

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C , where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

Model Solution**Scoring**

- (a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \leq t \leq 7$. Show the work that leads to your answer and include units of measure.

$C'(5) \approx \frac{C(7) - C(3)}{7 - 3} = \frac{69 - 85}{4} = -4$ degrees Celsius per minute	Estimate with supporting work	1 point
	Units	1 point

Scoring notes:

- To earn the first point a response must include a difference and a quotient as the supporting work.
- $\frac{-16}{7-3}$, $\frac{69-85}{7-3}$, or $\frac{69-85}{4}$ is sufficient to earn the first point.
- A response that presents only units without a numerical approximation for $C'(5)$ does not earn the second point.
- The second point is also earned for “degrees per minute” attached to a numerical value.

Total for part (a) 2 points

- (b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t) dt$. Interpret the meaning of $\frac{1}{12} \int_0^{12} C(t) dt$ in the context of the problem.

$\int_0^{12} C(t) dt \approx (3-0) \cdot C(0) + (7-3) \cdot C(3) + (12-7) \cdot C(7)$ $= 3 \cdot 100 + 4 \cdot 85 + 5 \cdot 69 = 985$	Form of left Riemann sum	1 point
	Estimate	1 point
$\frac{1}{12} \int_0^{12} C(t) dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	1 point

Scoring notes:

- Read “=” as “ $\approx$ ” for the first point.
- To earn the first point at least five of the six factors in the Riemann sum must be correct. If any of the six factors is incorrect, the response does not earn the second point.
- A response of $(3-0) \cdot C(0) + (7-3) \cdot C(3) + (12-7) \cdot C(7)$ earns the first point. Values must be pulled from the table to earn the second point.
- A response of $3 \cdot 100 + 4 \cdot 85 + 5 \cdot 69$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A completely correct right Riemann sum (e.g., $3 \cdot 85 + 4 \cdot 69 + 5 \cdot 55$) earns 1 of the first 2 points. An unsupported answer of 806 does not earn either of the first 2 points.
- Units will not affect scoring for the second point.
- To earn the third point the interpretation must include both “average temperature” and the time interval. The response need not include a reference to units. However, if incorrect units are given in the interpretation, the response does not earn the third point.

Total for part (b) 3 points

- (c) For $12 \leq t \leq 20$, the rate of change of the temperature of the coffee is modeled by

$C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

$C(20) = C(12) + \int_{12}^{20} C'(t) dt$	Integral	1 point
$= 55 - 14.670812 = 40.329188$	Uses initial condition	1 point
	Answer	1 point
The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.		

Scoring notes:

- The first point is earned for a definite integral with integrand $C'(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A linkage error such as $C(20) = \int_{12}^{20} C'(t) dt = 55 - 14.670812$ or $\int_{12}^{20} C'(t) dt = -14.670812 = 40.329188$ earns the first 2 points but does not earn the third point.
- Missing differential (dt):
 - Unambiguous responses of $C(20) = C(12) + \int_{12}^{20} C'(t)$ or $C(20) = 55 + \int_{12}^{20} C'(t)$ earn the first 2 points and are eligible for the third point.
 - Ambiguous responses of $C(20) = \int_{12}^{20} C'(t) + C(12)$ or $C(20) = \int_{12}^{20} C'(t) + 55$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, these responses do not earn the third point.
- The second point is earned for adding $C(12)$ or 55 to a definite integral with a lower limit of 12, either symbolically or numerically.
- The third point is earned for an answer of $55 - 14.671$ or $-14.671 + 55$ with no additional simplification, provided there is some supporting work for these values.
- An answer of just 40.329 with no supporting work does not earn any points.

Total for part (c) 3 points

- (d) For the model defined in part (c), it can be shown that $C''(t) = \frac{0.2455e^{0.01t}(100-t)}{t^2}$. For

$12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

Because $C''(t) > 0$ on the interval $12 < t < 20$, the rate of change in the temperature of the coffee, $C'(t)$, is increasing on this interval.	Answer with reason	1 point
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That is, on the interval $12 < t < 20$, the temperature of the coffee is changing at an increasing rate.

Scoring notes:

- This point is earned only for a correct answer with a correct reason that references the sign of the second derivative of C .
- A response that provides a reason based on the evaluation of $C''(t)$ at a single point does not earn this point.
- A response that uses ambiguous pronouns (such as “It is positive, so increasing”) does not earn this point.
- A response does not need to reference the interval $12 < t < 20$ to earn the point.

Total for part (d) 1 point**Total for question 1 9 points****Part B (AB or BC): Graphing calculator not allowed****Question 3****9 points****General Scoring Notes**

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A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t , where $M(t)$ is measured in degrees Celsius ($^{\circ}\text{C}$) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C . It can be shown that $M(t) < 40$ for all values of t .

Model Solution	Scoring
(a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.	1 point

Scoring notes:

- The solution curve must pass through the point $(0, 5)$, extend reasonably close to the left and right edges of the rectangle, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.
- The solution curve must lie entirely below the horizontal line segments at $M = 40$.

Total for question 3 9 points

- (b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

$\left. \frac{dM}{dt} \right _{t=0} = \frac{1}{4}(40 - 5) = \frac{35}{4}$	$\left. \frac{dM}{dt} \right _{t=0}$	1 point
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The tangent line equation is $y = 5 + \frac{35}{4}(t - 0)$.

$$M(2) \approx 5 + \frac{35}{4} \cdot 2 = 22.5$$

The temperature of the milk at time $t = 2$ minutes is approximately 22.5° Celsius.

Scoring notes:

- The value of the slope may appear in a tangent line equation or approximation.
- A response of $5 + \frac{35}{4} \cdot 2$ is the minimal response to earn both points.
- A response of $\frac{1}{4}(40 - 5)$ earns the first point. If there are any subsequent errors in simplification, the response does not earn the second point.
- In order to earn the second point the response must present an approximation found by using a tangent line that:
 - passes through the point $(0, 5)$ and
 - has slope $\frac{35}{4}$ or a nonzero slope that is declared to be the value of $\frac{dM}{dt}$.
- An unsupported approximation does not earn the second point.
- The approximation need not be simplified, but the response does not earn the second point if the approximation is simplified incorrectly.

