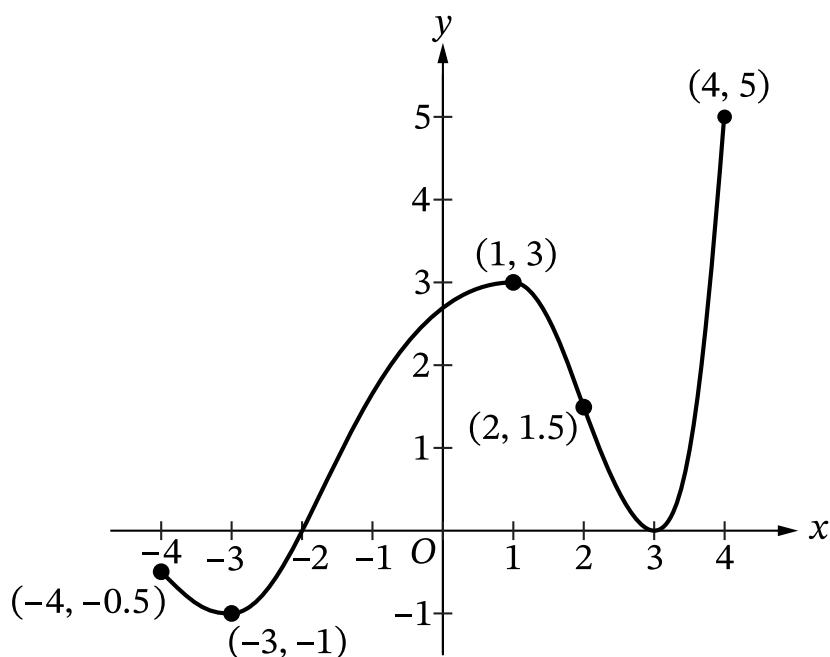


4. Let f be a twice-differentiable function on the closed interval $[-4, 4]$ with $f(2) = 3$. The graph of f' , the derivative of f , is shown.

Graph of f' **Part A**

For $x > 0$, the function g is defined by $g(x) = f(x) - \ln x$. Find $g'(2)$. Show the work that leads to your answer.

Part B

Find all values of x on the open interval $0 < x < 3$ at which the graph of f has a point of inflection. Give a reason for your answer.

Part C

For $-4 \leq x \leq 4$, on what open intervals, if any, is the graph of f both increasing and concave down? Give a reason for your answer.

