ with
supporting work **1 point**

$$= \frac{0.6}{3} = 0.2 \text{ centimeter per day per day}$$

Units **1 point**

Scoring notes:

- To earn the first point the supporting work must include at least a difference and a quotient.
- Simplification is not required to earn the first point. If the numerical value is simplified, it must be correct.
- The second point can be earned with an incorrect approximation for $r''(8.5)$ but cannot be earned without some value for $r''(8.5)$ presented.
- Units may be written in any equivalent form (such as cm/day^2).

Total for part (a) 2 points

(b) Is there a time t , $0 \leq t \leq 3$, for which $r'(t) = -6$? Justify your answer.

$r(t)$ is twice-differentiable. $\Rightarrow r'(t)$ is differentiable. $\Rightarrow r'(t)$ is continuous.	$r'(0) < -6 < r'(3)$	1 point
$r'(0) = -6.1 < -6 < -5.0 = r'(3)$ Therefore, by the Intermediate Value Theorem, there is a time t , $0 < t < 3$, such that $r'(t) = -6$.	Conclusion using Intermediate Value Theorem	1 point

Scoring notes:

- To earn the first point, the response must establish that -6 is between $r'(0)$ and $r'(3)$ (or -6.1 and -5). This statement may be represented symbolically (with or without including one or both endpoints in an inequality) or verbally. A response of “ $r'(t) = -6$ because $r'(0) = -6.1$ and $r'(3) = -5$ ” does not state that -6 is between -6.1 and -5 . Thus this response does not earn the first point.
- To earn the second point:
 - The response must state that $r'(t)$ is continuous because $r'(t)$ is differentiable (or because $r(t)$ is twice differentiable).
 - The response must have earned the first point.
 - Exception: A response of “ $r'(t) = -6$ because $r'(0) = -6.1$ and $r'(3) = -5$ ” does not earn the first point because of imprecise communication but may nonetheless earn the second point if all other criteria for the second point are met.
 - The response must conclude that there is a time t such that $r'(t) = -6$. (A statement of “yes” would be sufficient.)
- To earn the second point, the response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

Total for part (b) 2 points

- (c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of

$$\int_0^{12} r'(t) dt.$$

$\int_0^{12} r'(t) dt \approx 3r'(3) + 4r'(7) + 3r'(10) + 2r'(12)$	Form of right Riemann sum	1 point
$= 3(-5.0) + 4(-4.4) + 3(-3.8) + 2(-3.5)$ $= -51$	Answer	1 point

Scoring notes:

- To earn the first point, at least seven of the eight factors in the Riemann sum must be correct. If there is any error in the Riemann sum, the response does not earn the second point.
- A response of $3(-5.0) + 4(-4.4) + 3(-3.8) + 2(-3.5)$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A response that presents the correct answer, with accompanying work that shows the four products in the Riemann sum (without explicitly showing all of the factors and/or the sum process) does not earn the first point but earns the second point. For example, $-15 + 4(-4.4) + 3(-3.8) + -7$ does not earn the first point but earns the second point. Similarly, $-15, -17.6, -11.4, -7 \rightarrow -51$ does not earn the first point but earns the second point.
- A response that presents the correct answer (-51) with no supporting work earns no points.
- A response that provides a completely correct left Riemann sum and approximation $\int_0^{12} r'(t) dt$ (i.e., $3r'(0) + 4r'(3) + 3r'(7) + 2r'(10) = 3(-6.1) + 4(-5.0) + 3(-4.4) + 2(-3.8) = -59.1$) earns 1 of the 2 points. A response that has any error in a left Riemann sum or evaluation for $\int_0^{12} r'(t) dt$ earns no points.
- Units are not required or read in this part.

Total for part (c) 2 points

- (d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

$\frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$	Product rule	1 point
	Chain rule	1 point
$\frac{dV}{dt}\bigg _{t=3} = \frac{2}{3}\pi(100)(50)(-5) + \frac{1}{3}\pi(100)^2(-2) = -\frac{70,000\pi}{3}$	Answer	1 point
The rate of change of the volume of the sculpture at $t = 3$ is approximately $-\frac{70,000\pi}{3}$ cubic centimeters per day.		

Scoring notes:

- The first 2 points could be earned in either order.
- A response with a completely correct product rule, missing one or both of the correct differentials, earns the product rule point, but not the chain rule point. For example, $\frac{dV}{dt} = \frac{2}{3}\pi r h + \frac{1}{3}\pi r^2$ earns the first point, but not the second.
- A response that treats r or h (but not both) as a constant is eligible for the chain rule point but not the product rule point. For example, $\frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt}$ is correct if h is constant, and thus earns the chain rule point.
- Note: Neither $\frac{dV}{dt} = \frac{2}{3}\pi r \frac{dh}{dt}$ nor $\frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt} \frac{dh}{dt}$ earns any points.
- A response that assumes a functional relationship between r and h (such as $r = 2h$), and uses this relationship to create a function for volume in terms of one variable, is eligible for at most the chain rule point. For example, $r = 2h \rightarrow V = \frac{1}{3}\pi(2h)^2 h = \frac{4}{3}\pi h^3 \rightarrow \frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$ earns only the chain rule point.
- A response that mishandles the constant $\frac{1}{3}\pi$ cannot earn the third point but is eligible for the first 2 points.
- The third point cannot be earned without both of the first 2 points.
- $\frac{dV}{dt} = \frac{2}{3}\pi(100)(50)(-5) + \frac{1}{3}\pi(100)^2(-2)$ earns all 3 points.
- Units are not required or read in this part.

Total for part (d) 3 points

Total for question 4 9 points

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f , where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

Model Solution**Scoring**

- (a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{0.5} = 8$$

Estimate **1 point**

At a distance of $r = 2.25$ centimeters from the center of the petri dish, the density of the bacteria population is increasing at a rate of 8 milligrams per square centimeter per centimeter.

Interpretation with units **1 point**

Scoring notes:

- To earn the first point the response must provide both a difference and a quotient and must explicitly use values of f from the table.
- Simplification of the numerical value is not required to earn the first point. If the numerical value is simplified, it must be correct.
- The interpretation requires all of the following: distance $r = 2.25$, density of bacteria (population) is increasing or changing, at a rate of 8, and units of milligrams per square centimeter per centimeter.
- The second point (interpretation) cannot be earned without a nonzero presented value for $f'(2.25)$.
- To earn the second point the interpretation must be consistent with the presented nonzero value for $f'(2.25)$. In particular, if the presented value for $f'(2.25)$ is negative, the interpretation must include “decreasing at a rate of $|f'(2.25)|$ ” or “changing at a rate of $f'(2.25)$.” The second point cannot be earned for an incorrect statement such as “the bacteria density is decreasing at a rate of $-8 \dots$ ” even for a presented $f'(2.25) = -8$.
- The units ($\text{mg}/\text{cm}^2/\text{cm}$) may be attached to the estimate of $f'(2.25)$ and, if so, do not need to be repeated in the interpretation.
- If units attached to the estimate do not agree with units in the interpretation, read the units in the interpretation.

