

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring
A	Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.	
	$\frac{1}{4 - 0} \int_0^4 C(t) dt$	Average value formula Point 1 (P1)
	$= \frac{1}{4}(11.112896) = 2.778224$	Answer Point 2 (P2)
	From time $t = 0$ to $t = 4$ weeks, the average number of acres affected by the invasive species was 2.778 acres.	

Scoring Notes for Part A

- P1** is earned for the correct integral, with or without the differential, along with evidence of division by 4. In the presence of the correct integral, the correct answer will suffice as evidence of division by 4. These may appear all in one step, as in the model solution, or in multiple steps.
- P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect or unclear communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring. For example:
 - $\int_0^4 C(t) dt = 11.112896$ so the average velocity is 2.778224.
Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer.
 - $\int_0^4 C(t) dt = \frac{11.112896}{4} = 2.778224$
Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
 - $\int_0^4 C(t) dt = 2.778224$
Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- Note that the values $\frac{1}{4}(11.112)$ and $\frac{1}{4}(11.113)$ are accurate to three digits after the decimal and therefore earn **P2**.

- B** Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.

$$\frac{C(4) - C(0)}{4 - 0} = 1.282008$$

Uses average rate of change **Point 3 (P3)**

$$C'(t) = \frac{38}{25 + t^2} = 1.282008 \Rightarrow t = 2.154298$$

Answer with supporting work **Point 4 (P4)**

The instantaneous rate of change of C equals the average rate of change of C over the interval $0 \leq t \leq 4$ at time $t = 2.154$.

Scoring Notes for Part B

- P3** may be earned by presenting the expression or value for the average rate of change. Note that because $C(0) = 0$ and the interval is $0 \leq t \leq 4$, any of the following will earn **P3**: $\frac{\int_0^4 C'(t) dt}{4}$, $\frac{C(4) - C(0)}{4 - 0}$, $\frac{C(4)}{4}$, $\frac{5.128 - 0}{4 - 0}$, $\frac{5.128}{4}$, or 1.282. However, neither **P3** nor **P4** is earned by just presenting $t = 1.282$.
 - P4** is earned for the correct answer supported by the appropriate equation. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- The following response, for example, earns both **P3** and **P4**: $C'(t) = \frac{C(4) - C(0)}{4}$ when $t = 2.154$.

- C** Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

$$\lim_{t \rightarrow \infty} C'(t) = \lim_{t \rightarrow \infty} \frac{38}{25 + t^2}$$

Limit expression **Point 5 (P5)**

$$= 0$$

Value **Point 6 (P6)**

Scoring Notes for Part C

- P5** can be earned for either $\lim_{t \rightarrow \infty} C'(t)$ or $\lim_{t \rightarrow \infty} C(t)$.
- A response that includes $\lim_{t \rightarrow \infty} C(t)$ is not eligible to earn **P6**.
- For **P6**, arithmetic with infinity, e.g., $\frac{38}{25 + \infty^2} = 0$, will be considered as scratch work and will not be considered in scoring.

- D** At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A , defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x) dx$, models the number of acres affected by the species over the time interval $4 \leq t \leq 36$. At what time t , for $4 \leq t \leq 36$, does A attain its maximum value? Justify your answer.

$$A'(t) = C'(t) - 0.1 \cdot \ln t$$

Considers $A'(t) = 0$ **Point 7 (P7)**

For $4 \leq t \leq 36$, the maximum value of $A(t)$ occurs when $A'(t) = 0$ or at an endpoint.

$$A'(t) = C'(t) - 0.1 \cdot \ln t = 0 \Rightarrow C'(t) = 0.1 \cdot \ln t$$

$$\Rightarrow t = 11.441700$$

Justification **Point 8 (P8)**

t	$A(t)$
4	5.128031
11.441700	7.316978
36	1.743056

Therefore, the number of acres affected by the species is a maximum at time $t = 11.442$ (or 11.441) weeks.

Answer with supporting work **Point 9 (P9)**

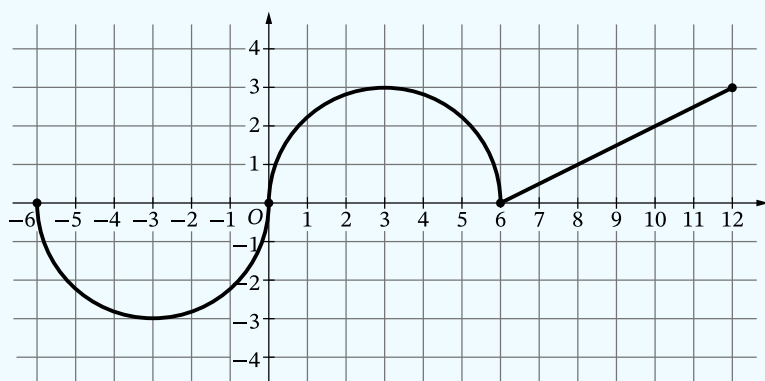
Scoring Notes for Part D

- P7** is earned for considering $A'(t) = 0$, $C'(t) - 0.1 \cdot \ln t = 0$, or $C'(t) = 0.1 \cdot \ln t$. **P7** is not earned by just presenting $t = 11.441700$. A response that discusses the sign of $A'(t)$ changing or uses the phrase “critical points of A ” also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by correctly evaluating $A(t)$ at $t = 4$, $t = 11.441700$, and $t = 36$. The evaluations must be correct to the first digit after the decimal, rounded or truncated.
- Alternate justifications:
 - $A'(t) > 0$ for $4 < t < 11.442$, and $A'(t) < 0$ for $11.442 < t < 36$. Therefore, $t = 11.442$ is the location of the absolute maximum for A on the interval $4 \leq t \leq 36$.
 - Because $A'(t)$ changes sign from positive to negative at $t = 11.442$ (this might be presented as “ $A'(t) > 0$ for $t < 11.442$, and $A'(t) < 0$ for $t > 11.442$ ”), it is the location of a relative maximum for A . And because $t = 11.442$ is the only critical point of A in the interval $4 \leq t \leq 36$, it is the location of the absolute maximum for A on the interval.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

Part A (AB or BC): Graphing calculator not allowed**Question 4****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f , consisting of two semicircles and one line segment, is shown in the figure.

Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t) dt$.

Model Solution**Scoring**

- A** Find $g'(8)$. Give a reason for your answer.

$$g'(x) = f(x)$$

Considers
 $g'(x) = f(x)$

Point 1 (P1)

$$g'(8) = f(8) = 1$$

Answer

Point 2 (P2)**Scoring Notes for Part A**

- P1** is earned for $g' = f$, $g'(x) = f(x)$, or $g'(8) = f(8)$ in part A.
- A response of $g'(8) = f(8) = 1$ earns both **P1** and **P2**.
- A response that does not earn **P1** can earn **P2** with an implied application of the Fundamental Theorem of Calculus (e.g., $g'(8) = 1$ or $f(8) = 1$).
- A response of $g'(8) = f(8) - f(6) = 1$ earns **P2** but not **P1**.