Total for part (b) 2 points

- (c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M . Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

$\frac{d^2M}{dt^2} = -\frac{1}{4} \frac{dM}{dt} = -\frac{1}{4} \left(\frac{1}{4}(40 - M) \right) = -\frac{1}{16}(40 - M)$	$\frac{d^2M}{dt^2}$	1 point
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Because $M(t) < 40$, $\frac{d^2M}{dt^2} < 0$, so the graph of M is concave down. Therefore, the tangent line approximation of $M(2)$ is an overestimate.

Overestimate with reason **1 point**

Scoring notes:

- The first point is earned for either $\frac{d^2M}{dt^2} = -\frac{1}{4} \left(\frac{1}{4}(40 - M) \right)$ or $\frac{d^2M}{dt^2} = -\frac{1}{16}(40 - M)$ (or equivalent). A response that presents any subsequent simplification error does not earn the second point.
- A response that presents an expression for $\frac{d^2M}{dt^2}$ in terms of $\frac{dM}{dt}$ but fails to continue to an expression in terms of M (i.e., $\frac{d^2M}{dt^2} = -\frac{1}{4} \frac{dM}{dt}$) does not earn the first point but is eligible for the second point.
- If the response presents an expression for $\frac{d^2M}{dt^2}$ that is incorrect, the response is eligible for the second point only if the expression is a nonconstant linear function that is negative for $5 < M < 40$.
 - Special case: A response that presents $\frac{d^2M}{dt^2} = \frac{1}{16}(40 - M)$ does not earn the first point but is eligible to earn the second point for a consistent answer and reason.
- To earn the second point a response must include $\frac{d^2M}{dt^2} < 0$, or $\frac{dM}{dt}$ is decreasing, or the graph of M is concave down, as well as the conclusion that the approximation is an overestimate.
- A response that presents an argument based on $\frac{d^2M}{dt^2}$ or concavity at a single point does not earn the second point.

Total for part (c) 2 points

- (d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

$\frac{dM}{40 - M} = \frac{1}{4} dt$	Separates variables	1 point
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$$\int \frac{dM}{40 - M} = \int \frac{1}{4} dt$$

$-\ln 40 - M = \frac{1}{4}t + C$	Finds antiderivatives	1 point
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$-\ln 40 - 5 = 0 + C \Rightarrow C = -\ln 35$ $M(t) < 40 \Rightarrow 40 - M > 0 \Rightarrow 40 - M = 40 - M$	Constant of integration and uses initial condition	1 point
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$$-\ln(40 - M) = \frac{1}{4}t - \ln 35$$

$$\ln(40 - M) = -\frac{1}{4}t + \ln 35$$

$$40 - M = 35e^{-t/4}$$

$$M = 40 - 35e^{-t/4}$$

Solves for M

1 point

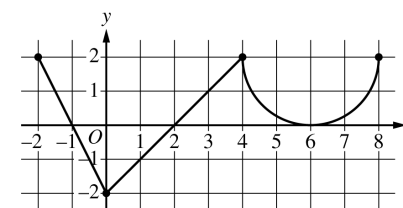
Scoring notes:

- A response with no separation of variables earns 0 out of 4 points.
- A response that presents an antiderivative of $-\ln(40 - M)$ without absolute value symbols is eligible for all 4 points.
- A response with no constant of integration can earn at most the first 2 points.
- A response is eligible for the third point only if it has earned the first 2 points.
 - Special Case: A response that presents $+\ln(40 - M) = \frac{t}{4} + C$ (or equivalent) does not earn the second point, is eligible for the third point, but not eligible for the fourth.
- An eligible response earns the third point by correctly including the constant of integration in an equation and substituting 0 for t and 5 for M .
- A response is eligible for the fourth point only if it has earned the first 3 points.
- A response earns the fourth point only for an answer of $M = 40 - 35e^{-t/4}$ or equivalent.

Total for part (d) 4 points**Total for question 3 9 points****Part B (AB or BC): Graphing calculator not allowed****Question 4****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

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Graph of f'

The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f' , the derivative of f , consists of two line segments and a semicircle, as shown in the figure.

Model Solution**Scoring**

- (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

$$f'(x) > 0 \text{ on } (2, 6) \text{ and } f'(x) > 0 \text{ on } (6, 8).$$

Answer with reason **1 point**

$f'(x)$ does not change sign at $x = 6$, so there is neither a relative maximum nor a relative minimum at this location.

Scoring notes:

- A response that declares $f'(x)$ does not change sign at $x = 6$, so neither, is sufficient to earn the point.
- A response does not have to present intervals on which $f'(x)$ is positive or negative, but if any are given, they must be correct. Any presented intervals may include none, one, or both endpoints.
- A response that declares $f'(x) > 0$ before and after $x = 6$ does not earn the point.

Total for part (a) 1 point

- (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

The graph of f is concave down on $(-2, 0)$ and $(4, 6)$ because f' is decreasing on these intervals.	Intervals	1 point
	Reason	1 point

Scoring notes:

- The first point is earned only by an answer of $(-2, 0)$ and $(4, 6)$, or these intervals including one or both endpoints.
- A response must earn the first point to be eligible for second point.
- To earn the second point a response must correctly discuss the behavior of f' or the slopes of f' .
- Special case: A response that presents exactly one of the two correct intervals with a correct reason earns 1 out of 2 points.