Total for part (a) 2 points

- (b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression

$2\pi \int_0^4 r f(r) dr$. Approximate the value of $2\pi \int_0^4 r f(r) dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

$2\pi \int_0^4 r f(r) dr \approx 2\pi(1 \cdot f(1) \cdot (1 - 0) + 2 \cdot f(2) \cdot (2 - 1) + 2.5 \cdot f(2.5) \cdot (2.5 - 2) + 4 \cdot f(4) \cdot (4 - 2.5))$	Right Riemann sum setup	1 point
$= 2\pi(1 \cdot 2 \cdot 1 + 2 \cdot 6 \cdot 1 + 2.5 \cdot 10 \cdot 0.5 + 4 \cdot 18 \cdot 1.5)$ $= 269\pi = 845.088$	Approximation	1 point

Scoring notes:

- The presence or absence of 2π has no bearing on earning the first point.
- The first point is earned for a sum of four products with at most one error in any single value among the four products. Multiplication by 1 in any term does not need to be shown, but all other products must be explicitly shown.
- A response of $1 \cdot f(1) \cdot (1 - 0) + 2 \cdot f(2) \cdot (2 - 1) + 2.5 \cdot f(2.5) \cdot (2.5 - 2) + 4 \cdot f(4) \cdot (4 - 2.5)$ earns the first point but not the second.
- A response with any error in the Riemann sum is not eligible for the second point.
- A response that provides a completely correct left Riemann sum for $2\pi \int_0^4 r f(r) dr$ and approximation (91π) earns one of the two points. A response that has any error in a left Riemann sum or evaluation for $2\pi \int_0^4 r f(r) dr$ earns no points.
- A response that provides a completely correct right Riemann sum for $2\pi \int_0^4 r f(r) dr$ and approximation (80π) earns one of the two points. A response that has any error in a right Riemann sum or evaluation for $2\pi \int_0^4 r f(r) dr$ earns no points.
- Simplification of the numerical value is not required to earn the second point. If a numerical value is given, it must be correct to three decimal places.

Total for part (b) 2 points

- (c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

$\frac{d}{dr}(rf(r)) = f(r) + rf'(r)$	Product rule expression for $\frac{d}{dr}(rf(r))$	1 point
Because f is nonnegative and increasing, $\frac{d}{dr}(rf(r)) > 0$ on the interval $0 \leq r \leq 4$. Thus, the integrand $rf(r)$ is strictly increasing. Therefore, the right Riemann sum approximation of $2\pi \int_0^4 rf(r) dr$ is an overestimate.	Answer with explanation	1 point

Scoring notes:

- To earn the second point a response must explain that $rf(r)$ is increasing and, therefore, the right Riemann sum is an overestimate. The second point can be earned without having earned the first point.
- A response that attempts to explain based on a left Riemann sum for $2\pi \int_0^4 rf(r) dr$ from part (b) earns no points.
- A response that attempts to explain based on a right Riemann sum for $2\pi \int_0^4 f(r) dr$ from part (b) earns no points.

Total for part (c) 2 points

- (d) The density of bacteria in the petri dish, for $1 \leq r \leq 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k , $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \leq r \leq 4$?

Average value = $g_{\text{avg}} = \frac{1}{4-1} \int_1^4 g(r) dr$	Definite integral	1 point
$\frac{1}{4-1} \int_1^4 g(r) dr = 9.875795$	Average value	1 point
$g(k) = g_{\text{avg}} \Rightarrow k = 2.497$	Answer	1 point

Scoring notes:

- The first point is earned for a definite integral, with or without $\frac{1}{4-1}$ or $\frac{1}{3}$.
- A response that presents a definite integral with incorrect limits but a correct integrand earns the first point.
- Presentation of the numerical value 9.875795 is not required to earn the second point. This point can be earned by the average value setup: $\frac{1}{3} \int_1^4 g(r) dr$.
- Once earned for the average value setup, the second point cannot be lost. Subsequent errors will result in not earning the third point.
- The third point is earned only for the value $k = 2.497$.
- The third point cannot be earned without the second.
- Special case: A response that does not provide the average value setup but presents an average value of -13.955 is using degree mode on their calculator. This response would not earn the second point but could earn the third point for an answer of $k = 2.5$ (or 2.499).

Total for part (d) 3 points

Total for question 1 9 points

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Scoring Guidelines

2019 SCORING GUIDELINES

Question 1

(a) $\int_0^5 E(t) dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b) $\frac{1}{5-0} \int_0^5 L(t) dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

- (c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$$E(t) - L(t) = 0 \Rightarrow t = 6.20356$$

$E(t) - L(t) > 0$ for $0 \leq t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$. Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

$$3 : \begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$$A(t) = \int_0^t (E(s) - L(s)) ds$$

$$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

$$2 : \begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$$

2019 SCORING GUIDELINES

Question 2

- (a)
- v_P
- is differentiable
- $\Rightarrow v_P$
- is continuous on
- $0.3 \leq t \leq 2.8$
- .

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value c , $0.3 < c < 2.8$, such that

$$v_P'(c) = 0.$$

— OR —

v_P is differentiable $\Rightarrow v_P$ is continuous on $0.3 \leq t \leq 2.8$.

By the Extreme Value Theorem, v_P has a minimum on $[0.3, 2.8]$.

$$v_P(0.3) = 55 > -29 = v_P(1.7) \text{ and } v_P(1.7) = -29 < 55 = v_P(2.8).$$

Thus v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P'(t)$ must equal 0 at this minimum.

$$\begin{aligned} \text{(b)} \quad \int_0^{2.8} v_P(t) dt &\approx 0.3 \left(\frac{v_P(0) + v_P(0.3)}{2} \right) + 1.4 \left(\frac{v_P(0.3) + v_P(1.7)}{2} \right) \\ &\quad + 1.1 \left(\frac{v_P(1.7) + v_P(2.8)}{2} \right) \\ &= 0.3 \left(\frac{0 + 55}{2} \right) + 1.4 \left(\frac{55 + (-29)}{2} \right) + 1.1 \left(\frac{-29 + 55}{2} \right) \\ &= 40.75 \end{aligned}$$

- (c)
- $v_Q(t) = 60 \Rightarrow t = A = 1.866181$
- or
- $t = B = 3.519174$

$$v_Q(t) \geq 60 \text{ for } A \leq t \leq B$$

$$\int_A^B |v_Q(t)| dt = 106.108754$$

The distance traveled by particle Q during the interval $A \leq t \leq B$ is 106.109 (or 106.108) meters.