- B** Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

The graph of g has a point of inflection where $g'' = f'$ changes sign, which is where $g' = f$ changes from decreasing to increasing or vice versa.

Answer

Point 3 (P3)

Reason

Point 4 (P4)

The graph of g has points of inflection at $x = -3$ and $x = 6$ because f changes from decreasing to increasing there.

The graph of g also has a point of inflection at $x = 3$ because f changes from increasing to decreasing there.

Scoring Notes for Part B

- P3** is earned only for an answer of $x = -3$, $x = 3$, and $x = 6$. If any other/additional values of x in $-6 < x < 12$ are declared to be points of inflection, the response does not earn either **P3** or **P4**. Consideration of $x = -6$ or of $x = 12$ does not impact scoring.
- To earn **P4**, a response must tie the reason to the given graph of f .
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f changes from increasing to decreasing or decreasing to increasing there” earns both **P3** and **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because the slope of f changes sign there” earns both **P3** and **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f attains relative extrema there” earns both **P3** and **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because g changes concavity there” earns **P3** but not **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $g'' = f'$ changes sign there” earns **P3** but not **P4**.
 - A response that relies upon an ambiguous term such as “the function” or “the graph” does not earn **P4**.
- Special case:** A response with two of the three correct x -values with correct reasoning and no other/additional values of x declared to be points of inflection earns **P4** but not **P3**.

C Find $g(12)$ and $g(0)$. Label your answers.

$g(12) = \int_6^{12} f(t) dt = \frac{1}{2} \cdot 6 \cdot 3 = 9$	$g(12)$	Point 5 (P5)
$g(0) = \int_6^0 f(x) dx = -\int_0^6 f(x) dx = -\frac{\pi}{2} 3^2 = -\frac{9\pi}{2}$	$g(0)$	Point 6 (P6)

Scoring Notes for Part C

- Unlabeled values do not earn either **P5** or **P6**.
- P5** is earned for a response of $g(12) = 9$, with or without supporting work.
- P6** is earned for a response of $g(0) = -\frac{9\pi}{2}$, with or without supporting work.

Note: Incorrect communication between the label “ $g(0)$ ” and the answer will be treated as scratch work and will not impact scoring. For example, $g(0) = \int_0^6 f(x) dx = -\frac{9\pi}{2}$ earns **P6**.

D Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

For $-6 \leq x \leq 12$, g attains a minimum either when
 $g'(x) = f(x) = 0$ or at an endpoint.

Considers $g'(x) = 0$ **Point 7 (P7)**

$$g'(x) = f(x) = 0$$

$$\Rightarrow x = 0, x = 6$$

Justification **Point 8 (P8)**

x	$g(x)$
-6	0
0	$-\frac{9\pi}{2}$
6	0
12	9

Therefore, on the closed interval $-6 \leq x \leq 12$, g attains an absolute minimum value at $x = 0$.

Answer **Point 9 (P9)**

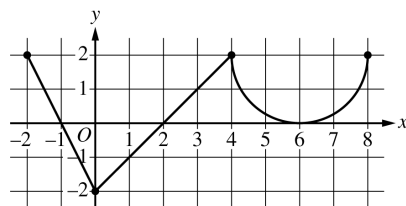
Scoring Notes for Part D

- P7** is earned for considering $g'(x) = 0$ or $f(x) = 0$. **P7** is not earned by just presenting $x = 0$ and $x = 6$.
 A response that discusses the sign of $g'(x)$ or $f(x)$ changing OR uses the phrase “critical points of g ” also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by providing evaluations or reasoning for each of $g(-6)$, $g(0)$, $g(6)$, and $g(12)$ (and no other x -values).
- Alternate justification and answer:
 Because $g'(x) \leq 0$ (or $f(x) \leq 0$) for $-6 \leq x < 0$ and $g'(x) \geq 0$ (or $f(x) \geq 0$) for $0 < x \leq 12$, the absolute minimum of g occurs at $x = 0$.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer of $x = 0$.
- For **P8**, values of $g(0)$ and $g(12)$ can be imported from part C. A response can earn **P9** with an answer that is consistent with the imported values.

Part B (AB or BC): Graphing calculator not allowed**Question 4****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Graph of f'

The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f' , the derivative of f , consists of two line segments and a semicircle, as shown in the figure.

Model Solution**Scoring**

- (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

$$f'(x) > 0 \text{ on } (2, 6) \text{ and } f'(x) > 0 \text{ on } (6, 8).$$

$f'(x)$ does not change sign at $x = 6$, so there is neither a relative maximum nor a relative minimum at this location.

Answer with reason

1 point**Scoring notes:**

- A response that declares $f'(x)$ does not change sign at $x = 6$, so neither, is sufficient to earn the point.
- A response does not have to present intervals on which $f'(x)$ is positive or negative, but if any are given, they must be correct. Any presented intervals may include none, one, or both endpoints.
- A response that declares $f'(x) > 0$ before and after $x = 6$ does not earn the point.

Total for part (a)**1 point**

- (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

The graph of f is concave down on $(-2, 0)$ and $(4, 6)$ because f' is decreasing on these intervals.

Intervals

1 point

Reason

1 point**Scoring notes:**

- The first point is earned only by an answer of $(-2, 0)$ and $(4, 6)$, or these intervals including one or both endpoints.
- A response must earn the first point to be eligible for second point.
- To earn the second point a response must correctly discuss the behavior of f' or the slopes of f' .
- Special case: A response that presents exactly one of the two correct intervals with a correct reason earns 1 out of 2 points.

Total for part (b)**2 points**

- (c) Find the value of $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

Because f is differentiable at $x = 2$, f is continuous at $x = 2$, so $\lim_{x \rightarrow 2} f(x) = f(2) = 1$.

$$\lim_{x \rightarrow 2} (6f(x) - 3x) = 6 \cdot 1 - 3 \cdot 2 = 0$$

$$\lim_{x \rightarrow 2} (x^2 - 5x + 6) = 0$$

Limits of numerator and denominator

1 point

Because $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$ is of indeterminate form $\frac{0}{0}$,

L'Hospital's Rule can be applied.

Uses L'Hospital's Rule

1 point

Answer

1 point

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6} = \lim_{x \rightarrow 2} \frac{6f'(x) - 3}{2x - 5} = \frac{6 \cdot 0 - 3}{2 \cdot 2 - 5} = 3.$$

Scoring notes:

- The first point is earned by the presentation of two separate limits for the numerator and denominator.
- A response that presents a limit explicitly equal to $\frac{0}{0}$ does not earn the first point.
- The second point is earned by applying L'Hospital's Rule, that is, by presenting at least one correct derivative in the limit of a ratio of derivatives.
- The third point is earned for the correct answer with supporting work.