Total for part (b) 2 points

- (c) Find the value of $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

Because f is differentiable at $x = 2$, f is continuous at $x = 2$, so $\lim_{x \rightarrow 2} f(x) = f(2) = 1$.	Limits of numerator and denominator	1 point
$\lim_{x \rightarrow 2} (6f(x) - 3x) = 6 \cdot 1 - 3 \cdot 2 = 0$		
$\lim_{x \rightarrow 2} (x^2 - 5x + 6) = 0$		

Because $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$ is of indeterminate form $\frac{0}{0}$, L'Hospital's Rule can be applied.	Uses L'Hospital's Rule	1 point
	Answer	1 point

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6} = \lim_{x \rightarrow 2} \frac{6f'(x) - 3}{2x - 5} = \frac{6 \cdot 0 - 3}{2 \cdot 2 - 5} = 3.$$

Scoring notes:

- The first point is earned by the presentation of two separate limits for the numerator and denominator.
- A response that presents a limit explicitly equal to $\frac{0}{0}$ does not earn the first point.
- The second point is earned by applying L'Hospital's Rule, that is, by presenting at least one correct derivative in the limit of a ratio of derivatives.
- The third point is earned for the correct answer with supporting work.

Total for part (c) 3 points

- (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

$f'(x) = 0 \Rightarrow x = -1, x = 2, x = 6$	Considers $f'(x) = 0$	1 point
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The function f is continuous on $[-2, 8]$, so the candidates for the location of an absolute minimum for f are $x = -2$, $x = -1$, $x = 2$, $x = 6$, and $x = 8$.	Justification	1 point
	Answer	1 point

x	$f(x)$
-2	3
-1	4
2	1
6	$7 - \pi$
8	$11 - 2\pi$

The absolute minimum value of f is $f(2) = 1$.

Scoring notes:

- To earn the first point a response must state $f' = 0$ or equivalent. Listing the zeros of f' is not sufficient.
- A response that presents any error in evaluating f at any critical point or endpoint will not earn the justification point.
- A response need not present the value of $f(-1)$ provided $x = -1$ is eliminated because it is the location of a local maximum. A response need not present the value of $f(6)$ provided $x = 6$ is eliminated by reference to part (a) or eliminated through analysis.
- A response need not present the value of $f(8)$ provided there is a presentation that argues $f'(x) \geq 0$ for $x > 2$ and, therefore, $f(8) > f(2)$.
- A response that does not consider both endpoints does not earn the justification point.
- The answer point is earned only for indicating that the minimum value is 1. It is not earned for noting that the minimum occurs at $x = 2$.

Total for part (d) 3 points

Total for question 4 9 points

Part B (AB): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x .

Model Solution	Scoring
(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer. $h'(x) = f'(g(x)) \cdot g'(x)$ $h'(7) = f'(g(7)) \cdot g'(7)$ $= f'(0) \cdot 8 = \frac{3}{2} \cdot 8 = 12$	Chain rule 1 point Answer 1 point
Scoring notes: - The first point is earned for either $h'(x) = f'(g(x)) \cdot g'(x)$ or $h'(7) = f'(g(7)) \cdot g'(7)$. - If the first point is earned, the second point is earned only for an answer of 12 (or equivalent). - If the first point is not earned, the second point can be earned only for a response of either $f'(0) \cdot 8 = 12$ or $\frac{3}{2} \cdot 8$. - A response of 12 with no supporting work does not earn either point.	
Total for part (a) 2 points	

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer. $k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$	Product or chain rule 1 point
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$k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$ $= 2 \cdot 4 \cdot 3 \cdot (-3) + 4^2 \cdot 2 = -72 + 32 = -40$	$k''(4)$ 1 point
The graph of k is concave down at the point where $x = 4$ because $k''(4) < 0$ and k'' is continuous.	Answer with reason 1 point

Scoring notes:

- The first point is earned for either $k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$.
- The first point is also earned by any of the following incorrect expressions, each of which has a single error in the application of the product rule or the chain rule:
 - $2f(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
 - $2f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
 - $f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
 - $2f(x) \cdot f'(x) \cdot g'(x)$ or $2f(4) \cdot f'(4) \cdot g'(4)$
 - Note: A response that presents one of these expressions cannot earn the second point.
- To earn the second point a response must correctly find $k''(4) = -40$ (or equivalent) with supporting work.
- The third point is earned for an answer and reason that are consistent with any declared nonzero value of $k''(4)$.

Total for part (b) 3 points

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t) dt$. Find $m(2)$. Show the work that leads to your answer. $m(2) = 5 \cdot 8 + \int_0^2 f'(t) dt = 40 + (f(2) - f(0))$ $= 40 + (7 - 10) = 37$	Answer with supporting work 1 point
Scoring notes: - The point is earned only for an answer of 37 (or equivalent) with supporting work equivalent to $5 \cdot 8 + (f(2) - f(0))$, $40 + (f(2) - f(0))$, $5 \cdot 8 + (7 - 10)$, or $40 + (7 - 10)$. - An answer of 37 with no supporting work does not earn the point.	
Total for part (c) 1 point	

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer. $m'(x) = 15x^2 + f'(x)$ $m'(2) = 15 \cdot 4 + f'(2) = 60 + (-8) = 52$	Considers $m'(x)$ 1 point $m'(2)$ with supporting work 1 point
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The graph of m is increasing at $x = 2$ because $m'(2) > 0$.

Answer with
justification

1 point

Scoring notes:

- The first point is earned for considering $m'(x)$, $m'(2)$, or m' . This consideration may appear in a justification statement.
- The second point is earned for $m'(2) = 15 \cdot 2^2 + f'(2)$, $m'(2) = 60 + f'(2)$, or $m'(2) = 60 - 8$ but is not earned for an unsupported response of $m'(2) = 52$.
- The third point is earned for an answer and justification consistent with any declared value of $m'(2)$.

Total for part (d) 3 points

Total for question 5 9 points

Part B (AB or BC): Graphing calculator not allowed

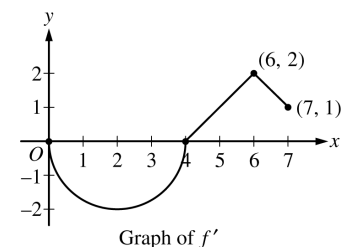
Question 3

9 points

General Scoring Notes

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Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f' , the derivative of f , consists of a semicircle and two line segments, as shown in the figure above.