- (d) From part (b), the position of particle
- P
- at time
- $t = 2.8$
- is

$$x_P(2.8) = \int_0^{2.8} v_P(t) dt \approx 40.75.$$

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t) dt = -90 + 135.937653 = 45.937653$$

Therefore at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

$$2 : \begin{cases} 1 : v_P(2.8) - v_P(0.3) = 0 \\ 1 : \text{justification, using} \\ \text{Mean Value Theorem} \end{cases}$$

— OR —

$$2 : \begin{cases} 1 : v_P(0.3) > v_P(1.7) \\ \text{and } v_P(1.7) < v_P(2.8) \\ 1 : \text{justification, using} \\ \text{Extreme Value Theorem} \end{cases}$$

1 : answer, using trapezoidal sum

$$3 : \begin{cases} 1 : \text{interval} \\ 1 : \text{definite integral} \\ 1 : \text{distance} \end{cases}$$

$$3 : \begin{cases} 1 : \int_0^{2.8} v_Q(t) dt \\ 1 : \text{position of particle } Q \\ 1 : \text{answer} \end{cases}$$

2019 SCORING GUIDELINES

Question 3

$$\begin{aligned}
 \text{(a)} \quad \int_{-6}^5 f(x) \, dx &= \int_{-6}^{-2} f(x) \, dx + \int_{-2}^5 f(x) \, dx \\
 &\Rightarrow 7 = \int_{-6}^{-2} f(x) \, dx + 2 + \left(9 - \frac{9\pi}{4}\right) \\
 &\Rightarrow \int_{-6}^{-2} f(x) \, dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \int_3^5 (2f'(x) + 4) \, dx &= 2 \int_3^5 f'(x) \, dx + \int_3^5 4 \, dx \\
 &= 2(f(5) - f(3)) + 4(5 - 3) \\
 &= 2(0 - (3 - \sqrt{5})) + 8 \\
 &= 2(-3 + \sqrt{5}) + 8 = 2 + 2\sqrt{5}
 \end{aligned}$$

— OR —

$$\begin{aligned}
 \int_3^5 (2f'(x) + 4) \, dx &= [2f(x) + 4x]_{x=3}^{x=5} \\
 &= (2f(5) + 20) - (2f(3) + 12) \\
 &= (2 \cdot 0 + 20) - (2(3 - \sqrt{5}) + 12) \\
 &= 2 + 2\sqrt{5}
 \end{aligned}$$

$$\text{(c)} \quad g'(x) = f(x) = 0 \Rightarrow x = -1, x = \frac{1}{2}, x = 5$$

x	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval $-2 \leq x \leq 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$.

$$\begin{aligned}
 \text{(d)} \quad \lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} &= \frac{10^1 - 3f'(1)}{f(1) - \arctan 1} \\
 &= \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}}
 \end{aligned}$$

$$3 : \begin{cases} 1 : \int_{-6}^5 f(x) \, dx = \int_{-6}^{-2} f(x) \, dx + \int_{-2}^5 f(x) \, dx \\ 1 : \int_{-2}^5 f(x) \, dx \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -1 \text{ as a candidate} \\ 1 : \text{answer with justification} \end{cases}$$

$$1 : \text{answer}$$

2019 SCORING GUIDELINES

Question 4

(a) $V = \pi r^2 h = \pi(1)^2 h = \pi h$
 $\left. \frac{dV}{dt} \right|_{h=4} = \pi \left. \frac{dh}{dt} \right|_{h=4} = \pi \left(-\frac{1}{10} \sqrt{4} \right) = -\frac{\pi}{5}$ cubic feet per second

$$2 : \begin{cases} 1 : \frac{dV}{dt} = \pi \frac{dh}{dt} \\ 1 : \text{answer with units} \end{cases}$$

(b) $\frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10} \sqrt{h} \right) = \frac{1}{200}$
 Because $\frac{d^2 h}{dt^2} = \frac{1}{200} > 0$ for $h > 0$, the rate of change of the height is increasing when the height of the water is 3 feet.

$$3 : \begin{cases} 1 : \frac{d}{dh} \left(-\frac{1}{10} \sqrt{h} \right) = -\frac{1}{20\sqrt{h}} \\ 1 : \frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} \\ 1 : \text{answer with explanation} \end{cases}$$

(c) $\frac{dh}{\sqrt{h}} = -\frac{1}{10} dt$
 $\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10} dt$
 $2\sqrt{h} = -\frac{1}{10} t + C$
 $2\sqrt{5} = -\frac{1}{10} \cdot 0 + C \Rightarrow C = 2\sqrt{5}$
 $2\sqrt{h} = -\frac{1}{10} t + 2\sqrt{5}$
 $h(t) = \left(-\frac{1}{20} t + \sqrt{5} \right)^2$

$$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : h(t) \end{cases}$$

Note: 0/4 if no separation of variables

Note: max 2/4 [1-1-0-0] if no constant of integration

2019 SCORING GUIDELINES

Question 5

$$\begin{aligned}
 \text{(a)} \quad \int_0^2 (h(x) - g(x)) \, dx &= \int_0^2 \left((6 - 2(x-1)^2) - \left(-2 + 3\cos\left(\frac{\pi}{2}x\right) \right) \right) dx \\
 &= \left[6x - \frac{2}{3}(x-1)^3 - \left(-2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right) \right) \right]_{x=0}^{x=2} \\
 &= \left(\left(12 - \frac{2}{3} \right) - (-4 + 0) \right) - \left(\left(0 + \frac{2}{3} \right) - (0 + 0) \right) \\
 &= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}
 \end{aligned}$$

The area of R is $\frac{44}{3}$.

$$\begin{aligned}
 \text{(b)} \quad \int_0^2 A(x) \, dx &= \int_0^2 \frac{1}{x+3} \, dx \\
 &= [\ln(x+3)]_{x=0}^{x=2} = \ln 5 - \ln 3
 \end{aligned}$$

The volume of the solid is $\ln 5 - \ln 3$.

$$\text{(c)} \quad \pi \int_0^2 ((6 - g(x))^2 - (6 - h(x))^2) \, dx$$

4 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative of } 3\cos\left(\frac{\pi}{2}x\right) \\ 1 : \text{antiderivative of} \\ \quad \text{remaining terms} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : \text{limits and constant} \\ 1 : \text{form of integrand} \\ 1 : \text{integrand} \end{cases}$

2019 SCORING GUIDELINES

Question 6

(a) $h'(2) = \frac{2}{3}$

(b) $a'(x) = 9x^2h(x) + 3x^3h'(x)$

$$a'(2) = 9 \cdot 2^2 h(2) + 3 \cdot 2^3 h'(2) = 36 \cdot 4 + 24 \cdot \frac{2}{3} = 160$$

(c) Because h is differentiable, h is continuous, so $\lim_{x \rightarrow 2} h(x) = h(2) = 4$.