Total for part (c)**3 points**

- (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

$f'(x) = 0 \Rightarrow x = -1, \ x = 2, \ x = 6$	Considers $f'(x) = 0$	1 point
The function f is continuous on $[-2, 8]$, so the candidates for the location of an absolute minimum for f are $x = -2, \ x = -1, \ x = 2, \ x = 6$, and $x = 8$.	Justification	1 point
	Answer	1 point

x	$f(x)$
-2	3
-1	4
2	1
6	$7 - \pi$
8	$11 - 2\pi$

The absolute minimum value of f is $f(2) = 1$.

Scoring notes:

- To earn the first point a response must state $f' = 0$ or equivalent. Listing the zeros of f' is not sufficient.
- A response that presents any error in evaluating f at any critical point or endpoint will not earn the justification point.
- A response need not present the value of $f(-1)$ provided $x = -1$ is eliminated because it is the location of a local maximum. A response need not present the value of $f(6)$ provided $x = 6$ is eliminated by reference to part (a) or eliminated through analysis.
- A response need not present the value of $f(8)$ provided there is a presentation that argues $f'(x) \geq 0$ for $x > 2$ and, therefore, $f(8) > f(2)$.
- A response that does not consider both endpoints does not earn the justification point.
- The answer point is earned only for indicating that the minimum value is 1. It is not earned for noting that the minimum occurs at $x = 2$.

Total for part (d) 3 points

Total for question 4 9 points

Part A (AB or BC): Graphing calculator required

Question 1

9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by

$A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

Model Solution	Scoring
(a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).	
The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by $\int_1^5 A(t) dt$.	Answer 1 point

Scoring notes:

- The response must be a definite integral with correct lower and upper limits to earn this point.
- Because $|A(t)| = A(t)$ for $1 \leq t \leq 5$, a response of $\int_1^5 |450\sqrt{\sin(0.62t)}| dt$ or $\int_1^5 |A(t)| dt$ earns the point.
- A response missing dt or using dx is eligible to earn the point.
- A response with a copy error in the expression for $A(t)$ will earn the point only in the presence of $\int_1^5 A(t) dt$.

Total for part (a) 1 point

- (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

$$\text{Average} = \frac{1}{5-1} \int_1^5 A(t) dt = 375.536966$$

Uses average value formula: **1 point**

$$\frac{1}{b-a} \int_a^b A(t) dt$$

The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is 375.537 (or 375.536) vehicles per hour.

Answer **1 point**

Scoring notes:

- The use of the average value formula, indicating that $a = 1$ and $b = 5$, can be presented in single or multiple steps to earn the first point. For example, the following response earns both points:

$$\int_1^5 A(t) dt = 1502.147865, \text{ so the average value is } 375.536966.$$

- A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points: $\int_1^5 A(t) dt = 1502.147865 = 375.536966$.

- The answer must be correct to three decimal places. For example,

$$\frac{1}{5-1} \int_1^5 A(t) dt = 375.536966 \approx 376 \text{ earns only the first point.}$$

- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{4} \int_1^5 A(t) dt = 79.416068$.

- Special case: $\frac{1}{5} \int_1^5 A(t) dt = 300.429573$ earns 1 out of 2 points.

Total for part (b) 2 points

- (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

$$A'(1) = 148.947272$$

Considers $A'(1)$ **1 point**

Because $A'(1) > 0$, the rate at which the vehicles arrive at the toll plaza is increasing.

Answer with reason **1 point**

Scoring notes:

- The response need not present the value of $A'(1)$. The second line of the model solution earns both points.
- An incorrect value assigned to $A'(1)$ earns the first point (but will not earn the second point).
- Without a reference to $t = 1$, the first point is earned by any of the following:
 - 148.947 accurate to the number of decimals presented, with zero up to three decimal places (i.e., 149, 148, 148.9, 148.95, or 148.94)
 - $A'(t) = 148.947$ by itself
- To be eligible for the second point, the first point must be earned.
- To earn the second point, there must be a reference to $t = 1$.
- Degree mode: $A'(1) = 23.404311$

Total for part (c) 2 points

- (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t , for $a \leq t \leq 4$, is given by

$N(t) = \int_a^t (A(x) - 400) dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

$$N'(t) = A(t) - 400 = 0$$

Considers $N'(t) = 0$ **1 point**

$$\Rightarrow A(t) = 400 \Rightarrow t = 1.469372, t = 3.597713$$

$t = a$ and $t = b$ **1 point**

$$a = 1.469372$$

$$b = 3.597713$$

t	$N(t) = \int_a^t (A(x) - 400) dx$
a	0
b	71.254129
4	62.338346

Answer **1 point**

Justification **1 point**

The greatest number of vehicles in line is 71.

Scoring notes:

- It is not necessary to indicate that $A(t) = 400$ to earn the first point, although this statement alone would earn the first point.
- A response of “ $A(t) \geq 400$ when $1.469372 \leq t \leq 3.597713$ ” will earn the first 2 points. A response of “ $A(t) \geq 400$ ” along with the presence of exactly one of the two numbers above will earn the first point, but not the second. A response of “ $A(t) \geq 400$ ” by itself will not earn either of the first 2 points.
- To earn the second point the values for a and b must be accurate to the number of decimals presented, with at least one and up to three decimal places. These may appear only in a candidates table, as limits of integration, or on a number line.
- A response with incorrect notation involving t or x is eligible to earn all 4 points.
- A response that does not earn the first point is still eligible for the remaining 3 points.
- To earn the third point, a response must present the greatest number of vehicles. This point is earned for answers of either 71 or 71.254*** only.
- A correct justification earns the fourth point, even if the third point is not earned because of a decimal presentation error.
- When using a Candidates Test, the response must include the values for $N(a)$, $N(b)$, and $N(4)$ to earn the fourth point. These values must be correct to the number of decimals presented, with up to three decimal places. (Correctly rounded integer values are acceptable.)
- Alternate solution for the third and fourth points:

For $a \leq t \leq b$, $A(t) \geq 400$. For $b \leq t \leq 4$, $A(t) \leq 400$.

Thus, $N(t) = \int_a^t (A(x) - 400) dx$ is greatest at $t = b$.

$N(b) = 71.254129$, and the greatest number of vehicles in line is 71.

- Degree mode: The response is only eligible to earn the first point because in degree mode $A(t) < 400$.

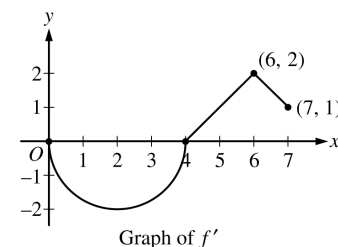
Total for part (d) 4 points

Total for question 1 9 points

Part B (AB or BC): Graphing calculator not allowed**Question 3****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f' , the derivative of f , consists of a semicircle and two line segments, as shown in the figure above.