Model Solution

Scoring

- (a) Find $f(0)$ and $f(5)$.

$$f(0) = f(4) + \int_4^0 f'(x) dx = 3 - \int_0^4 f'(x) dx = 3 + 2\pi$$

$$f(5) = f(4) + \int_4^5 f'(x) dx = 3 + \frac{1}{2} = \frac{7}{2}$$

Area of either region

1 point

– OR – $\int_0^4 f'(x) dx$

– OR – $\int_4^5 f'(x) dx$

$f(0)$

1 point

$f(5)$

1 point

Scoring notes:

- A response with answers of only $f(0) = \pm 2\pi$, or only $f(5) = \frac{1}{2}$, or both earns 1 of the 3 points.
- A response displaying $f(5) = \frac{7}{2}$ and a missing or incorrect value for $f(0)$ earns 2 of the 3 points.
- The second and third points can be earned in either order.
- Read unlabeled values from left to right and from top to bottom as $f(0)$ and $f(5)$. A single value must be labeled as either $f(0)$ or $f(5)$ in order to earn any points.

Total for part (a) 3 points

- (b) Find the x -coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

The graph of f has a point of inflection at each of $x = 2$ and $x = 6$, because $f'(x)$ changes from decreasing to increasing at $x = 2$ and from increasing to decreasing at $x = 6$.	Answer	1 point
	Justification	1 point

Scoring notes:

- A response that gives only one of $x = 2$ or $x = 6$, along with a correct justification, earns 1 of the 2 points.
- A response that claims that there is a point of inflection at any value other than $x = 2$ or $x = 6$ earns neither point.
- To earn the second point a response must use correct reasoning based on the graph of f' . Examples of correct reasoning include:
 - Correctly discussing the signs of the slopes of the graph of f'
 - Citing $x = 2$ and $x = 6$ as the locations of local extrema on the graph of f'
- Examples of reasoning not (sufficiently) connected to the graph of f' include:
 - Reasoning based on sign changes in f'' unless the connection is made between the sign of f'' and the slopes of the graph of f'
 - Reasoning based only on the concavity of the graph of f
- The second point cannot be earned by use of vague or undefined terms such as “it” or “the function” or “the derivative.”
- Responses that report inflection points as ordered pairs must report the points $(2, 3 + \pi)$ and $(6, 5)$ in order to earn the first point. If the y -coordinates are reported incorrectly, the response remains eligible for the second point.

Total for part (b) 2 points

- (c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.

$g'(x) = f'(x) - 1$	$g'(x) = f'(x) - 1$	1 point
$f'(x) - 1 \leq 0 \Rightarrow f'(x) \leq 1$	Interval with reason	1 point
The graph of g is decreasing on the interval $0 \leq x \leq 5$ because $g'(x) \leq 0$ on this interval.		

Scoring notes:

- The first point can be earned for $f'(x) \leq 1$ or the equivalent, in words or symbols.
- Endpoints do not need to be included in the interval to be eligible for the second point.

Total for part (c) 2 points

- (d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

g is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$.	Considers $g'(x) = 0$	1 point
	Answer with justification	1 point

Therefore, the absolute minimum occurs at $x = 5$, and

$g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$ is the absolute minimum value of g .

Scoring notes:

- A justification that uses a local argument, such as “ g' changes from negative to positive (or g changes from decreasing to increasing) at $x = 5$ ” must also state that $x = 5$ is the only critical point.
- If $g'(x) = 0$ (or equivalent) is not declared explicitly, a response that isolates $x = 5$ as the only critical number belonging to $(0, 7)$ earns the first point.
- A response that imports $g'(x) = f'(x)$ from part (c) is eligible for the first point but not the second.
 - In this case, consideration of $x = 4$ as the only critical number on $(0, 7)$ earns the first point.
- Solution using Candidates Test:

$$g'(x) = f'(x) - 1 = 0 \Rightarrow x = 5, x = 7$$

x	$g(x)$
0	$3 + 2\pi$
5	$-\frac{3}{2}$
7	$-\frac{1}{2}$

The absolute minimum value of g on the interval $0 \leq x \leq 7$ is $-\frac{3}{2}$.

- When using a Candidates Test, a response may import an incorrect value of $f(0) = g(0) > -\frac{3}{2}$ from part (a). The second point can only be earned for an answer of $-\frac{3}{2}$.

Total for part (d) 2 points

Total for question 3 9 points

Part B (AB): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

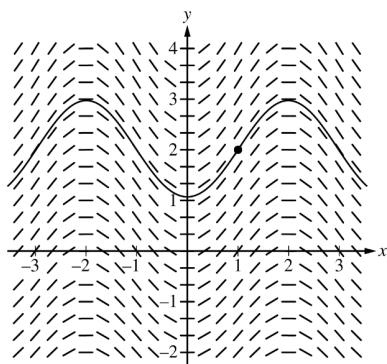
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2} \sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

Model Solution**Scoring**

- (a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.



Solution curve

1 point**Scoring notes:**

- The solution curve must pass through the point $(1, 2)$, extend reasonably close to the left and right edges of the square and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.
- The solution curve must indicate $f(x) > 0$ for all points on the curve.
- All local maximum/minimum points on the solution curve must occur at horizontal line segments in the slope field.

Total for part (a) 1 point

- (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

$$\left. \frac{dy}{dx} \right|_{(x,y)=(1,2)} = \frac{1}{2} \cdot 3 \cdot \sin\left(\frac{\pi}{2}\right) = \frac{3}{2}$$

Tangent line equation **1 point**

$$\text{An equation for the tangent line is } y = 2 + \frac{3}{2}(x - 1).$$

$$f(0.8) \approx 2 + \frac{3}{2}(0.8 - 1) = 1.7$$

Approximation **1 point****Scoring notes:**

- The tangent line equation can be presented in any equivalent form.
- An incorrect tangent line equation with a slope of $\frac{3}{2}$ is eligible to earn the second point for a consistent answer.
- A response of only $2 + \frac{3}{2}(0.8 - 1)$ earns the second point but not the first.

