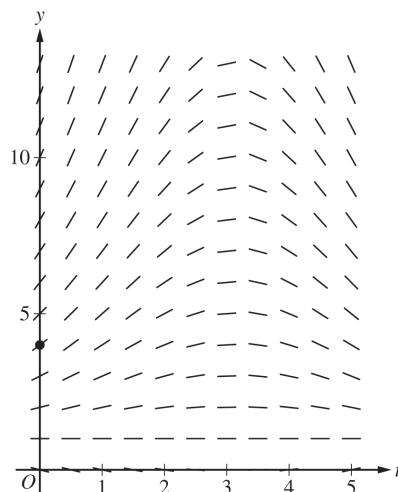


3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation

$$\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right), \text{ where } H(t) \text{ is measured in feet and } t \text{ is measured in hours after noon } (t = 0). \text{ It is}$$

known that $H(0) = 4$.

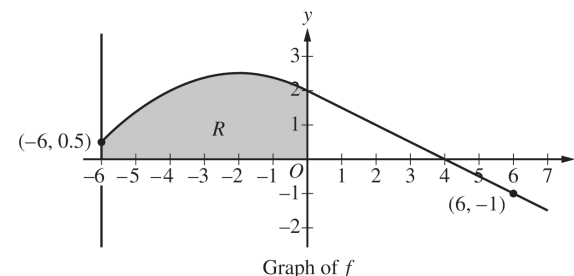
- (a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



- (b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t , for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.
- (c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation

$$\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right) \text{ with initial condition } H(0) = 4.$$

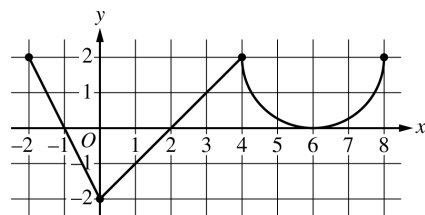
Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



4. The graph of the differentiable function f , shown for $-6 \leq x \leq 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \leq x \leq 7$. Let R be the region in the second quadrant bounded by the graph of f , the vertical line $x = -6$, and the x - and y -axes. Region R has area 12.

- (a) The function g is defined by $g(x) = \int_0^x f(t) dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.
- (b) For the function g defined in part (a), find all values of x in the interval $0 \leq x \leq 6$ at which the graph of g has a critical point. Give a reason for your answer.
- (c) The function h is defined by $h(x) = \int_{-6}^x f'(t) dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

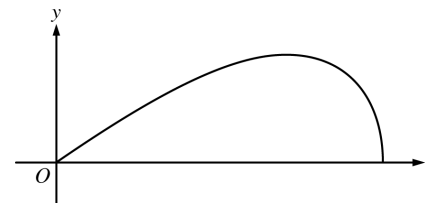
Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f' , the derivative of f , consists of two line segments and a semicircle, as shown in the figure.

- Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.
- On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.
- Find the value of $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.
- Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c . Each spinning toy is in the shape of the solid generated when such a region is revolved about the x -axis. Both x and y are measured in inches.

- Find the area of the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.
- It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?
- For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^4 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

(c) For $0 \leq x \leq \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. People enter a line for an escalator at a rate modeled by the function r given by

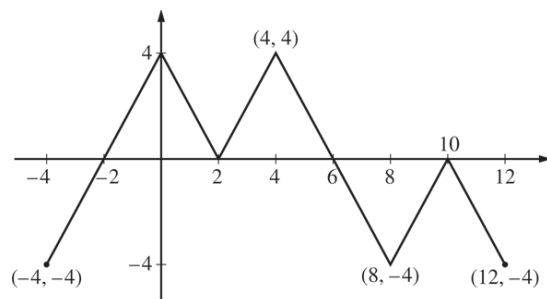
$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \leq t \leq 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

- How many people enter the line for the escalator during the time interval $0 \leq t \leq 300$?
- During the time interval $0 \leq t \leq 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?
- For $t > 300$, what is the first time t that there are no people in line for the escalator?
- For $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



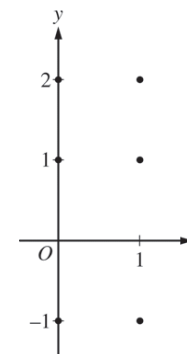
Graph of f

3. The figure above shows the graph of the piecewise-linear function f . For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_2^x f(t) dt$.

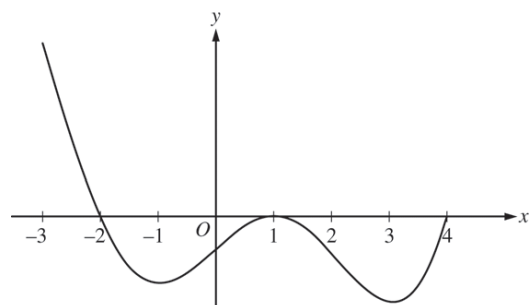
- Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.
- Does the graph of g have a point of inflection at $x = 4$? Justify your answer.
- Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.
- For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

- On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



- Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.
- Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

Graph of f'

5. The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.
- Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
 - On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
 - Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
 - Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.
- Find the x -coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.
 - Explain why there must be a value c , for $-1 < c < 1$, such that $f''(c) = 0$.
 - The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.
 - Evaluate $\int_{-2}^3 f'(g(x))g'(x) dx$.