Also, $\lim_{x \rightarrow 2} h(x) = \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3}$, so $\lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$.

Because $\lim_{x \rightarrow 2} (x^2 - 4) = 0$, we must also have $\lim_{x \rightarrow 2} (1 - (f(x))^3) = 0$.

Thus $\lim_{x \rightarrow 2} f(x) = 1$.

Because f is differentiable, f is continuous, so $f(2) = \lim_{x \rightarrow 2} f(x) = 1$.

Also, because f is twice differentiable, f' is continuous, so

$\lim_{x \rightarrow 2} f'(x) = f'(2)$ exists.

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = \lim_{x \rightarrow 2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{4}{-3(1)^2 \cdot f'(2)} = 4.$$

Thus $f'(2) = -\frac{1}{3}$.

(d) Because g and h are differentiable, g and h are continuous, so

$$\lim_{x \rightarrow 2} g(x) = g(2) = 4 \text{ and } \lim_{x \rightarrow 2} h(x) = h(2) = 4.$$

Because $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$, it follows from the squeeze theorem that $\lim_{x \rightarrow 2} k(x) = 4$.

Also, $4 = g(2) \leq k(2) \leq h(2) = 4$, so $k(2) = 4$.

Thus k is continuous at $x = 2$.

1 : answer

3 : $\begin{cases} 1 : \text{form of product rule} \\ 1 : a'(x) \\ 1 : a'(2) \end{cases}$

4 : $\begin{cases} 1 : \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4 \\ 1 : f(2) \\ 1 : \text{L'Hospital's Rule} \\ 1 : f'(2) \end{cases}$

1 : continuous with justification

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AP Calculus AB

Scoring Guidelines

2018 SCORING GUIDELINES

Question 1

(a) $\int_0^{300} r(t) dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $20 + \int_0^{300} (r(t) - 0.7) dt = 20 + \int_0^{300} r(t) dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

2 : $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or 414.285) seconds.}$$

1 : answer

(d) The total number of people in line at time t , $0 \leq t \leq 300$, is modeled by

$$20 + \int_0^t r(x) dx - 0.7t.$$

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

4 : $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

2018 SCORING GUIDELINES

Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118 .

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760 .

3 : $\begin{cases} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{3.5} v(t) dt = 2.844$ (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral $\int_0^{3.5} v(t) dt$ is the displacement of the particle over the time interval $0 \leq t \leq 3.5$.

The integral $\int_0^{3.5} |v(t)| dt$ is the total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

3 : $\begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{cases}$

(d) $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

2 : $\begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$

2018 SCORING GUIDELINES

Question 3

$$\begin{aligned} \text{(a)} \quad f(-5) &= f(1) + \int_1^{-5} g(x) \, dx = f(1) - \int_{-5}^1 g(x) \, dx \\ &= 3 - \left(-9 - \frac{3}{2} + 1 \right) = 3 - \left(-\frac{19}{2} \right) = \frac{25}{2} \end{aligned}$$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b)} \quad \int_1^6 g(x) \, dx &= \int_1^3 g(x) \, dx + \int_3^6 g(x) \, dx \\ &= \int_1^3 2 \, dx + \int_3^6 2(x-4)^2 \, dx \\ &= 4 + \left[\frac{2}{3}(x-4)^3 \right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3} \right) = 10 \end{aligned}$$

3 : $\begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f''(x) = g'(x)$ is increasing on those intervals.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

2018 SCORING GUIDELINES

Question 4

$$(a) \quad H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$$(b) \quad \frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$$

Because H is differentiable on $3 \leq t \leq 5$, H is continuous on $3 \leq t \leq 5$.

By the Mean Value Theorem, there exists a value c , $3 < c < 5$, such that $H'(c) = 2$.

$$(c) \quad \text{The average height of the tree over the time interval } 2 \leq t \leq 10 \text{ is given by } \frac{1}{10 - 2} \int_2^{10} H(t) \, dt.$$

$$\begin{aligned} \frac{1}{8} \int_2^{10} H(t) \, dt &\approx \frac{1}{8} \left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3 \right) \\ &= \frac{1}{8} (65.75) = \frac{263}{32} \end{aligned}$$

The average height of the tree over the time interval $2 \leq t \leq 10$ is $\frac{263}{32}$ meters.

$$(d) \quad G(x) = 50 \Rightarrow x = 1$$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$$

$$\left. \frac{d}{dt}(G(x)) \right|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\frac{3}{4}$ meter per year.

$$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$$

$$2 : \begin{cases} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$$

$$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$$

$$3 : \begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$$

Note: max 1/3 [1-0] if no chain rule

2018 SCORING GUIDELINES

Question 5

- (a) The average rate of change of
- f
- on the interval
- $0 \leq x \leq \pi$
- is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

1 : answer

- (b)
- $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

2 : $\begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

- (c)
- $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

3 : $\begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$

The absolute minimum value of f on $0 \leq x \leq 2\pi$ is $-\frac{1}{\sqrt{2}}e^{5\pi/4}$.