Total for part (b) 2 points

- (c) It is known that $f''(x) > 0$ for $-1 \leq x \leq 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

Because $f''(x) > 0$, f is concave up on $-1 \leq x \leq 1$, the tangent line lies below the graph of $y = f(x)$ at $x = 0.8$, and the approximation for $f(0.8)$ is an underestimate.

Answer with reason **1 point****Scoring notes:**

- The reason must include $f''(x) > 0$, $f'(x)$ is increasing, or $f(x)$ is concave up.

Total for part (c) 1 point

- (d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation

$$\frac{dy}{dx} = \frac{1}{2} \sin\left(\frac{\pi}{2}x\right) \sqrt{y+7} \text{ with the initial condition } f(1) = 2.$$

$$\int \frac{dy}{\sqrt{y+7}} = \int \frac{1}{2} \sin\left(\frac{\pi}{2}x\right) dx$$

Separation of variables **1 point**

$$2\sqrt{y+7} = -\frac{1}{\pi} \cos\left(\frac{\pi}{2}x\right) + C$$

One correct antiderivative **1 point**

The other correct antiderivative **1 point**

$$f(1) = 2 \Rightarrow 2\sqrt{2+7} = -\frac{1}{\pi} \cos\left(\frac{\pi}{2} \cdot 1\right) + C$$

$$\Rightarrow 6 = -\frac{1}{\pi} \cos\left(\frac{\pi}{2}\right) + C \Rightarrow C = 6$$

Constant of integration and uses initial condition **1 point**

$$\sqrt{y+7} = 3 - \frac{1}{2\pi} \cos\left(\frac{\pi}{2}x\right)$$

$$y = \left(3 - \frac{1}{2\pi} \cos\left(\frac{\pi}{2}x\right)\right)^2 - 7$$

Solves for y **1 point**

Scoring notes:

- A response with no separation of variables earns 0 out of 5 points.
- A response with no constant of integration can earn at most the first 3 points.
- A response is eligible for the fourth point only if it has earned the first point and at least 1 of the 2 antiderivative points.
 - Special case: The incorrect separation of $\sqrt{y+7} \, dy = \frac{1}{2} \sin\left(\frac{\pi}{2}x\right) dx$ does not earn the first point, is only eligible for the antiderivative point for $-\frac{1}{\pi} \cos\left(\frac{\pi}{2}x\right)$, and is eligible for the fourth point.
- An eligible response earns the fourth point by correctly including the constant of integration in an equation and substituting 1 for x and 2 for y .
- A response is eligible for the fifth point only if it has earned the first 4 points.

Total for part (d) 5 points

Total for question 5 9 points

Part B (AB or BC): Graphing calculator not allowed

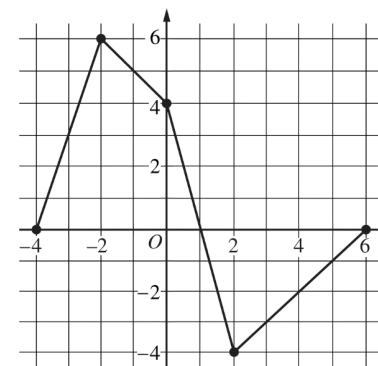
Question 4

9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.



Graph of f

Let f be a continuous function defined on the closed interval $-4 \leq x \leq 6$. The graph of f , consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t) \, dt$.

Model Solution	Scoring
$G'(x) = f(x)$ in any part of the response.	$G'(x) = f(x)$ 1 point

Scoring notes:

- This “global point” can be earned in any one part. Expressions that show this connection and therefore earn this point include: $G' = f$, $G'(x) = f(x)$, $G''(x) = f'(x)$ in part (a), $G'(3) = f(3)$ in part (b), or $G'(2) = f(2)$ in part (c).

Total 1 point

- (a) On what open intervals is the graph of
- G
- concave up? Give a reason for your answer.

$$G'(x) = f(x)$$

The graph of G is concave up for $-4 < x < -2$ and $2 < x < 6$, because $G' = f$ is increasing on these intervals.

Answer with reason

1 point**Scoring notes:**

- Intervals may also include one or both endpoints.

Total for part (a)**1 point**

- (b) Let
- P
- be the function defined by
- $P(x) = G(x) \cdot f(x)$
- . Find
- $P'(3)$
- .

$$P'(x) = G(x) \cdot f'(x) + f(x) \cdot G'(x)$$

$$P'(3) = G(3) \cdot f'(3) + f(3) \cdot G'(3)$$

Product rule

1 point

Substituting $G(3) = \int_0^3 f(t) dt = -3.5$ and $G'(3) = f(3) = -3$ into the above expression for $P'(3)$ gives the following:

 $G(3)$ or $G'(3)$ **1 point**

$$P'(3) = -3.5 \cdot 1 + (-3) \cdot (-3) = 5.5$$

Answer

1 point**Scoring notes:**

- The first point is earned for the correct application of the product rule in terms of x or in the evaluation of $P'(3)$. Once earned, this point cannot be lost.
- The second point is earned by correctly evaluating $G(3) = -3.5$, $G'(3) = -3$, or $f(3) = -3$.
- To be eligible to earn the third point a response must have earned the first two points.
- Simplification of the numerical value is not required to earn the third point.