Model Solution	Scoring
(a) Find $f(0)$ and $f(5)$.	
$f(0) = f(4) + \int_4^0 f'(x) dx = 3 - \int_0^4 f'(x) dx = 3 + 2\pi$	Area of either region 1 point – OR – $\int_0^4 f'(x) dx$
$f(5) = f(4) + \int_4^5 f'(x) dx = 3 + \frac{1}{2} = \frac{7}{2}$	– OR – $\int_4^5 f'(x) dx$
	$f(0)$ 1 point
	$f(5)$ 1 point

Scoring notes:

- A response with answers of only $f(0) = \pm 2\pi$, or only $f(5) = \frac{1}{2}$, or both earns 1 of the 3 points.
- A response displaying $f(5) = \frac{7}{2}$ and a missing or incorrect value for $f(0)$ earns 2 of the 3 points.
- The second and third points can be earned in either order.
- Read unlabeled values from left to right and from top to bottom as $f(0)$ and $f(5)$. A single value must be labeled as either $f(0)$ or $f(5)$ in order to earn any points.

Total for part (a) 3 points

- (b) Find the x -coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

The graph of f has a point of inflection at each of $x = 2$ and $x = 6$, because $f'(x)$ changes from decreasing to increasing at $x = 2$ and from increasing to decreasing at $x = 6$.	Answer	1 point
	Justification	1 point

Scoring notes:

- A response that gives only one of $x = 2$ or $x = 6$, along with a correct justification, earns 1 of the 2 points.
- A response that claims that there is a point of inflection at any value other than $x = 2$ or $x = 6$ earns neither point.
- To earn the second point a response must use correct reasoning based on the graph of f' . Examples of correct reasoning include:
 - Correctly discussing the signs of the slopes of the graph of f'
 - Citing $x = 2$ and $x = 6$ as the locations of local extrema on the graph of f'
- Examples of reasoning not (sufficiently) connected to the graph of f' include:
 - Reasoning based on sign changes in f'' unless the connection is made between the sign of f'' and the slopes of the graph of f'
 - Reasoning based only on the concavity of the graph of f
- The second point cannot be earned by use of vague or undefined terms such as “it” or “the function” or “the derivative.”
- Responses that report inflection points as ordered pairs must report the points $(2, 3 + \pi)$ and $(6, 5)$ in order to earn the first point. If the y -coordinates are reported incorrectly, the response remains eligible for the second point.

Total for part (b) 2 points

- (c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.

$g'(x) = f'(x) - 1$	$g'(x) = f'(x) - 1$	1 point
$f'(x) - 1 \leq 0 \Rightarrow f'(x) \leq 1$	Interval with reason	1 point
The graph of g is decreasing on the interval $0 \leq x \leq 5$ because $g'(x) \leq 0$ on this interval.		

Scoring notes:

- The first point can be earned for $f'(x) \leq 1$ or the equivalent, in words or symbols.
- Endpoints do not need to be included in the interval to be eligible for the second point.

Total for part (c) 2 points

- (d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

g is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$.	Considers $g'(x) = 0$	1 point
	Answer with justification	1 point

Therefore, the absolute minimum occurs at $x = 5$, and

$g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$ is the absolute minimum value of g .

Scoring notes:

- A justification that uses a local argument, such as “ g' changes from negative to positive (or g changes from decreasing to increasing) at $x = 5$ ” must also state that $x = 5$ is the only critical point.
- If $g'(x) = 0$ (or equivalent) is not declared explicitly, a response that isolates $x = 5$ as the only critical number belonging to $(0, 7)$ earns the first point.
- A response that imports $g'(x) = f'(x)$ from part (c) is eligible for the first point but not the second.
 - In this case, consideration of $x = 4$ as the only critical number on $(0, 7)$ earns the first point.
- Solution using Candidates Test:

$$g'(x) = f'(x) - 1 = 0 \Rightarrow x = 5, x = 7$$

x	$g(x)$
0	$3 + 2\pi$
5	$-\frac{3}{2}$
7	$-\frac{1}{2}$

The absolute minimum value of g on the interval $0 \leq x \leq 7$ is $-\frac{3}{2}$.

- When using a Candidates Test, a response may import an incorrect value of $f(0) = g(0) > -\frac{3}{2}$ from part (a). The second point can only be earned for an answer of $-\frac{3}{2}$.

Total for part (d) 2 points

Total for question 3 9 points

AP[®] Calculus AB

Scoring Guidelines

2019 SCORING GUIDELINES

Question 1

(a) $\int_0^5 E(t) dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{1}{5-0} \int_0^5 L(t) dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$$E(t) - L(t) = 0 \Rightarrow t = 6.20356$$

$E(t) - L(t) > 0$ for $0 \leq t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$. Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

3 : $\begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$$A(t) = \int_0^t (E(s) - L(s)) ds$$

$$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

2 : $\begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$

2019 SCORING GUIDELINES

Question 2

- (a)
- v_P
- is differentiable
- $\Rightarrow v_P$
- is continuous on
- $0.3 \leq t \leq 2.8$
- .

$$\frac{v_P(2.8) - v_P(0.3)}{2.8 - 0.3} = \frac{55 - 55}{2.5} = 0$$

By the Mean Value Theorem, there is a value c , $0.3 < c < 2.8$, such that $v_P'(c) = 0$.

— OR —

v_P is differentiable $\Rightarrow v_P$ is continuous on $0.3 \leq t \leq 2.8$.

By the Extreme Value Theorem, v_P has a minimum on $[0.3, 2.8]$.

$$v_P(0.3) = 55 > -29 = v_P(1.7) \text{ and } v_P(1.7) = -29 < 55 = v_P(2.8).$$

Thus v_P has a minimum on the interval $(0.3, 2.8)$.

Because v_P is differentiable, $v_P'(t)$ must equal 0 at this minimum.

$$\begin{aligned} \text{(b)} \quad \int_0^{2.8} v_P(t) dt &\approx 0.3 \left(\frac{v_P(0) + v_P(0.3)}{2} \right) + 1.4 \left(\frac{v_P(0.3) + v_P(1.7)}{2} \right) \\ &\quad + 1.1 \left(\frac{v_P(1.7) + v_P(2.8)}{2} \right) \\ &= 0.3 \left(\frac{0 + 55}{2} \right) + 1.4 \left(\frac{55 + (-29)}{2} \right) + 1.1 \left(\frac{-29 + 55}{2} \right) \\ &= 40.75 \end{aligned}$$

- (c)
- $v_Q(t) = 60 \Rightarrow t = A = 1.866181$
- or
- $t = B = 3.519174$

$$v_Q(t) \geq 60 \text{ for } A \leq t \leq B$$

$$\int_A^B |v_Q(t)| dt = 106.108754$$

The distance traveled by particle Q during the interval $A \leq t \leq B$ is 106.109 (or 106.108) meters.

