

**2019****AP<sup>®</sup>** **CollegeBoard**

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# **AP<sup>®</sup> Calculus AB**

## **Scoring Guidelines**

## 2019 SCORING GUIDELINES

## Question 1

(a)  $\int_0^5 E(t) dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b)  $\frac{1}{5-0} \int_0^5 L(t) dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

- (c) The rate of change in the number of fish in the lake at time  $t$  is given by  $E(t) - L(t)$ .

$$E(t) - L(t) = 0 \Rightarrow t = 6.20356$$

$E(t) - L(t) > 0$  for  $0 \leq t < 6.20356$ , and  $E(t) - L(t) < 0$  for  $6.20356 < t \leq 8$ . Therefore the greatest number of fish in the lake is at time  $t = 6.204$  (or 6.203).

$$3 : \begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

— OR —

Let  $A(t)$  be the change in the number of fish in the lake from midnight to  $t$  hours after midnight.

$$A(t) = \int_0^t (E(s) - L(s)) ds$$

$$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$$

| $t$ | $A(t)$    |
|-----|-----------|
| 0   | 0         |
| $C$ | 135.01492 |
| 8   | 80.91998  |

Therefore the greatest number of fish in the lake is at time  $t = 6.204$  (or 6.203).

(d)  $E'(5) - L'(5) = -10.7228 < 0$

Because  $E'(5) - L'(5) < 0$ , the rate of change in the number of fish is decreasing at time  $t = 5$ .

$$2 : \begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$$

## 2019 SCORING GUIDELINES

## Question 2

- (a)
- $v_P$
- is differentiable
- $\Rightarrow v_P$
- is continuous on
- $0.3 \leq t \leq 2.8$
- .

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value  $c$ ,  $0.3 < c < 2.8$ , such that

$$v_P'(c) = 0.$$

— OR —

$v_P$  is differentiable  $\Rightarrow v_P$  is continuous on  $0.3 \leq t \leq 2.8$ .

By the Extreme Value Theorem,  $v_P$  has a minimum on  $[0.3, 2.8]$ .

$$v_P(0.3) = 55 > -29 = v_P(1.7) \text{ and } v_P(1.7) = -29 < 55 = v_P(2.8).$$

Thus  $v_P$  has a minimum on the interval  $(0.3, 2.8)$ .

Because  $v_P$  is differentiable,  $v_P'(t)$  must equal 0 at this minimum.

$$\begin{aligned} \text{(b)} \quad \int_0^{2.8} v_P(t) dt &\approx 0.3 \left( \frac{v_P(0) + v_P(0.3)}{2} \right) + 1.4 \left( \frac{v_P(0.3) + v_P(1.7)}{2} \right) \\ &\quad + 1.1 \left( \frac{v_P(1.7) + v_P(2.8)}{2} \right) \\ &= 0.3 \left( \frac{0 + 55}{2} \right) + 1.4 \left( \frac{55 + (-29)}{2} \right) + 1.1 \left( \frac{-29 + 55}{2} \right) \\ &= 40.75 \end{aligned}$$

- (c)
- $v_Q(t) = 60 \Rightarrow t = A = 1.866181$
- or
- $t = B = 3.519174$

$$v_Q(t) \geq 60 \text{ for } A \leq t \leq B$$

$$\int_A^B |v_Q(t)| dt = 106.108754$$

The distance traveled by particle  $Q$  during the interval  $A \leq t \leq B$  is 106.109 (or 106.108) meters.

- (d) From part (b), the position of particle
- $P$
- at time
- $t = 2.8$
- is

$$x_P(2.8) = \int_0^{2.8} v_P(t) dt \approx 40.75.$$

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t) dt = -90 + 135.937653 = 45.937653$$

Therefore at time  $t = 2.8$ , particles  $P$  and  $Q$  are approximately  $45.937653 - 40.75 = 5.188$  (or 5.187) meters apart.

$$2 : \begin{cases} 1 : v_P(2.8) - v_P(0.3) = 0 \\ 1 : \text{justification, using} \\ \text{Mean Value Theorem} \end{cases}$$

— OR —

$$2 : \begin{cases} 1 : v_P(0.3) > v_P(1.7) \\ \text{and } v_P(1.7) < v_P(2.8) \\ 1 : \text{justification, using} \\ \text{Extreme Value Theorem} \end{cases}$$