Total for part (b)**3 points**

- (c) Find
- $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$
- .

$$\lim_{x \rightarrow 2} (x^2 - 2x) = 0$$

Because G is continuous for $-4 \leq x \leq 6$,

$$\lim_{x \rightarrow 2} G(x) = \int_0^2 f(t) dt = 0.$$

Therefore, the limit $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$ is an indeterminate form of

$$\text{type } \frac{0}{0}.$$

Uses L'Hospital's Rule

1 point

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x} = \lim_{x \rightarrow 2} \frac{G'(x)}{2x - 2}$$

$$= \lim_{x \rightarrow 2} \frac{f(x)}{2x - 2} = \frac{f(2)}{2} = \frac{-4}{2} = -2$$

Answer with justification

1 point**Scoring notes:**

- To earn the first point the response must show $\lim_{x \rightarrow 2} (x^2 - 2x) = 0$ and $\lim_{x \rightarrow 2} G(x) = 0$ and must show a ratio of the two derivatives, $G'(x)$ and $2x - 2$. The ratio may be shown as evaluations of the derivatives at $x = 2$, such as $\frac{G'(2)}{2}$.
- To earn the second point the response must evaluate correctly with appropriate limit notation. In particular the response must include $\lim_{x \rightarrow 2} \frac{G'(x)}{2x - 2}$ or $\lim_{x \rightarrow 2} \frac{f(x)}{2x - 2}$.
- With any linkage errors (such as $\frac{G'(x)}{2x - 2} = \frac{f(2)}{2}$), the response does not earn the second point.

Total for part (c)**2 points**

- (d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c , $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

$$G(2) = \int_0^2 f(t) dt = 0 \text{ and } G(-4) = \int_0^{-4} f(t) dt = -16$$

Average rate of change

1 point

$$\text{Average rate of change} = \frac{G(2) - G(-4)}{2 - (-4)} = \frac{0 - (-16)}{6} = \frac{8}{3}$$

Yes, $G'(x) = f(x)$ so G is differentiable on $(-4, 2)$ and continuous on $[-4, 2]$. Therefore, the Mean Value Theorem applies and guarantees a value c , $-4 < c < 2$, such that

$$G'(c) = \frac{8}{3}.$$

Answer with justification

1 point

Scoring notes:

- To earn the first point a response must present at least a difference and a quotient and a correct evaluation. For example, $\frac{0+16}{6}$ or $\frac{G(2) - G(-4)}{6} = \frac{16}{6}$.
- Simplification of the numerical value is not required to earn the first point, but any simplification must be correct.
- The second point can be earned without the first point if the response has the correct setup but an incorrect or no evaluation of the average rate of change. The Mean Value Theorem need not be explicitly stated provided the conditions and conclusion are stated.

Total for part (d) 2 points**Total for question 4 9 points****Part B (AB): Graphing calculator not allowed****Question 6****9 points****General Scoring Notes**

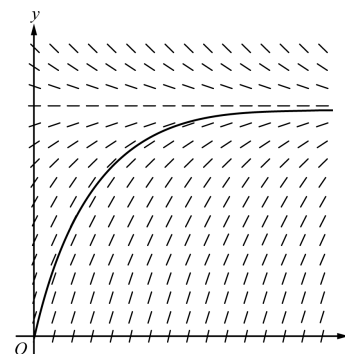
Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12-y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

Model Solution**Scoring**

- (a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12-y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



Solution curve

1 point

Scoring notes:

- To earn the point the solution curve must pass through the point $(0, 0)$, be generally increasing and concave down, and approach the horizontal asymptote from below as t increases. The point is not earned if two or more solution curves are presented.

Total for part (a) 1 point

- (b) Using correct units, interpret the statement $\lim_{t \rightarrow \infty} A(t) = 12$ in the context of this problem.

Over time the amount of medication in the patient approaches 12 milligrams.	Interpretation	1 point
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Scoring notes:

- To earn the point the interpretation must include “medication in the patient,” “approaches 12,” and units (milligrams), or their equivalents.

Total for part (b) 1 point

- (c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation

$$\frac{dy}{dt} = \frac{12 - y}{3} \text{ with initial condition } A(0) = 0.$$

$\frac{dy}{dt} = \frac{12 - y}{3} \Rightarrow \frac{dy}{12 - y} = \frac{dt}{3}$	Separation of variables	1 point
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$\int \frac{dy}{12 - y} = \int \frac{dt}{3} \Rightarrow -\ln 12 - y = \frac{t}{3} + C$	Antiderivatives	1 point
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$\ln 12 - y = -\frac{t}{3} - C \Rightarrow 12 - y = e^{-t/3 - C}$ $\Rightarrow y = 12 + Ke^{-t/3}$	Constant of integration and uses initial condition	1 point
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$$0 = 12 + K \Rightarrow K = -12$$

$y = A(t) = 12 - 12e^{-t/3}$	Solves for y	1 point
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Scoring notes:

- A response of $\frac{dy}{12 - y} = 3 dt$ is a bad separation and does not earn the first point. However, this response is eligible for the second and third points. It cannot earn the fourth point.
- Absolute value bars are not required in this part.
- A response that correctly separates to $\frac{3 dy}{12 - y} = dt$ but then incorrectly simplifies to $\frac{dy}{4 - y} = dt$ earns the first point (for the initial correct separation), is eligible for the second point (for $-\ln|4 - y| = t$, with or without $+C$), but is not eligible for the third or fourth points.
- $+\ln|12 - y| = \frac{t}{3}$ (with or without $+C$) does not earn the second point and is not eligible for the fourth point; $+\ln|12 - y| = \frac{t}{3} + C$ is eligible for the third point.
- In all other cases, the points are earned consecutively—the second point cannot be earned without the first, the third without the second, etc.