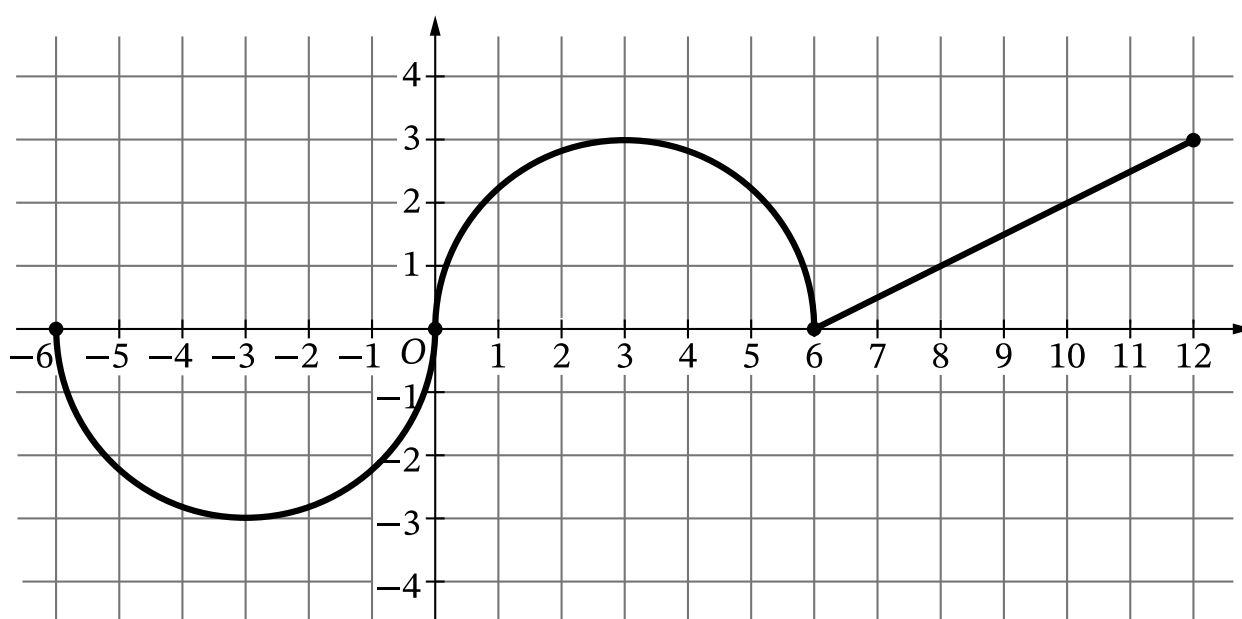
Part D

For $-4 \leq x \leq 4$, find the value of x at which f has an absolute minimum and the value of x at which f has an absolute maximum. Give reasons for your answers.

1. An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

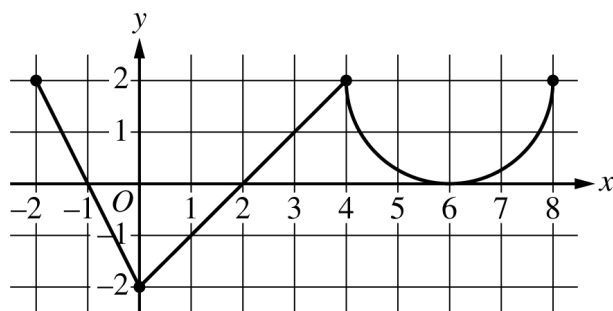
- A. Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.
- B. Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.
- C. Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.
- D. At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A , defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x) dx$, models the number of acres affected by the species over the time interval $4 \leq t \leq 36$. At what time t , for $4 \leq t \leq 36$, does A attain its maximum value? Justify your answer.
4. The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f , consisting of two semicircles and one line segment, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t) dt$.

- A. Find $g'(8)$. Give a reason for your answer.
- B. Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.
- C. Find $g(12)$ and $g(0)$. Label your answers.
- D. Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f' , the derivative of f , consists of two line segments and a semicircle, as shown in the figure.
- (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.
- (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.
- (c) Find the value of $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.
- (d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

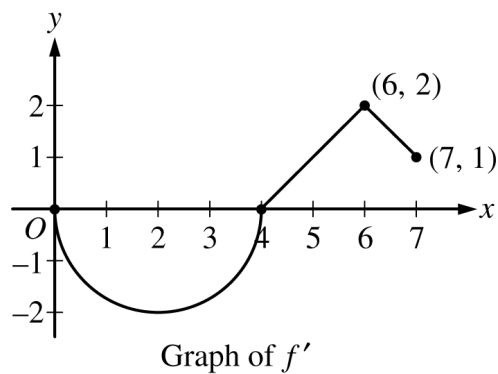
1. From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by

$A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour.

Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

- (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).
- (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).
- (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.
- (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t , for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400) dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



3. Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f' , the derivative of f , consists of a semicircle and two line segments, as shown in the figure above.
- (a) Find $f(0)$ and $f(5)$.
- (b) Find the x -coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.
- (c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.
- (d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

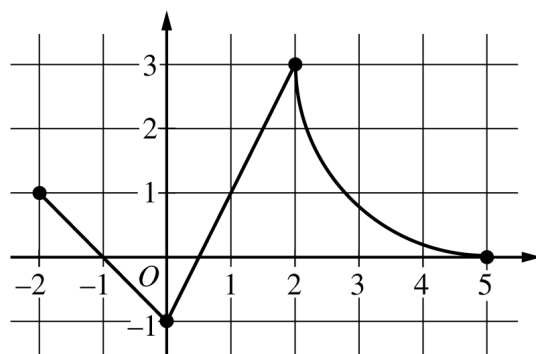
1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15 \sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).
- (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.
- (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?
- (c) At what time t , for $0 \leq t \leq 8$, is the greatest number of fish in the lake? Justify your answer.
- (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.
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CALCULUS AB
SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f