- (d) From part (b), the position of particle
- P
- at time
- $t = 2.8$
- is

$$x_P(2.8) = \int_0^{2.8} v_P(t) dt \approx 40.75.$$

$$x_Q(2.8) = x_Q(0) + \int_0^{2.8} v_Q(t) dt = -90 + 135.937653 = 45.937653$$

Therefore at time $t = 2.8$, particles P and Q are approximately $45.937653 - 40.75 = 5.188$ (or 5.187) meters apart.

$$2 : \begin{cases} 1 : v_P(2.8) - v_P(0.3) = 0 \\ 1 : \text{justification, using} \\ \quad \text{Mean Value Theorem} \end{cases}$$

— OR —

$$2 : \begin{cases} 1 : v_P(0.3) > v_P(1.7) \\ \quad \text{and } v_P(1.7) < v_P(2.8) \\ 1 : \text{justification, using} \\ \quad \text{Extreme Value Theorem} \end{cases}$$

1 : answer, using trapezoidal sum

$$3 : \begin{cases} 1 : \text{interval} \\ 1 : \text{definite integral} \\ 1 : \text{distance} \end{cases}$$

$$3 : \begin{cases} 1 : \int_0^{2.8} v_Q(t) dt \\ 1 : \text{position of particle } Q \\ 1 : \text{answer} \end{cases}$$

2019 SCORING GUIDELINES

Question 3

$$\begin{aligned} \text{(a)} \quad \int_{-6}^5 f(x) dx &= \int_{-6}^{-2} f(x) dx + \int_{-2}^5 f(x) dx \\ &\Rightarrow 7 = \int_{-6}^{-2} f(x) dx + 2 + \left(9 - \frac{9\pi}{4} \right) \\ &\Rightarrow \int_{-6}^{-2} f(x) dx = 7 - \left(11 - \frac{9\pi}{4} \right) = \frac{9\pi}{4} - 4 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \int_3^5 (2f'(x) + 4) dx &= 2 \int_3^5 f'(x) dx + \int_3^5 4 dx \\ &= 2(f(5) - f(3)) + 4(5 - 3) \\ &= 2(0 - (3 - \sqrt{5})) + 8 \\ &= 2(-3 + \sqrt{5}) + 8 = 2 + 2\sqrt{5} \end{aligned}$$

— OR —

$$\begin{aligned} \int_3^5 (2f'(x) + 4) dx &= [2f(x) + 4x]_{x=3}^{x=5} \\ &= (2f(5) + 20) - (2f(3) + 12) \\ &= (2 \cdot 0 + 20) - (2(3 - \sqrt{5}) + 12) \\ &= 2 + 2\sqrt{5} \end{aligned}$$

- (c)
- $g'(x) = f(x) = 0 \Rightarrow x = -1, x = \frac{1}{2}, x = 5$

x	$g(x)$
-2	0
-1	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{4}$
5	$11 - \frac{9\pi}{4}$

On the interval $-2 \leq x \leq 5$, the absolute maximum value of g is $g(5) = 11 - \frac{9\pi}{4}$.

$$\begin{aligned} \text{(d)} \quad \lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x} &= \frac{10^1 - 3f'(1)}{f(1) - \arctan 1} \\ &= \frac{10 - 3 \cdot 2}{1 - \arctan 1} = \frac{4}{1 - \frac{\pi}{4}} \end{aligned}$$

$$3 : \begin{cases} 1 : \int_{-6}^5 f(x) dx = \int_{-6}^{-2} f(x) dx + \int_{-2}^5 f(x) dx \\ 1 : \int_{-2}^5 f(x) dx \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -1 \text{ as a candidate} \\ 1 : \text{answer with justification} \end{cases}$$

1 : answer

2019 SCORING GUIDELINES

Question 4

(a) $V = \pi r^2 h = \pi(1)^2 h = \pi h$

$$\left. \frac{dV}{dt} \right|_{h=4} = \pi \left. \frac{dh}{dt} \right|_{h=4} = \pi \left(-\frac{1}{10} \sqrt{4} \right) = -\frac{\pi}{5} \text{ cubic feet per second}$$

(b) $\frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10} \sqrt{h} \right) = \frac{1}{200}$

Because $\frac{d^2 h}{dt^2} = \frac{1}{200} > 0$ for $h > 0$, the rate of change of the height is increasing when the height of the water is 3 feet.

(c) $\frac{dh}{\sqrt{h}} = -\frac{1}{10} dt$

$$\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10} dt$$

$$2\sqrt{h} = -\frac{1}{10}t + C$$

$$2\sqrt{5} = -\frac{1}{10} \cdot 0 + C \Rightarrow C = 2\sqrt{5}$$

$$2\sqrt{h} = -\frac{1}{10}t + 2\sqrt{5}$$

$$h(t) = \left(-\frac{1}{20}t + \sqrt{5} \right)^2$$

$$2 : \begin{cases} 1 : \frac{dV}{dt} = \pi \frac{dh}{dt} \\ 1 : \text{answer with units} \end{cases}$$

$$3 : \begin{cases} 1 : \frac{d}{dh} \left(-\frac{1}{10} \sqrt{h} \right) = -\frac{1}{20\sqrt{h}} \\ 1 : \frac{d^2 h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} \\ 1 : \text{answer with explanation} \end{cases}$$

$$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : h(t) \end{cases}$$

Note: 0/4 if no separation of variables

Note: max 2/4 [1-1-0-0] if no constant of integration

2019 SCORING GUIDELINES

Question 5

$$\begin{aligned} \text{(a)} \quad \int_0^2 (h(x) - g(x)) \, dx &= \int_0^2 \left((6 - 2(x-1)^2) - \left(-2 + 3 \cos\left(\frac{\pi}{2}x\right) \right) \right) dx \\ &= \left[6x - \frac{2}{3}(x-1)^3 - \left(-2x + \frac{6}{\pi} \sin\left(\frac{\pi}{2}x\right) \right) \right]_{x=0}^{x=2} \\ &= \left(\left(12 - \frac{2}{3} \right) - (-4 + 0) \right) - \left(\left(0 + \frac{2}{3} \right) - (0 + 0) \right) \\ &= 12 - \frac{2}{3} + 4 - \frac{2}{3} = \frac{44}{3} \end{aligned}$$

The area of R is $\frac{44}{3}$.

$$\begin{aligned} \text{(b)} \quad \int_0^2 A(x) \, dx &= \int_0^2 \frac{1}{x+3} \, dx \\ &= \left[\ln(x+3) \right]_{x=0}^{x=2} = \ln 5 - \ln 3 \end{aligned}$$

The volume of the solid is $\ln 5 - \ln 3$.

$$\text{(c)} \quad \pi \int_0^2 \left((6 - g(x))^2 - (6 - h(x))^2 \right) dx$$

$$4 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative of } 3 \cos\left(\frac{\pi}{2}x\right) \\ 1 : \text{antiderivative of} \\ \text{remaining terms} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{form of integrand} \\ 1 : \text{integrand} \end{cases}$$

2019 SCORING GUIDELINES

Question 6

(a) $h'(2) = \frac{2}{3}$

1 : answer

(b) $a'(x) = 9x^2h(x) + 3x^3h'(x)$

$$a'(2) = 9 \cdot 2^2 h(2) + 3 \cdot 2^3 h'(2) = 36 \cdot 4 + 24 \cdot \frac{2}{3} = 160$$