Total for part (c) 4 points

- (d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

$\frac{dy}{dt} = 3 - \frac{y}{t+2} \Rightarrow \frac{d^2y}{dt^2} = (-1) \frac{\frac{dy}{dt}(t+2) - y}{(t+2)^2}$	Quotient rule	1 point
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$B'(1) = 3 - \frac{B(1)}{3} = 3 - \frac{2.5}{3} = \frac{6.5}{3}$	$B''(1) < 0$	1 point
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$B''(1) = -\frac{B'(1) \cdot 3 - B(1)}{3^2} = -\frac{6.5 - 2.5}{9} = -\frac{4}{9} < 0$		
--	--	--

The rate of change of the amount of medication is decreasing at time $t = 1$ because $B''(1) < 0$ and $\frac{d^2y}{dt^2}$ is continuous in an interval containing $t = 1$.	Answer with reason	1 point
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Scoring notes:

- The first point is for correctly applying the quotient rule to $\frac{y}{t+2}$ or applying the product rule to $y(t+2)^{-1}$. Errors in differentiating the constant, 3, or handling the sign of the second term of $\frac{dy}{dt}$ will result in not earning the second point.
- The second point cannot be earned unless the second derivative $\frac{d^2y}{dt^2}$ is correct.
- For the second point it is sufficient to state the sign of $B''(1)$ is negative with supporting work. If a value is declared for $B''(1)$, it must be correct in order to earn the second point.
- Eligibility for the third point: An attempt at using the quotient rule (or product rule) to find $B''(1)$. In this case the third point will be earned for a consistent conclusion based on the declared value (or sign) of $B''(1)$.

Total for part (d) 3 points

Total for question 6 9 points

AP[®] Calculus AB

Scoring Guidelines

2019 SCORING GUIDELINES

Question 1

(a) $\int_0^5 E(t) dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

(b) $\frac{1}{5-0} \int_0^5 L(t) dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

- (c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$$E(t) - L(t) = 0 \Rightarrow t = 6.20356$$

$E(t) - L(t) > 0$ for $0 \leq t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$. Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$$A(t) = \int_0^t (E(s) - L(s)) ds$$

$$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

2 : $\begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$

2019 SCORING GUIDELINES

Question 2

- (a)
- v_P
- is differentiable
- $\Rightarrow v_P$
- is continuous on
- $0.3 \leq t \leq 2.8$
- .

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value c , $0.3 < c < 2.8$, such that $v_P'(c) = 0$.

— OR —

v_P is differentiable $\Rightarrow v_P$ is continuous on $0.3 \leq t \leq 2.8$.

By the Extreme Value Theorem, v_P has a minimum on $[0.3, 2.8]$.

$$v_P(0.3) = 55 > -29 = v_P(1.7) \text{ and } v_P(1.7) = -29 < 55 = v_P(2.8).$$

Thus v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P'(t)$ must equal 0 at this minimum.

$$\begin{aligned} \text{(b)} \quad \int_0^{2.8} v_P(t) dt &\approx 0.3 \left(\frac{v_P(0) + v_P(0.3)}{2} \right) + 1.4 \left(\frac{v_P(0.3) + v_P(1.7)}{2} \right) \\ &\quad + 1.1 \left(\frac{v_P(1.7) + v_P(2.8)}{2} \right) \\ &= 0.3 \left(\frac{0 + 55}{2} \right) + 1.4 \left(\frac{55 + (-29)}{2} \right) + 1.1 \left(\frac{-29 + 55}{2} \right) \\ &= 40.75 \end{aligned}$$

- (c)
- $v_Q(t) = 60 \Rightarrow t = A = 1.866181$
- or
- $t = B = 3.519174$

$$v_Q(t) \geq 60 \text{ for } A \leq t \leq B$$

$$\int_A^B |v_Q(t)| dt = 106.108754$$

The distance traveled by particle Q during the interval $A \leq t \leq B$ is 106.109 (or 106.108) meters.

- (d) From part (b), the position of particle
- P
- at time
- $t = 2.8$
- is

$$x_P(2.8) = \int_0^{2.8} v_P(t) dt \approx 40.75.$$

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t) dt = -90 + 135.937653 = 45.937653$$

Therefore at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

$$2 : \begin{cases} 1 : v_P(2.8) - v_P(0.3) = 0 \\ 1 : \text{justification, using} \\ \quad \text{Mean Value Theorem} \end{cases}$$

— OR —

$$2 : \begin{cases} 1 : v_P(0.3) > v_P(1.7) \\ \quad \text{and } v_P(1.7) < v_P(2.8) \\ 1 : \text{justification, using} \\ \quad \text{Extreme Value Theorem} \end{cases}$$

1 : answer, using trapezoidal sum

$$3 : \begin{cases} 1 : \text{interval} \\ 1 : \text{definite integral} \\ 1 : \text{distance} \end{cases}$$

$$3 : \begin{cases} 1 : \int_0^{2.8} v_Q(t) dt \\ 1 : \text{position of particle } Q \\ 1 : \text{answer} \end{cases}$$

2019 SCORING GUIDELINES

Question 3

$$\begin{aligned} \text{(a)} \quad \int_{-6}^5 f(x) dx &= \int_{-6}^{-2} f(x) dx + \int_{-2}^5 f(x) dx \\ &\Rightarrow 7 = \int_{-6}^{-2} f(x) dx + 2 + \left(9 - \frac{9\pi}{4} \right) \\ &\Rightarrow \int_{-6}^{-2} f(x) dx = 7 - \left(11 - \frac{9\pi}{4} \right) = \frac{9\pi}{4} - 4 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \int_3^5 (2f'(x) + 4) dx &= 2 \int_3^5 f'(x) dx + \int_3^5 4 dx \\ &= 2(f(5) - f(3)) + 4(5 - 3) \\ &= 2(0 - (3 - \sqrt{5})) + 8 \\ &= 2(-3 + \sqrt{5}) + 8 = 2 + 2\sqrt{5} \end{aligned}$$

— OR —

$$\begin{aligned} \int_3^5 (2f'(x) + 4) dx &= [2f(x) + 4x]_{x=3}^{x=5} \\ &= (2f(5) + 20) - (2f(3) + 12) \\ &= (2 \cdot 0 + 20) - (2(3 - \sqrt{5}) + 12) \\ &= 2 + 2\sqrt{5} \end{aligned}$$