- (d)
- $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

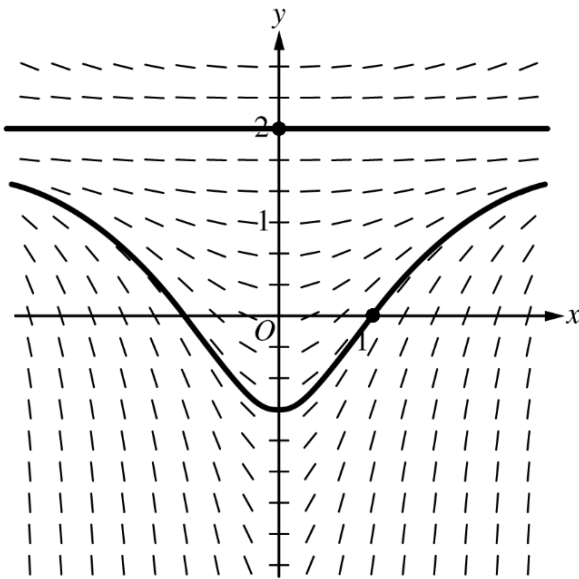
3 : $\begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \text{and limits equal 0} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

2018 SCORING GUIDELINES

Question 6

(a)



$$2 : \begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

$$(b) \left. \frac{dy}{dx} \right|_{(x, y)=(1, 0)} = \frac{4}{3}$$

An equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ is $y = \frac{4}{3}(x - 1)$.

$$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$$

$$2 : \begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$$

$$(c) \frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$$

$$\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$$

$$y = 2 - \frac{6}{x^2 + 2}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: this solution is valid for $-\infty < x < \infty$.

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2016 SCORING GUIDELINES

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

- (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

$$(a) \quad R'(2) \approx \frac{R(3) - R(1)}{3 - 1} = \frac{950 - 1190}{3 - 1} = -120 \text{ liters/hr}^2$$

$$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$$

(b) The total amount of water removed is given by $\int_0^8 R(t) dt$.

$$\begin{aligned} \int_0^8 R(t) dt &\approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters} \end{aligned}$$

This is an overestimate since R is a decreasing function.

$$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$$

$$\begin{aligned} (c) \quad \text{Total} &\approx 50000 + \int_0^8 W(t) dt - 8050 \\ &= 50000 + 7836.195325 - 8050 \approx 49786 \text{ liters} \end{aligned}$$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$$

(d) $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

$$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$$

Therefore, the Intermediate Value Theorem guarantees at least one time t , $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

For this value of t , the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

2016 SCORING GUIDELINES

Question 2

For $t \geq 0$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

- At time $t = 4$, is the particle speeding up or slowing down?
- Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.
- Find the position of the particle at time $t = 0$.
- Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

(a) $v(4) = 2.978716 > 0$
 $v'(4) = -1.164000 < 0$

The particle is slowing down since the velocity and acceleration have different signs.

(b) $v(t) = 0 \Rightarrow t = 2.707468$

$v(t)$ changes from positive to negative at $t = 2.707$.
 Therefore, the particle changes direction at this time.

(c) $x(0) = x(4) + \int_4^0 v(t) dt$
 $= 2 + (-5.815027) = -3.815$

(d) Distance $= \int_0^3 |v(t)| dt = 5.301$

2 : conclusion with reason

2 : $\begin{cases} 1 : t = 2.707 \\ 1 : \text{justification} \end{cases}$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

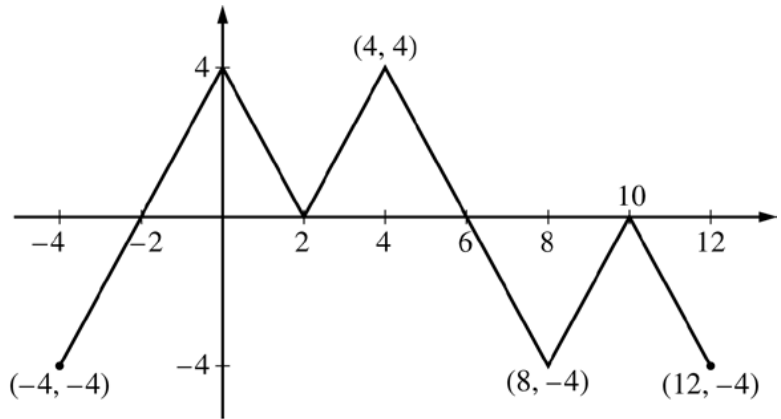
2016 SCORING GUIDELINES

Question 3

The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by

$$g(x) = \int_2^x f(t) \, dt.$$

- (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
- (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
- (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.
- (d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

Graph of f

- (a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \leq 0$ for $8 \leq x \leq 12$.
- (b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \leq x \leq 4$ and decreasing for $4 \leq x \leq 8$.
- (c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
-4	-4
-2	-8
6	8
12	-4

On the interval $-4 \leq x \leq 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

- (d) $g(x) \leq 0$ for $-4 \leq x \leq 2$ and $10 \leq x \leq 12$.

1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

1 : answer with justification

1 : answer with justification

4 : $\left\{ \begin{array}{l} 1 : \text{considers } x = -2 \text{ and } x = 6 \\ \quad \text{as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{array} \right.$

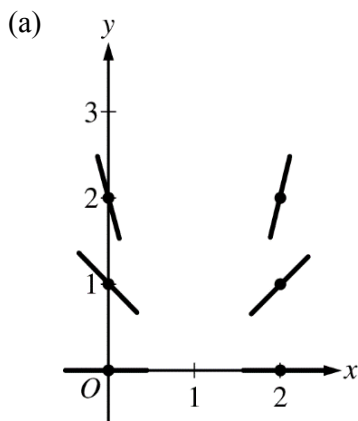
2 : intervals

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Question 4

Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.



2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = \frac{3^2}{2-1} = 9$

2 : $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

An equation for the tangent line is $y = 9(x - 2) + 3$.

$$f(2.1) \approx 9(2.1 - 2) + 3 = 3.9$$

(c) $\frac{1}{y^2} dy = \frac{1}{x-1} dx$

$$\int \frac{1}{y^2} dy = \int \frac{1}{x-1} dx$$

$$-\frac{1}{y} = \ln|x-1| + C$$

$$-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$$

$$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$$

$$y = \frac{1}{\frac{1}{3} - \ln(x-1)}$$

5 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and} \\ \quad \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

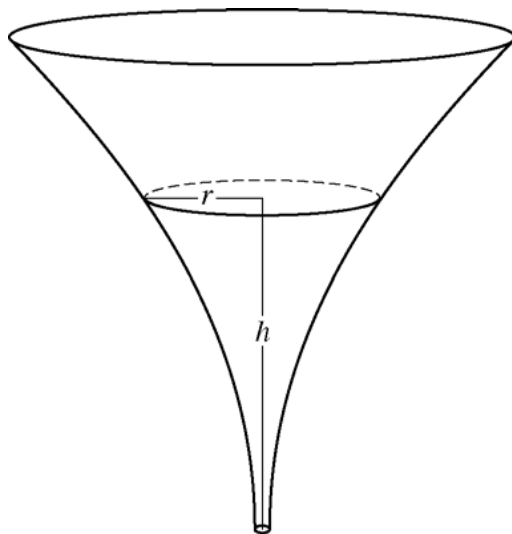
Note: max 3/5 [1-2-0-0] if no constant of integration