3 : $\begin{cases} 1 : \text{form of product rule} \\ 1 : a'(x) \\ 1 : a'(2) \end{cases}$ (c) Because h is differentiable, h is continuous, so $\lim_{x \rightarrow 2} h(x) = h(2) = 4$.

$$\text{Also, } \lim_{x \rightarrow 2} h(x) = \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3}, \text{ so } \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4.$$

$$\text{Because } \lim_{x \rightarrow 2} (x^2 - 4) = 0, \text{ we must also have } \lim_{x \rightarrow 2} (1 - (f(x))^3) = 0.$$

$$\text{Thus } \lim_{x \rightarrow 2} f(x) = 1.$$

$$\text{Because } f \text{ is differentiable, } f \text{ is continuous, so } f(2) = \lim_{x \rightarrow 2} f(x) = 1.$$

Also, because f is twice differentiable, f' is continuous, so

$$\lim_{x \rightarrow 2} f'(x) = f'(2) \text{ exists.}$$

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = \lim_{x \rightarrow 2} \frac{2x}{-3(f(x))^2 f'(x)} = \frac{4}{-3(1)^2 \cdot f'(2)} = 4.$$

$$\text{Thus } f'(2) = -\frac{1}{3}.$$

4 : $\begin{cases} 1 : \lim_{x \rightarrow 2} \frac{x^2 - 4}{1 - (f(x))^3} = 4 \\ 1 : f(2) \\ 1 : \text{L'Hospital's Rule} \\ 1 : f'(2) \end{cases}$ (d) Because g and h are differentiable, g and h are continuous, so

$$\lim_{x \rightarrow 2} g(x) = g(2) = 4 \text{ and } \lim_{x \rightarrow 2} h(x) = h(2) = 4.$$

Because $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$, it follows from the squeeze theorem that $\lim_{x \rightarrow 2} k(x) = 4$.

$$\text{Also, } 4 = g(2) \leq k(2) \leq h(2) = 4, \text{ so } k(2) = 4.$$

Thus k is continuous at $x = 2$.

1 : continuous with justification

2018

AP[®]

CollegeBoard

AP Calculus AB

Scoring Guidelines

2018 SCORING GUIDELINES

Question 1

(a) $\int_0^{300} r(t) dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$.

(b) $20 + \int_0^{300} (r(t) - 0.7) dt = 20 + \int_0^{300} r(t) dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or } 414.285 \text{) seconds.}$$

(d) The total number of people in line at time t , $0 \leq t \leq 300$, is modeled by

$$20 + \int_0^t r(x) dx - 0.7t.$$

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

1 : answer

4 : $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

2018 SCORING GUIDELINES

Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118 .

(b) $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760 .

(c) $\int_0^{3.5} v(t) dt = 2.844$ (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral $\int_0^{3.5} v(t) dt$ is the displacement of the particle over the time interval $0 \leq t \leq 3.5$.

The integral $\int_0^{3.5} |v(t)| dt$ is the total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

(d) $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

1 : answer

3 : $\begin{cases} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{cases}$

2 : $\begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$

2018 SCORING GUIDELINES

Question 3

$$\begin{aligned} \text{(a)} \quad f(-5) &= f(1) + \int_1^{-5} g(x) \, dx = f(1) - \int_{-5}^1 g(x) \, dx \\ &= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2} \end{aligned}$$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b)} \quad \int_1^6 g(x) \, dx &= \int_1^3 g(x) \, dx + \int_3^6 g(x) \, dx \\ &= \int_1^3 2 \, dx + \int_3^6 2(x-4)^2 \, dx \\ &= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10 \end{aligned}$$

3 : $\begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

2018 SCORING GUIDELINES

Question 4

$$\text{(a)} \quad H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$$\text{(b)} \quad \frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$$

Because H is differentiable on $3 \leq t \leq 5$, H is continuous on $3 \leq t \leq 5$.

By the Mean Value Theorem, there exists a value c , $3 < c < 5$, such that $H'(c) = 2$.

(c) The average height of the tree over the time interval $2 \leq t \leq 10$ is given by $\frac{1}{10-2} \int_2^{10} H(t) \, dt$.

$$\begin{aligned} \frac{1}{8} \int_2^{10} H(t) \, dt &\approx \frac{1}{8} \left(\frac{1.5+2}{2} \cdot 1 + \frac{2+6}{2} \cdot 2 + \frac{6+11}{2} \cdot 2 + \frac{11+15}{2} \cdot 3 \right) \\ &= \frac{1}{8} (65.75) = \frac{263}{32} \end{aligned}$$

The average height of the tree over the time interval $2 \leq t \leq 10$ is $\frac{263}{32}$ meters.

$$\text{(d)} \quad G(x) = 50 \Rightarrow x = 1$$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$$

$$\left. \frac{d}{dt}(G(x)) \right|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\frac{3}{4}$ meter per year.

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

2 : $\begin{cases} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

2 : $\begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

3 : $\begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0] if no chain rule

2018 SCORING GUIDELINES

Question 5

- (a) The average rate of change of
- f
- on the interval
- $0 \leq x \leq \pi$
- is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

- (b)
- $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

- (c)
- $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \leq x \leq 2\pi$ is $-\frac{1}{\sqrt{2}}e^{5\pi/4}$.

- (d)
- $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

1 : answer

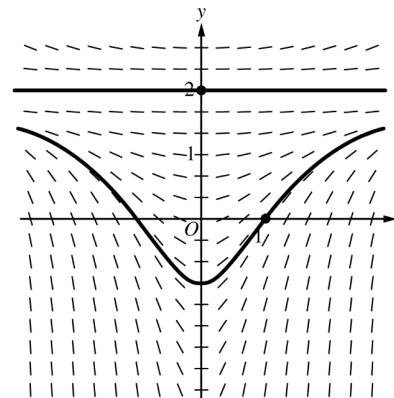
2 : $\begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$ 3 : $\begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$ 3 : $\begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \text{and limits equal 0} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

2018 SCORING GUIDELINES

Question 6

- (a)



- 2 :
- $\begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

- (b)
- $\frac{dy}{dx}\bigg|_{(x,y)=(1,0)} = \frac{4}{3}$

An equation for the line tangent to the graph of $y = f(x)$ at

$$x = 1 \text{ is } y = \frac{4}{3}(x - 1).$$

$$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$$

- 2 :
- $\begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

- (c)
- $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$

$$\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$$

$$y = 2 - \frac{6}{x^2 + 2}$$

Note: this solution is valid for $-\infty < x < \infty$.

- 5 :
- $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

AP[®] Calculus AB

2016 Scoring Guidelines

© 2016 The College Board. College Board, Advanced Placement Program, AP, AP Central, and the acorn logo are registered trademarks of the College Board.

Visit the College Board on the Web: www.collegeboard.org.