1 : answer, using trapezoidal sum

$$3 : \begin{cases} 1 : \text{interval} \\ 1 : \text{definite integral} \\ 1 : \text{distance} \end{cases}$$

$$3 : \begin{cases} 1 : \int_0^{2.8} v_Q(t) dt \\ 1 : \text{position of particle } Q \\ 1 : \text{answer} \end{cases}$$

## 2019 SCORING GUIDELINES

## Question 3

$$\begin{aligned}
 \text{(a)} \quad \int_{-6}^5 f(x) \, dx &= \int_{-6}^{-2} f(x) \, dx + \int_{-2}^5 f(x) \, dx \\
 &\Rightarrow 7 = \int_{-6}^{-2} f(x) \, dx + 2 + \left(9 - \frac{9\pi}{4}\right) \\
 &\Rightarrow \int_{-6}^{-2} f(x) \, dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \int_3^5 (2f'(x) + 4) \, dx &= 2 \int_3^5 f'(x) \, dx + \int_3^5 4 \, dx \\
 &= 2(f(5) - f(3)) + 4(5 - 3) \\
 &= 2(0 - (3 - \sqrt{5})) + 8 \\
 &= 2(-3 + \sqrt{5}) + 8 = 2 + 2\sqrt{5}
 \end{aligned}$$

— OR —

$$\begin{aligned}
 \int_3^5 (2f'(x) + 4) \, dx &= [2f(x) + 4x]_{x=3}^{x=5} \\
 &= (2f(5) + 20) - (2f(3) + 12) \\
 &= (2 \cdot 0 + 20) - (2(3 - \sqrt{5}) + 12) \\
 &= 2 + 2\sqrt{5}
 \end{aligned}$$

$$\text{(c)} \quad g'(x) = f(x) = 0 \Rightarrow x = -1, x = \frac{1}{2}, x = 5$$

| $x$           | $g(x)$                |
|---------------|-----------------------|
| -2            | 0                     |
| -1            | $\frac{1}{2}$         |
| $\frac{1}{2}$ | $-\frac{1}{4}$        |
| 5             | $11 - \frac{9\pi}{4}$ |

On the interval  $-2 \leq x \leq 5$ , the absolute maximum value of  $g$  is  $g(5) = 11 - \frac{9\pi}{4}$ .

$$\begin{aligned}
 \text{(d)} \quad \lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} &= \frac{10^1 - 3f'(1)}{f(1) - \arctan 1} \\
 &= \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}}
 \end{aligned}$$

$$3 : \begin{cases} 1 : \int_{-6}^5 f(x) \, dx = \int_{-6}^{-2} f(x) \, dx + \int_{-2}^5 f(x) \, dx \\ 1 : \int_{-2}^5 f(x) \, dx \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -1 \text{ as a candidate} \\ 1 : \text{answer with justification} \end{cases}$$

$$1 : \text{answer}$$

## 2019 SCORING GUIDELINES

## Question 4

(a)  $V = \pi r^2 h = \pi(1)^2 h = \pi h$   
 $\left. \frac{dV}{dt} \right|_{h=4} = \pi \left. \frac{dh}{dt} \right|_{h=4} = \pi \left( -\frac{1}{10} \sqrt{4} \right) = -\frac{\pi}{5}$  cubic feet per second

$$2 : \begin{cases} 1 : \frac{dV}{dt} = \pi \frac{dh}{dt} \\ 1 : \text{answer with units} \end{cases}$$

(b)  $\frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left( -\frac{1}{10} \sqrt{h} \right) = \frac{1}{200}$   
 Because  $\frac{d^2 h}{dt^2} = \frac{1}{200} > 0$  for  $h > 0$ , the rate of change of the height is increasing when the height of the water is 3 feet.

$$3 : \begin{cases} 1 : \frac{d}{dh} \left( -\frac{1}{10} \sqrt{h} \right) = -\frac{1}{20\sqrt{h}} \\ 1 : \frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} \\ 1 : \text{answer with explanation} \end{cases}$$