Note: 0/5 if no separation of variables

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

2016 SCORING GUIDELINES

Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

- (a) Find the average value of the radius of the funnel.
- (b) Find the volume of the funnel.
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

$$\begin{aligned} \text{(a) Average radius} &= \frac{1}{10} \int_0^{10} \frac{1}{20}(3 + h^2) dh = \frac{1}{200} \left[3h + \frac{h^3}{3} \right]_0^{10} \\ &= \frac{1}{200} \left(\left(30 + \frac{1000}{3} \right) - 0 \right) = \frac{109}{60} \text{ in} \end{aligned}$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_0^{10} \left(\left(\frac{1}{20} \right) (3 + h^2) \right)^2 dh = \frac{\pi}{400} \int_0^{10} (9 + 6h^2 + h^4) dh \\ &= \frac{\pi}{400} \left[9h + 2h^3 + \frac{h^5}{5} \right]_0^{10} \\ &= \frac{\pi}{400} \left(\left(90 + 2000 + \frac{100000}{5} \right) - 0 \right) = \frac{2209\pi}{40} \text{ in}^3 \end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(c) } \frac{dr}{dt} &= \frac{1}{20}(2h) \frac{dh}{dt} \\ -\frac{1}{5} &= \frac{3}{10} \frac{dh}{dt} \\ \frac{dh}{dt} &= -\frac{1}{5} \cdot \frac{10}{3} = -\frac{2}{3} \text{ in/sec} \end{aligned}$$

3 : $\begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

2016 SCORING GUIDELINES

Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x) dx$.

$$\begin{aligned} \text{(a)} \quad k(3) &= f(g(3)) = f(6) = 4 \\ k'(3) &= f'(g(3)) \cdot g'(3) = f'(6) \cdot 2 = 5 \cdot 2 = 10 \end{aligned}$$

An equation for the tangent line is $y = 10(x - 3) + 4$.

$$\begin{aligned} \text{(b)} \quad h'(1) &= \frac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{(f(1))^2} \\ &= \frac{(-6) \cdot 8 - 2 \cdot 3}{(-6)^2} = \frac{-54}{36} = -\frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \int_1^3 f''(2x) dx &= \frac{1}{2} [f'(2x)]_1^3 = \frac{1}{2} [f'(6) - f'(2)] \\ &= \frac{1}{2} [5 - (-2)] = \frac{7}{2} \end{aligned}$$

$$3 : \begin{cases} 2 : \text{slope at } x = 3 \\ 1 : \text{equation for tangent line} \end{cases}$$

$$3 : \begin{cases} 2 : \text{expression for } h'(1) \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

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2015 SCORING GUIDELINES

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
- (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
- (c) At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t) dt = 76.570$

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$
 Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

2 : $\begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t , $0 \leq t \leq 8$, is $30 + \int_0^t [R(x) - D(x)] dx$.

3 : $\begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

$R(t) - D(t) = 0 \Rightarrow t = 0, 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

(d) $30 + \int_0^w [R(t) - D(t)] dt = 50$

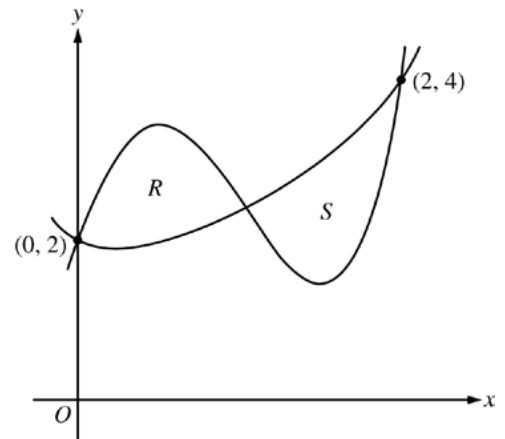
2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

2015 SCORING GUIDELINES

Question 2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

- (a) Find the sum of the areas of regions R and S .
- (b) Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.
- (c) Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.



- (a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$.

$$\begin{aligned}\text{Area} &= \int_0^A [g(x) - f(x)] dx + \int_A^2 [f(x) - g(x)] dx \\ &= 0.997427 + 1.006919 = 2.004\end{aligned}$$

- (b) $\text{Volume} = \int_A^2 [f(x) - g(x)]^2 dx = 1.283$

- (c) $h(x) = f(x) - g(x)$
 $h'(x) = f'(x) - g'(x)$
 $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

4 : $\begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

- (a) Use the data in the table to estimate the value of $v'(16)$.
- (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.
- Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.
- (c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.
- (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

(a) $v'(16) \approx \frac{240 - 200}{20 - 12} = 5 \text{ meters/min}^2$

1 : approximation

- (b) $\int_0^{40} |v(t)| dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \leq t \leq 40$ minutes.

3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned} \int_0^{40} |v(t)| dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters} \end{aligned}$$

- (c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.
 $B'(5) = 3(25) - 12(5) = 15 \text{ meters/min}^2$

2 : $\begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

(d) Avg vel = $\frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300) dt$

$$\begin{aligned} &= \frac{1}{10} \left[\frac{t^4}{4} - 2t^3 + 300t \right]_0^{10} \\ &= \frac{1}{10} \left[\frac{10000}{4} - 2000 + 3000 \right] = 350 \text{ meters/min} \end{aligned}$$

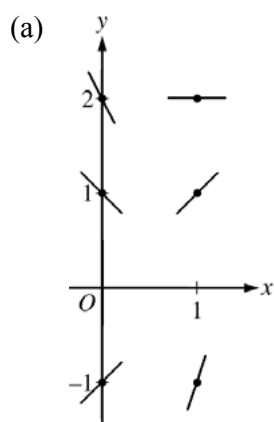
3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.
- (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.



$$2 : \begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$$

(b) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$

In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$.
Therefore, all solution curves are concave up in Quadrant II.

$$2 : \begin{cases} 1 : \frac{d^2y}{dx^2} \\ 1 : \text{concave up with reason} \end{cases}$$

(c) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = 2(2) - 3 = 1 \neq 0$

Therefore, f has neither a relative minimum nor a relative maximum at $x = 2$.

$$2 : \begin{cases} 1 : \text{considers } \left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} \\ 1 : \text{conclusion with justification} \end{cases}$$