AP Central is the official online home for the AP Program: apcentral.collegeboard.org.

2016 SCORING GUIDELINES

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

- Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

$$(a) \quad R'(2) \approx \frac{R(3) - R(1)}{3 - 1} = \frac{950 - 1190}{3 - 1} = -120 \text{ liters/hr}^2$$

$$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$$

- The total amount of water removed is given by $\int_0^8 R(t) dt$.

$$\begin{aligned} \int_0^8 R(t) dt &\approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters} \end{aligned}$$

This is an overestimate since R is a decreasing function.

$$\begin{aligned} (c) \quad \text{Total} &\approx 50000 + \int_0^8 W(t) dt - 8050 \\ &= 50000 + 7836.195325 - 8050 \approx 49786 \text{ liters} \end{aligned}$$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$$

- $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

$$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$$

Therefore, the Intermediate Value Theorem guarantees at least one time t , $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

For this value of t , the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

2016 SCORING GUIDELINES

Question 2

For $t \geq 0$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right).$$

The particle is at position $x = 2$ at time $t = 4$.

- (a) At time $t = 4$, is the particle speeding up or slowing down?
 (b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.
 (c) Find the position of the particle at time $t = 0$.
 (d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

(a) $v(4) = 2.978716 > 0$
 $v'(4) = -1.164000 < 0$

The particle is slowing down since the velocity and acceleration have different signs.

(b) $v(t) = 0 \Rightarrow t = 2.707468$

$v(t)$ changes from positive to negative at $t = 2.707$.
 Therefore, the particle changes direction at this time.

(c) $x(0) = x(4) + \int_4^0 v(t) dt$
 $= 2 + (-5.815027) = -3.815$

(d) Distance $= \int_0^3 |v(t)| dt = 5.301$

2 : conclusion with reason

2 : $\begin{cases} 1 : t = 2.707 \\ 1 : \text{justification} \end{cases}$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

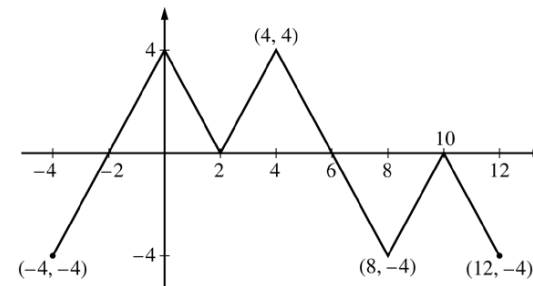
2016 SCORING GUIDELINES

Question 3

The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by

$$g(x) = \int_2^x f(t) dt.$$

- (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
 (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
 (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.



Graph of f

- (d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

- (a) The function g has neither a relative minimum nor a relative maximum at $x = 10$ since $g'(x) = f(x)$ and $f(x) \leq 0$ for $8 \leq x \leq 12$.

- (b) The graph of g has a point of inflection at $x = 4$ since $g'(x) = f(x)$ is increasing for $2 \leq x \leq 4$ and decreasing for $4 \leq x \leq 6$.

- (c) $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$.

x	$g(x)$
-4	-4
-2	-8
6	8
12	-4

On the interval $-4 \leq x \leq 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$.

- (d) $g(x) \leq 0$ for $-4 \leq x \leq 2$ and $10 \leq x \leq 12$.

1 : $g'(x) = f(x)$ in (a), (b), (c), or (d)

1 : answer with justification

1 : answer with justification

4 : $\begin{cases} 1 : \text{considers } x = -2 \text{ and } x = 6 \\ \quad \text{as candidates} \\ 1 : \text{considers } x = -4 \text{ and } x = 12 \\ 2 : \text{answers with justification} \end{cases}$

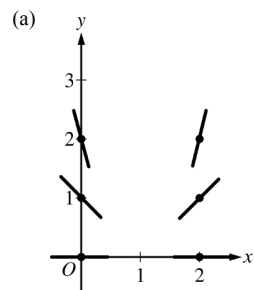
2 : intervals

2016 SCORING GUIDELINES

Question 4

Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.



(b) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = \frac{3^2}{2-1} = 9$

An equation for the tangent line is $y = 9(x - 2) + 3$.

$$f(2.1) \approx 9(2.1 - 2) + 3 = 3.9$$

(c) $\frac{1}{y^2} dy = \frac{1}{x-1} dx$

$$\int \frac{1}{y^2} dy = \int \frac{1}{x-1} dx$$

$$-\frac{1}{y} = \ln|x-1| + C$$

$$-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$$

$$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$$

$$y = \frac{1}{\frac{1}{3} - \ln(x-1)}$$

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

2 : $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

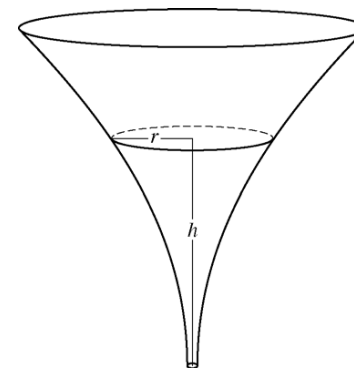
5 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: 0/5 if no separation of variables

2016 SCORING GUIDELINES

Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

- (a) Find the average value of the radius of the funnel.
- (b) Find the volume of the funnel.
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

(a) Average radius $= \frac{1}{10} \int_0^{10} \frac{1}{20}(3 + h^2) dh = \frac{1}{200} \left[3h + \frac{h^3}{3} \right]_0^{10}$

$$= \frac{1}{200} \left(\left(30 + \frac{1000}{3} \right) - 0 \right) = \frac{109}{60} \text{ in}$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_0^{10} \left(\left(\frac{1}{20}(3 + h^2) \right)^2 \right) dh = \frac{\pi}{400} \int_0^{10} (9 + 6h^2 + h^4) dh$

$$= \frac{\pi}{400} \left[9h + 2h^3 + \frac{h^5}{5} \right]_0^{10}$$

$$= \frac{\pi}{400} \left(\left(90 + 2000 + \frac{100000}{5} \right) - 0 \right) = \frac{2209\pi}{40} \text{ in}^3$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dr}{dt} = \frac{1}{20}(2h) \frac{dh}{dt}$

$$-\frac{1}{5} = \frac{3}{10} \frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{1}{5} \cdot \frac{10}{3} = -\frac{2}{3} \text{ in/sec}$$

3 : $\begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

2016 SCORING GUIDELINES

Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x) dx$.

$$\begin{aligned} \text{(a)} \quad k(3) &= f(g(3)) = f(6) = 4 \\ k'(3) &= f'(g(3)) \cdot g'(3) = f'(6) \cdot 2 = 5 \cdot 2 = 10 \end{aligned}$$

An equation for the tangent line is $y = 10(x - 3) + 4$.

$$\begin{aligned} \text{(b)} \quad h'(1) &= \frac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{(f(1))^2} \\ &= \frac{(-6) \cdot 8 - 2 \cdot 3}{(-6)^2} = \frac{-54}{36} = -\frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \int_1^3 f''(2x) dx &= \frac{1}{2} [f'(2x)]_1^3 = \frac{1}{2} [f'(6) - f'(2)] \\ &= \frac{1}{2} [5 - (-2)] = \frac{7}{2} \end{aligned}$$

$$3 : \begin{cases} 2 : \text{slope at } x = 3 \\ 1 : \text{equation for tangent line} \end{cases}$$

$$3 : \begin{cases} 2 : \text{expression for } h'(1) \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

AP[®] Calculus AB

2015 Scoring Guidelines

© 2015 The College Board. College Board, Advanced Placement Program, AP, AP Central, and the acorn logo are registered trademarks of the College Board.