- (c)
- $g'(x) = f(x) = 0 \Rightarrow x = -1, x = \frac{1}{2}, x = 5$

x	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval $-2 \leq x \leq 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$.

$$\begin{aligned} \text{(d)} \quad \lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} &= \frac{10^1 - 3f'(1)}{f(1) - \arctan 1} \\ &= \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}} \end{aligned}$$

$$3 : \begin{cases} 1 : \int_{-6}^5 f(x) dx = \int_{-6}^{-2} f(x) dx + \int_{-2}^5 f(x) dx \\ 1 : \int_{-2}^5 f(x) dx \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -1 \text{ as a candidate} \\ 1 : \text{answer with justification} \end{cases}$$

1 : answer

2019 SCORING GUIDELINES

Question 4

(a) $V = \pi r^2 h = \pi(1)^2 h = \pi h$

$$\left. \frac{dV}{dt} \right|_{h=4} = \pi \left. \frac{dh}{dt} \right|_{h=4} = \pi \left(-\frac{1}{10} \sqrt{4} \right) = -\frac{\pi}{5} \text{ cubic feet per second}$$

(b) $\frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10} \sqrt{h} \right) = \frac{1}{200}$

Because $\frac{d^2 h}{dt^2} = \frac{1}{200} > 0$ for $h > 0$, the rate of change of the height is increasing when the height of the water is 3 feet.

(c) $\frac{dh}{\sqrt{h}} = -\frac{1}{10} dt$

$$\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10} dt$$

$$2\sqrt{h} = -\frac{1}{10}t + C$$

$$2\sqrt{5} = -\frac{1}{10} \cdot 0 + C \Rightarrow C = 2\sqrt{5}$$

$$2\sqrt{h} = -\frac{1}{10}t + 2\sqrt{5}$$

$$h(t) = \left(-\frac{1}{20}t + \sqrt{5} \right)^2$$

$$2 : \begin{cases} 1 : \frac{dV}{dt} = \pi \frac{dh}{dt} \\ 1 : \text{answer with units} \end{cases}$$

$$3 : \begin{cases} 1 : \frac{d}{dh} \left(-\frac{1}{10} \sqrt{h} \right) = -\frac{1}{20\sqrt{h}} \\ 1 : \frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} \\ 1 : \text{answer with explanation} \end{cases}$$

$$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : h(t) \end{cases}$$

Note: 0/4 if no separation of variables

Note: max 2/4 [1-1-0-0] if no constant of integration

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Question 5

$$\begin{aligned} \text{(a)} \quad \int_0^2 (h(x) - g(x)) dx &= \int_0^2 \left((6 - 2(x-1)^2) - \left(-2 + 3 \cos\left(\frac{\pi}{2}x\right) \right) \right) dx \\ &= \left[6x - \frac{2}{3}(x-1)^3 - \left(-2x + \frac{6}{\pi} \sin\left(\frac{\pi}{2}x\right) \right) \right]_{x=0}^{x=2} \\ &= \left(\left(12 - \frac{2}{3} \right) - (-4 + 0) \right) - \left(\left(0 + \frac{2}{3} \right) - (0 + 0) \right) \\ &= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3} \end{aligned}$$

The area of R is $\frac{44}{3}$.

$$\begin{aligned} \text{(b)} \quad \int_0^2 A(x) dx &= \int_0^2 \frac{1}{x+3} dx \\ &= \left[\ln(x+3) \right]_{x=0}^{x=2} = \ln 5 - \ln 3 \end{aligned}$$

The volume of the solid is $\ln 5 - \ln 3$.

$$\text{(c)} \quad \pi \int_0^2 \left((6 - g(x))^2 - (6 - h(x))^2 \right) dx$$

$$4 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative of } 3 \cos\left(\frac{\pi}{2}x\right) \\ 1 : \text{antiderivative of} \\ \text{remaining terms} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{form of integrand} \\ 1 : \text{integrand} \end{cases}$$

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Question 6

(a) $h'(2) = \frac{2}{3}$

1 : answer

(b) $a'(x) = 9x^2h(x) + 3x^3h'(x)$

$$a'(2) = 9 \cdot 2^2 h(2) + 3 \cdot 2^3 h'(2) = 36 \cdot 4 + 24 \cdot \frac{2}{3} = 160$$

3 : $\begin{cases} 1 : \text{form of product rule} \\ 1 : a'(x) \\ 1 : a'(2) \end{cases}$ (c) Because h is differentiable, h is continuous, so $\lim_{x \rightarrow 2} h(x) = h(2) = 4$.

Also, $\lim_{x \rightarrow 2} h(x) = \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3}$, so $\lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4$.

Because $\lim_{x \rightarrow 2} (x^2 - 4) = 0$, we must also have $\lim_{x \rightarrow 2} (1 - (f(x))^3) = 0$.

Thus $\lim_{x \rightarrow 2} f(x) = 1$.

Because f is differentiable, f is continuous, so $f(2) = \lim_{x \rightarrow 2} f(x) = 1$.

Also, because f is twice differentiable, f' is continuous, so

$$\lim_{x \rightarrow 2} f'(x) = f'(2) \text{ exists.}$$

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = \lim_{x \rightarrow 2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{4}{-3(1)^2 \cdot f'(2)} = 4.$$

Thus $f'(2) = -\frac{1}{3}$.

4 : $\begin{cases} 1 : \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4 \\ 1 : f(2) \\ 1 : \text{L'Hospital's Rule} \\ 1 : f'(2) \end{cases}$ (d) Because g and h are differentiable, g and h are continuous, so

$$\lim_{x \rightarrow 2} g(x) = g(2) = 4 \text{ and } \lim_{x \rightarrow 2} h(x) = h(2) = 4.$$

Because $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$, it follows from the squeeze theorem that $\lim_{x \rightarrow 2} k(x) = 4$.

Also, $4 = g(2) \leq k(2) \leq h(2) = 4$, so $k(2) = 4$.

Thus k is continuous at $x = 2$.

1 : continuous with justification