3. The continuous function f is defined on the closed interval $-6 \leq x \leq 5$. The figure above shows a portion of the graph of f , consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f .
- (a) If $\int_{-6}^5 f(x) \, dx = 7$, find the value of $\int_{-6}^{-2} f(x) \, dx$. Show the work that leads to your answer.
- (b) Evaluate $\int_3^5 (2f'(x) + 4) \, dx$.
- (c) The function g is given by $g(x) = \int_{-2}^x f(t) \, dt$. Find the absolute maximum value of g on the interval $-2 \leq x \leq 5$. Justify your answer.
- (d) Find $\lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$.
-

5. Let f be the function defined by $f(x) = e^x \cos x$.

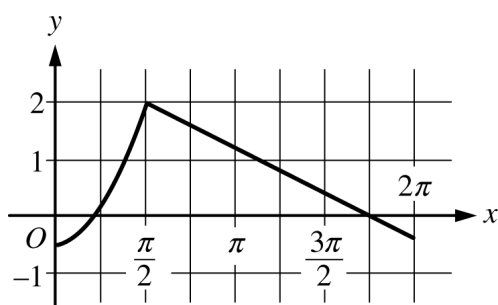
(a) Find the average rate of change of f on the interval $0 \leq x \leq \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \leq x \leq 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g' , the derivative of g , is shown

below. Find the value of $\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



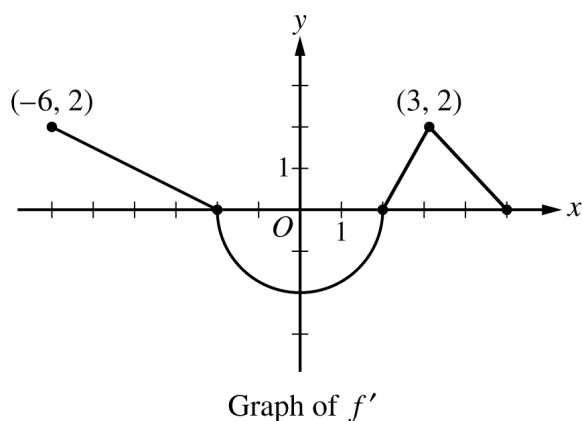
Graph of g'

CALCULUS AB
SECTION II, Part B

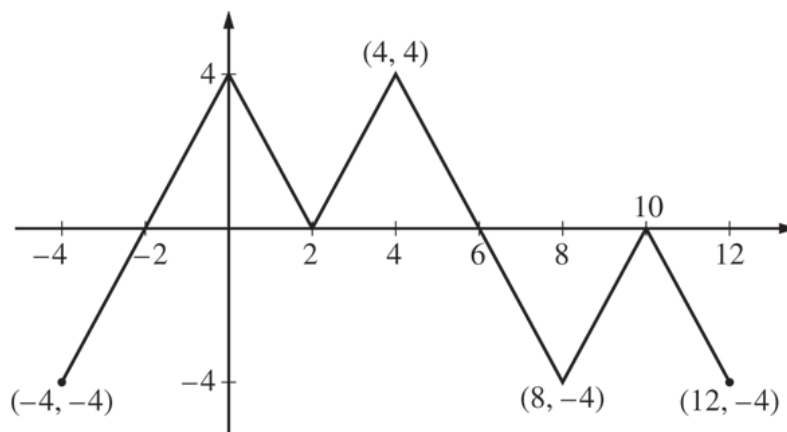
Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f' , the derivative of f , consists of a semicircle and three line segments, as shown in the figure above.
- (a) Find the values of $f(-6)$ and $f(5)$.
- (b) On what intervals is f increasing? Justify your answer.
- (c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
- (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.
-

CALCULUS AB
SECTION II, Part B**Time—60 minutes****Number of problems—4****No calculator is allowed for these problems.**Graph of f

3. The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_2^x f(t) dt$.
- (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
 - (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
 - (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.
 - (d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.
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CALCULUS AB
SECTION II, Part A**Time—30 minutes****Number of problems—2****A graphing calculator is required for these problems.**

1. The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.
- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
- (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
- (c) At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.
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