(d) $y = mx + b \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(mx + b) = m$

$$2x - y = m$$

$$2x - (mx + b) = m$$

$$(2 - m)x - (m + b) = 0$$

$$2 - m = 0 \Rightarrow m = 2$$

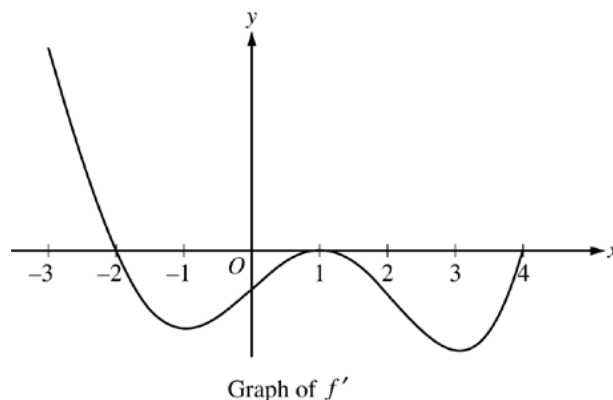
$$b = -m \Rightarrow b = -2$$

$$3 : \begin{cases} 1 : \frac{d}{dx}(mx + b) = m \\ 1 : 2x - y = m \\ 1 : \text{answer} \end{cases}$$

2015 SCORING GUIDELINES

Question 5

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



- (a) Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- (c) Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

- (a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$.
 $f'(x)$ changes from positive to negative at $x = -2$.
 Therefore, f has a relative maximum at $x = -2$.

2 : $\begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$

- (b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

- (c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points.

2 : $\begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$

The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.

- (d) $f(x) = 3 + \int_1^x f'(t) dt$

$$f(4) = 3 + \int_1^4 f'(t) dt = 3 + (-12) = -9$$

$$\begin{aligned} f(-2) &= 3 + \int_1^{-2} f'(t) dt = 3 - \int_{-2}^1 f'(t) dt \\ &= 3 - (-9) = 12 \end{aligned}$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$

2015 SCORING GUIDELINES

Question 6

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

- (a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.
- (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.
- (c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

$$(a) \left. \frac{dy}{dx} \right|_{(x,y)=(-1,1)} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$$

An equation for the tangent line is $y = \frac{1}{4}(x + 1) + 1$.

$$(b) 3y^2 - x = 0 \Rightarrow x = 3y^2$$

$$\text{So, } y^3 - xy = 2 \Rightarrow y^3 - (3y^2)(y) = 2 \Rightarrow y = -1$$

$$(-1)^3 - x(-1) = 2 \Rightarrow x = 3$$

The tangent line to the curve is vertical at the point $(3, -1)$.

$$(c) \frac{d^2y}{dx^2} = \frac{(3y^2 - x)\frac{dy}{dx} - y\left(6y\frac{dy}{dx} - 1\right)}{(3y^2 - x)^2}$$

$$\left. \frac{d^2y}{dx^2} \right|_{(x,y)=(-1,1)} = \frac{(3 \cdot 1^2 - (-1)) \cdot \frac{1}{4} - 1 \cdot \left(6 \cdot 1 \cdot \frac{1}{4} - 1\right)}{(3 \cdot 1^2 - (-1))^2}$$

$$= \frac{1 - \frac{1}{2}}{16} = \frac{1}{32}$$

2 : $\begin{cases} 1 : \text{slope} \\ 1 : \text{equation for tangent line} \end{cases}$

3 : $\begin{cases} 1 : \text{sets } 3y^2 - x = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{coordinates} \end{cases}$

4 : $\begin{cases} 2 : \text{implicit differentiation} \\ 1 : \text{substitution for } \frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$

AP[®] Calculus AB

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2014 SCORING GUIDELINES

Question 1

Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

- Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.
- Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.
- Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.
- For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$$(a) \quad \frac{A(30) - A(0)}{30 - 0} = -0.197 \text{ (or } -0.196) \text{ lbs/day}$$

1 : answer with units

$$(b) \quad A'(15) = -0.164 \text{ (or } -0.163)$$

The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.

2 : $\begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$

$$(c) \quad A(t) = \frac{1}{30} \int_0^{30} A(t) dt \Rightarrow t = 12.415 \text{ (or } 12.414)$$

2 : $\begin{cases} 1 : \frac{1}{30} \int_0^{30} A(t) dt \\ 1 : \text{answer} \end{cases}$

$$(d) \quad L(t) = A(30) + A'(30) \cdot (t - 30)$$

$$A'(30) = -0.055976$$

$$A(30) = 0.782928$$

$$L(t) = 0.5 \Rightarrow t = 35.054$$

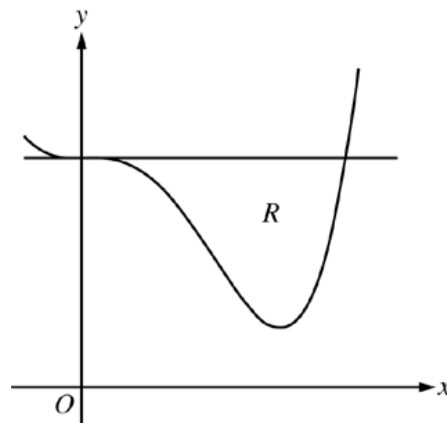
4 : $\begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

2014 SCORING GUIDELINES

Question 2

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

- (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
- (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in R . Find the volume of the solid.
- (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k .



(a) $f(x) = 4 \Rightarrow x = 0, 2.3$

$$\begin{aligned} \text{Volume} &= \pi \int_0^{2.3} \left[(4 + 2)^2 - (f(x) + 2)^2 \right] dx \\ &= 98.868 \text{ (or } 98.867) \end{aligned}$$

$$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$$

(b) $\text{Volume} = \int_0^{2.3} \frac{1}{2} (4 - f(x))^2 dx$
 $= 3.574 \text{ (or } 3.573)$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(c) $\int_0^k (4 - f(x)) dx = \int_k^{2.3} (4 - f(x)) dx$

$$2 : \begin{cases} 1 : \text{area of one region} \\ 1 : \text{equation} \end{cases}$$

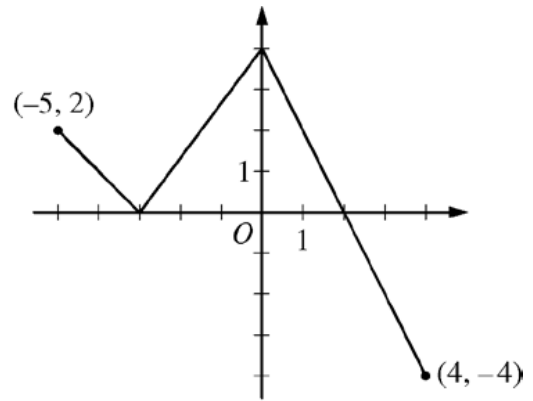
2014 SCORING GUIDELINES

Question 3

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above.