Visit the College Board on the Web: www.collegeboard.org.

AP Central is the official online home for the AP Program: apcentral.collegeboard.org.

2015 SCORING GUIDELINES

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
- (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
- (c) At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t) dt = 76.570$

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$
 Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

2 : $\begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t , $0 \leq t \leq 8$, is $30 + \int_0^t [R(x) - D(x)] dx$.

3 : $\begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

$$R(t) - D(t) = 0 \Rightarrow t = 0, 3.271658$$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

(d) $30 + \int_0^w [R(t) - D(t)] dt = 50$

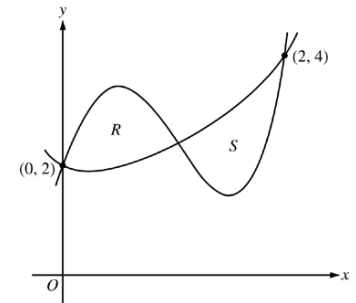
2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

2015 SCORING GUIDELINES

Question 2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

- (a) Find the sum of the areas of regions R and S .
- (b) Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.
- (c) Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.



- (a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$.

$$\begin{aligned} \text{Area} &= \int_0^A [g(x) - f(x)] dx + \int_A^2 [f(x) - g(x)] dx \\ &= 0.997427 + 1.006919 = 2.004 \end{aligned}$$

4 : $\begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$

(b) $\text{Volume} = \int_A^2 [f(x) - g(x)]^2 dx = 1.283$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $h(x) = f(x) - g(x)$
 $h'(x) = f'(x) - g'(x)$
 $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

2 : $\begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

- (a) Use the data in the table to estimate the value of $v'(16)$.
- (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.

- (c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

Find Bob's acceleration at time $t = 5$.

- (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

(a) $v'(16) \approx \frac{240 - 200}{20 - 12} = 5$ meters/min²

1 : approximation

- (b) $\int_0^{40} |v(t)| dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \leq t \leq 40$ minutes.

3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned} \int_0^{40} |v(t)| dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters} \end{aligned}$$

- (c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.

$$B'(5) = 3(25) - 12(5) = 15 \text{ meters/min}^2$$

2 : $\begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

- (d) Avg vel = $\frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300) dt$

$$\begin{aligned} &= \frac{1}{10} \left[\frac{t^4}{4} - 2t^3 + 300t \right]_0^{10} \\ &= \frac{1}{10} \left[\frac{10000}{4} - 2000 + 3000 \right] = 350 \text{ meters/min} \end{aligned}$$

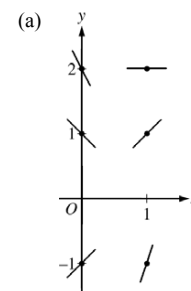
3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.
- (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.



2 : $\begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$

(b) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$

In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$.

Therefore, all solution curves are concave up in Quadrant II.

2 : $\begin{cases} 1 : \frac{d^2y}{dx^2} \\ 1 : \text{concave up with reason} \end{cases}$

(c) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = 2(2) - 3 = 1 \neq 0$

Therefore, f has neither a relative minimum nor a relative maximum at $x = 2$.

2 : $\begin{cases} 1 : \text{considers } \left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} \\ 1 : \text{conclusion with justification} \end{cases}$

(d) $y = mx + b \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(mx + b) = m$

$$2x - y = m$$

$$2x - (mx + b) = m$$

$$(2 - m)x - (m + b) = 0$$

$$2 - m = 0 \Rightarrow m = 2$$

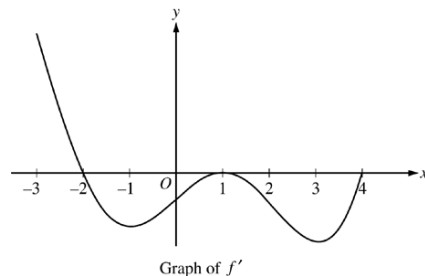
$$b = -m \Rightarrow b = -2$$

3 : $\begin{cases} 1 : \frac{d}{dx}(mx + b) = m \\ 1 : 2x - y = m \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 5

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



- (a) Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- (c) Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

- (a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$.
 $f'(x)$ changes from positive to negative at $x = -2$.
 Therefore, f has a relative maximum at $x = -2$.

2 : $\begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$

- (b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

- (c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points.

2 : $\begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$

The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.

- (d) $f(x) = 3 + \int_1^x f'(t) dt$

$$f(4) = 3 + \int_1^4 f'(t) dt = 3 + (-12) = -9$$

$$f(-2) = 3 + \int_1^{-2} f'(t) dt = 3 - \int_{-2}^1 f'(t) dt = 3 - (-9) = 12$$

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$

2015 SCORING GUIDELINES

Question 6

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

- (a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.
- (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.
- (c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

(a) $\frac{dy}{dx}\bigg|_{(x,y)=(-1,1)} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$

2 : $\begin{cases} 1 : \text{slope} \\ 1 : \text{equation for tangent line} \end{cases}$

An equation for the tangent line is $y = \frac{1}{4}(x + 1) + 1$.

(b) $3y^2 - x = 0 \Rightarrow x = 3y^2$

So, $y^3 - xy = 2 \Rightarrow y^3 - (3y^2)(y) = 2 \Rightarrow y = -1$

$(-1)^3 - x(-1) = 2 \Rightarrow x = 3$

The tangent line to the curve is vertical at the point $(3, -1)$.

3 : $\begin{cases} 1 : \text{sets } 3y^2 - x = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{coordinates} \end{cases}$

(c) $\frac{d^2y}{dx^2} = \frac{(3y^2 - x)\frac{dy}{dx} - y\left(6y\frac{dy}{dx} - 1\right)}{(3y^2 - x)^2}$

$$\frac{d^2y}{dx^2}\bigg|_{(x,y)=(-1,1)} = \frac{(3 \cdot 1^2 - (-1)) \cdot \frac{1}{4} - 1 \cdot \left(6 \cdot 1 \cdot \frac{1}{4} - 1\right)}{(3 \cdot 1^2 - (-1))^2} = \frac{1 - \frac{1}{2}}{16} = \frac{1}{32}$$

4 : $\begin{cases} 2 : \text{implicit differentiation} \\ 1 : \text{substitution for } \frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$