(c)  $\frac{dh}{\sqrt{h}} = -\frac{1}{10} dt$   
 $\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10} dt$   
 $2\sqrt{h} = -\frac{1}{10} t + C$   
 $2\sqrt{5} = -\frac{1}{10} \cdot 0 + C \Rightarrow C = 2\sqrt{5}$   
 $2\sqrt{h} = -\frac{1}{10} t + 2\sqrt{5}$   
 $h(t) = \left( -\frac{1}{20} t + \sqrt{5} \right)^2$

$$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : h(t) \end{cases}$$

Note: 0/4 if no separation of variables

Note: max 2/4 [1-1-0-0] if no constant of integration

## 2019 SCORING GUIDELINES

## Question 5

$$\begin{aligned}
 \text{(a)} \quad \int_0^2 (h(x) - g(x)) \, dx &= \int_0^2 \left( (6 - 2(x-1)^2) - \left( -2 + 3\cos\left(\frac{\pi}{2}x\right) \right) \right) dx \\
 &= \left[ 6x - \frac{2}{3}(x-1)^3 - \left( -2x + \frac{6}{\pi}\sin\left(\frac{\pi}{2}x\right) \right) \right]_{x=0}^{x=2} \\
 &= \left( \left( 12 - \frac{2}{3} \right) - (-4 + 0) \right) - \left( \left( 0 + \frac{2}{3} \right) - (0 + 0) \right) \\
 &= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3}
 \end{aligned}$$

The area of  $R$  is  $\frac{44}{3}$ .

$$\begin{aligned}
 \text{(b)} \quad \int_0^2 A(x) \, dx &= \int_0^2 \frac{1}{x+3} \, dx \\
 &= [\ln(x+3)]_{x=0}^{x=2} = \ln 5 - \ln 3
 \end{aligned}$$

The volume of the solid is  $\ln 5 - \ln 3$ .

$$\text{(c)} \quad \pi \int_0^2 \left( (6 - g(x))^2 - (6 - h(x))^2 \right) dx$$

4 :  $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative of } 3\cos\left(\frac{\pi}{2}x\right) \\ 1 : \text{antiderivative of} \\ \quad \text{remaining terms} \\ 1 : \text{answer} \end{cases}$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

3 :  $\begin{cases} 1 : \text{limits and constant} \\ 1 : \text{form of integrand} \\ 1 : \text{integrand} \end{cases}$

## 2019 SCORING GUIDELINES

## Question 6

(a)  $h'(2) = \frac{2}{3}$

1 : answer

(b)  $a'(x) = 9x^2h(x) + 3x^3h'(x)$

$$a'(2) = 9 \cdot 2^2 h(2) + 3 \cdot 2^3 h'(2) = 36 \cdot 4 + 24 \cdot \frac{2}{3} = 160$$

3 :  $\begin{cases} 1 : \text{form of product rule} \\ 1 : a'(x) \\ 1 : a'(2) \end{cases}$

(c) Because  $h$  is differentiable,  $h$  is continuous, so  $\lim_{x \rightarrow 2} h(x) = h(2) = 4$ .

Also,  $\lim_{x \rightarrow 2} h(x) = \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3}$ , so  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$ .

Because  $\lim_{x \rightarrow 2} (x^2 - 4) = 0$ , we must also have  $\lim_{x \rightarrow 2} (1 - (f(x))^3) = 0$ .

Thus  $\lim_{x \rightarrow 2} f(x) = 1$ .

4 :  $\begin{cases} 1 : \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4 \\ 1 : f(2) \\ 1 : \text{L'Hospital's Rule} \\ 1 : f'(2) \end{cases}$

Because  $f$  is differentiable,  $f$  is continuous, so  $f(2) = \lim_{x \rightarrow 2} f(x) = 1$ .

Also, because  $f$  is twice differentiable,  $f'$  is continuous, so

$\lim_{x \rightarrow 2} f'(x) = f'(2)$  exists.

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = \lim_{x \rightarrow 2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{4}{-3(1)^2 \cdot f'(2)} = 4.$$

Thus  $f'(2) = -\frac{1}{3}$ .

(d) Because  $g$  and  $h$  are differentiable,  $g$  and  $h$  are continuous, so

$$\lim_{x \rightarrow 2} g(x) = g(2) = 4 \text{ and } \lim_{x \rightarrow 2} h(x) = h(2) = 4.$$

Because  $g(x) \leq k(x) \leq h(x)$  for  $1 < x < 3$ , it follows from the squeeze theorem that  $\lim_{x \rightarrow 2} k(x) = 4$ .

Also,  $4 = g(2) \leq k(2) \leq h(2) = 4$ , so  $k(2) = 4$ .

Thus  $k$  is continuous at  $x = 2$ .

1 : continuous with justification