Let g be the function defined by $g(x) = \int_{-3}^x f(t) dt$.

- (a) Find $g(3)$.
- (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
- (c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.
- (d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

Graph of f

(a) $g(3) = \int_{-3}^3 f(t) dt = 6 + 4 - 1 = 9$

1 : answer

(b) $g'(x) = f(x)$

The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $h'(x) = \frac{5xg'(x) - g(x)5}{(5x)^2} = \frac{5xg'(x) - 5g(x)}{25x^2}$

3 : $\begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} h'(3) &= \frac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2} \\ &= \frac{15(-2) - 5(9)}{225} = \frac{-75}{225} = -\frac{1}{3} \end{aligned}$$

(d) $p'(x) = f'(x^2 - x)(2x - 1)$

3 : $\begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$

$$p'(-1) = f'(2)(-3) = (-2)(-3) = 6$$

2014 SCORING GUIDELINES

Question 4

Train A runs back and forth on an east-west section of railroad track. Train A 's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

- (a) Find the average acceleration of train A over the interval $2 \leq t \leq 8$.
- (b) Do the data in the table support the conclusion that train A 's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.
- (c) At time $t = 2$, train A 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A , in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.
- (d) A second train, train B , travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

(a) average accel = $\frac{v_A(8) - v_A(2)}{8 - 2} = \frac{-120 - 100}{6} = -\frac{110}{3} \text{ m/min}^2$

(b) v_A is differentiable $\Rightarrow v_A$ is continuous
 $v_A(8) = -120 < -100 < 40 = v_A(5)$

Therefore, by the Intermediate Value Theorem, there is a time t , $5 < t < 8$, such that $v_A(t) = -100$.

(c) $s_A(12) = s_A(2) + \int_2^{12} v_A(t) dt = 300 + \int_2^{12} v_A(t) dt$
 $\int_2^{12} v_A(t) dt \approx 3 \cdot \frac{100 + 40}{2} + 3 \cdot \frac{40 - 120}{2} + 4 \cdot \frac{-120 - 150}{2}$
 $= -450$

$$s_A(12) \approx 300 - 450 = -150$$

The position of Train A at time $t = 12$ minutes is approximately 150 meters west of Origin Station.

- (d) Let x be train A 's position, y train B 's position, and z the distance between train A and train B .

$$z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$x = 300, y = 400 \Rightarrow z = 500$$

$$v_B(2) = -20 + 120 + 25 = 125$$

$$500 \frac{dz}{dt} = (300)(100) + (400)(125)$$

$$\frac{dz}{dt} = \frac{80000}{500} = 160 \text{ meters per minute}$$

1 : average acceleration

2 : $\begin{cases} 1 : v_A(8) < -100 < v_A(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

3 : $\begin{cases} 1 : \text{position expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{position at time } t = 12 \end{cases}$

3 : $\begin{cases} 2 : \text{implicit differentiation of} \\ \text{distance relationship} \\ 1 : \text{answer} \end{cases}$

2014 SCORING GUIDELINES

Question 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.

- (a) Find the x -coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.
- (b) Explain why there must be a value c , for $-1 < c < 1$, such that $f''(c) = 0$.
- (c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.
- (d) Evaluate $\int_{-2}^3 f'(g(x))g'(x) dx$.

- (a) $x = 1$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a relative minimum at $x = 1$.

- (b) f' is differentiable $\Rightarrow f'$ is continuous on the interval $-1 \leq x \leq 1$

$$\frac{f'(1) - f'(-1)}{1 - (-1)} = \frac{0 - 0}{2} = 0$$

Therefore, by the Mean Value Theorem, there is at least one value c , $-1 < c < 1$, such that $f''(c) = 0$.

- (c) $h'(x) = \frac{1}{f(x)} \cdot f'(x)$

$$h'(3) = \frac{1}{f(3)} \cdot f'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}$$

- (d) $\int_{-2}^3 f'(g(x))g'(x) dx = \left[f(g(x)) \right]_{x=-2}^{x=3}$
 $= f(g(3)) - f(g(-2))$
 $= f(1) - f(-1)$
 $= 2 - 8 = -6$

1 : answer with justification

$$2 : \begin{cases} 1 : f'(1) - f'(-1) = 0 \\ 1 : \text{explanation, using Mean Value Theorem} \end{cases}$$

$$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 2 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$$

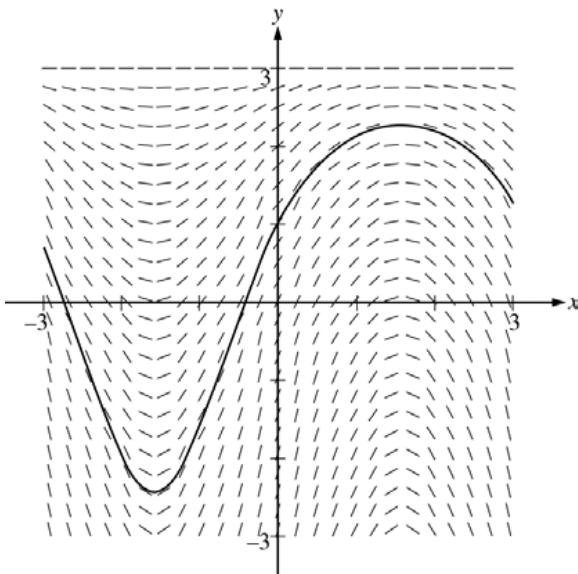
2014 SCORING GUIDELINES

Question 6

Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

- (a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.
- (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.
- (c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

(a) 1 : solution curve



(b) $\left. \frac{dy}{dx} \right|_{(x,y)=(0,1)} = 2 \cos 0 = 2$

An equation for the tangent line is $y = 2x + 1$.

$$f(0.2) \approx 2(0.2) + 1 = 1.4$$

2 : $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{dy}{dx} = (3 - y)\cos x$

$$\int \frac{dy}{3 - y} = \int \cos x \, dx$$

$$-\ln|3 - y| = \sin x + C$$

$$-\ln 2 = \sin 0 + C \Rightarrow C = -\ln 2$$

$$-\ln|3 - y| = \sin x - \ln 2$$

Because $y(0) = 1$, $y < 3$, so $|3 - y| = 3 - y$

$$3 - y = 2e^{-\sin x}$$

$$y = 3 - 2e^{-\sin x}$$

6 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables