

Part B (AB): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x .

Model Solution	Scoring
(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer. $h'(x) = f'(g(x)) \cdot g'(x)$ $h'(7) = f'(g(7)) \cdot g'(7)$ $= f'(0) \cdot 8 = \frac{3}{2} \cdot 8 = 12$	Chain rule 1 point Answer 1 point
Scoring notes: - The first point is earned for either $h'(x) = f'(g(x)) \cdot g'(x)$ or $h'(7) = f'(g(7)) \cdot g'(7)$. - If the first point is earned, the second point is earned only for an answer of 12 (or equivalent). - If the first point is not earned, the second point can be earned only for a response of either $f'(0) \cdot 8 = 12$ or $\frac{3}{2} \cdot 8$. - A response of 12 with no supporting work does not earn either point.	
Total for part (a) 2 points	

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer. $k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$	Product or chain rule 1 point
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$k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$ $= 2 \cdot 4 \cdot 3 \cdot (-3) + 4^2 \cdot 2 = -72 + 32 = -40$	$k''(4)$ 1 point
The graph of k is concave down at the point where $x = 4$ because $k''(4) < 0$ and k'' is continuous.	Answer with reason 1 point

Scoring notes:

- The first point is earned for either $k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$.
- The first point is also earned by any of the following incorrect expressions, each of which has a single error in the application of the product rule or the chain rule:
 - $2f(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
 - $2f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
 - $f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
 - $2f(x) \cdot f'(x) \cdot g'(x)$ or $2f(4) \cdot f'(4) \cdot g'(4)$
 - Note: A response that presents one of these expressions cannot earn the second point.
- To earn the second point a response must correctly find $k''(4) = -40$ (or equivalent) with supporting work.
- The third point is earned for an answer and reason that are consistent with any declared nonzero value of $k''(4)$.

Total for part (b) 3 points

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t) dt$. Find $m(2)$. Show the work that leads to your answer. $m(2) = 5 \cdot 8 + \int_0^2 f'(t) dt = 40 + (f(2) - f(0))$ $= 40 + (7 - 10) = 37$	Answer with supporting work 1 point
Scoring notes: - The point is earned only for an answer of 37 (or equivalent) with supporting work equivalent to $5 \cdot 8 + (f(2) - f(0))$, $40 + (f(2) - f(0))$, $5 \cdot 8 + (7 - 10)$, or $40 + (7 - 10)$. - An answer of 37 with no supporting work does not earn the point.	
Total for part (c) 1 point	

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer. $m'(x) = 15x^2 + f'(x)$ $m'(2) = 15 \cdot 4 + f'(2) = 60 + (-8) = 52$	Considers $m'(x)$ 1 point $m'(2)$ with supporting work 1 point
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The graph of m is increasing at $x = 2$ because $m'(2) > 0$.

Answer with
justification

1 point

Scoring notes:

- The first point is earned for considering $m'(x)$, $m'(2)$, or m' . This consideration may appear in a justification statement.
- The second point is earned for $m'(2) = 15 \cdot 2^2 + f'(2)$, $m'(2) = 60 + f'(2)$, or $m'(2) = 60 - 8$ but is not earned for an unsupported response of $m'(2) = 52$.
- The third point is earned for an answer and justification consistent with any declared value of $m'(2)$.

Total for part (d) 3 points

Total for question 5 9 points

Part A (AB or BC): Graphing calculator required

Question 1

9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by

$A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

Model Solution

Scoring

- (a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by $\int_1^5 A(t) dt$.

Answer

1 point

Scoring notes:

- The response must be a definite integral with correct lower and upper limits to earn this point.
- Because $|A(t)| = A(t)$ for $1 \leq t \leq 5$, a response of $\int_1^5 |450\sqrt{\sin(0.62t)}| dt$ or $\int_1^5 |A(t)| dt$ earns the point.
- A response missing dt or using dx is eligible to earn the point.
- A response with a copy error in the expression for $A(t)$ will earn the point only in the presence of $\int_1^5 A(t) dt$.

Total for part (a) 1 point

- (b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

$$\text{Average} = \frac{1}{5-1} \int_1^5 A(t) dt = 375.536966$$

Uses average value formula: **1 point**

$$\frac{1}{b-a} \int_a^b A(t) dt$$

The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is 375.537 (or 375.536) vehicles per hour.

Answer **1 point**

Scoring notes:

- The use of the average value formula, indicating that $a = 1$ and $b = 5$, can be presented in single or multiple steps to earn the first point. For example, the following response earns both points:

$$\int_1^5 A(t) dt = 1502.147865, \text{ so the average value is } 375.536966.$$

- A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points: $\int_1^5 A(t) dt = 1502.147865 = 375.536966$.

- The answer must be correct to three decimal places. For example,

$$\frac{1}{5-1} \int_1^5 A(t) dt = 375.536966 \approx 376 \text{ earns only the first point.}$$

- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{4} \int_1^5 A(t) dt = 79.416068$.

- Special case: $\frac{1}{5} \int_1^5 A(t) dt = 300.429573$ earns 1 out of 2 points.

Total for part (b) 2 points

- (c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

$$A'(1) = 148.947272$$

Considers $A'(1)$ **1 point**

Because $A'(1) > 0$, the rate at which the vehicles arrive at the toll plaza is increasing.

Answer with reason **1 point**

Scoring notes:

- The response need not present the value of $A'(1)$. The second line of the model solution earns both points.
- An incorrect value assigned to $A'(1)$ earns the first point (but will not earn the second point).
- Without a reference to $t = 1$, the first point is earned by any of the following:
 - 148.947 accurate to the number of decimals presented, with zero up to three decimal places (i.e., 149, 148, 148.9, 148.95, or 148.94)
 - $A'(t) = 148.947$ by itself
- To be eligible for the second point, the first point must be earned.
- To earn the second point, there must be a reference to $t = 1$.
- Degree mode: $A'(1) = 23.404311$

Total for part (c) 2 points

- (d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t , for $a \leq t \leq 4$, is given by

$N(t) = \int_a^t (A(x) - 400) dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

$$N'(t) = A(t) - 400 = 0$$

Considers $N'(t) = 0$ **1 point**

$$\Rightarrow A(t) = 400 \Rightarrow t = 1.469372, t = 3.597713$$

$t = a$ and $t = b$ **1 point**

$$a = 1.469372$$

$$b = 3.597713$$

t	$N(t) = \int_a^t (A(x) - 400) dx$
a	0
b	71.254129
4	62.338346

Answer **1 point**

Justification **1 point**

The greatest number of vehicles in line is 71.

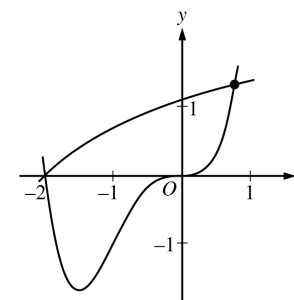
Scoring notes:

- It is not necessary to indicate that $A(t) = 400$ to earn the first point, although this statement alone would earn the first point.
- A response of “ $A(t) \geq 400$ when $1.469372 \leq t \leq 3.597713$ ” will earn the first 2 points. A response of “ $A(t) \geq 400$ ” along with the presence of exactly one of the two numbers above will earn the first point, but not the second. A response of “ $A(t) \geq 400$ ” by itself will not earn either of the first 2 points.
- To earn the second point the values for a and b must be accurate to the number of decimals presented, with at least one and up to three decimal places. These may appear only in a candidates table, as limits of integration, or on a number line.
- A response with incorrect notation involving t or x is eligible to earn all 4 points.
- A response that does not earn the first point is still eligible for the remaining 3 points.
- To earn the third point, a response must present the greatest number of vehicles. This point is earned for answers of either 71 or 71.254*** only.
- A correct justification earns the fourth point, even if the third point is not earned because of a decimal presentation error.
- When using a Candidates Test, the response must include the values for $N(a)$, $N(b)$, and $N(4)$ to earn the fourth point. These values must be correct to the number of decimals presented, with up to three decimal places. (Correctly rounded integer values are acceptable.)
- Alternate solution for the third and fourth points:
For $a \leq t \leq b$, $A(t) \geq 400$. For $b \leq t \leq 4$, $A(t) \leq 400$.
Thus, $N(t) = \int_a^t (A(x) - 400) dx$ is greatest at $t = b$.
 $N(b) = 71.254129$, and the greatest number of vehicles in line is 71.
- Degree mode: The response is only eligible to earn the first point because in degree mode $A(t) < 400$.

Total for part (d) 4 points**Total for question 1 9 points****Part A (AB): Graphing calculator required****Question 2****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Let f and g be the functions defined by $f(x) = \ln(x+3)$ and $g(x) = x^4 + 2x^3$. The graphs of f and g , shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.

Model Solution		Scoring
(a) Find the area of the region enclosed by the graphs of f and g .	$\ln(x+3) = x^4 + 2x^3 \Rightarrow x = -2, x = B = 0.781975$	Integrand 1 point
	$\int_{-2}^B (f(x) - g(x)) dx = 3.603548$	Limits of integration 1 point
	The area of the region is 3.604 (or 3.603).	Answer 1 point

Scoring notes:

- Other forms of the integrand in a definite integral, e.g., $|f(x) - g(x)|$, $|g(x) - f(x)|$, or $g(x) - f(x)$, earn the first point.
- To earn the second point, the response must have a lower limit of -2 and an upper limit expressed as either the letter B with no value attached, or a number that is correct to the number of digits presented, with at least one and up to three decimal places.
 - Case 1: If the response did not earn the second point because of an incorrect value of B , $0 < B < 1$, but used a lower limit of -2 , the response earns the third point only for a consistent answer.
 - Case 2: If the response did not earn the second point because the lower limit used was $x = 0$, but the response used a correct upper limit of B , the response earns the third point for a consistent answer of 0.708 (or 0.707).
 - Case 3: If a response uses any other incorrect limits it does not earn the second or third points.
- A response containing the integrand $g(x) - f(x)$ must interpret the value of the resulting integral correctly to earn the third point. For example, the following response earns all 3 points:
 $\int_{-2}^B (g(x) - f(x)) dx = -3.604$ so the area is 3.604 . However, the response
 “Area = $\int_{-2}^B (g(x) - f(x)) dx = 3.604$ ” presents an untrue statement and earns the first and second points but not the third point.
- A response must earn the first point in order to be eligible for the third point. If the response has earned the second point, then only the correct answer will earn the third point.
- Instructions for scoring a response that presents an integrand of $\ln(x+3) - x^4 + 2x^3$ and the correct answer are shown in the “Global Special Case” after part (d).

Total for part (a) 3 points

- (b) For $-2 \leq x \leq B$, let $h(x)$ be the vertical distance between the graphs of f and g . Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.

$h(x) = f(x) - g(x)$	Considers $h'(-0.5)$	1 point
$h'(x) = f'(x) - g'(x)$	– OR –	
$h'(-0.5) = f'(-0.5) - g'(-0.5) = -0.6$ (or -0.599)	$f'(x) - g'(x)$	
Since $h'(-0.5) < 0$, h is decreasing at $x = -0.5$.	Answer with reason	1 point

Scoring notes:

- The response need not present the value of $h'(-0.5)$. The last line earns both points. However, if a value is presented it must be correct for the digits reported up to three decimal places.
- A response that reports an incorrect value of $h'(-0.5)$ earns only the first point.
- A response that presents only $h'(x)$ does not earn either point.
- The only response that earns the second point for concluding “ h is increasing” is described in the “Global Special Case” provided after part (d).
- A response that compares the values of $f'(x)$ and $g'(x)$ at $x = -0.5$ earns the first point and is eligible for the second point. This comparison can be made symbolically or verbally; for example, the response “the rate of change of $f(x)$ is less than the rate of change of $g(x)$ at $x = -0.5$ ” earns the first point.

Total for part (b) 2 points

- (c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x -axis are squares. Find the volume of the solid.

$\int_{-2}^8 (f(x) - g(x))^2 dx = 5.340102$	Integrand	1 point
The volume of the solid is 5.340.	Answer	1 point

Scoring notes:

- The first point is earned for an integrand of $k(f(x) - g(x))^2$ or its equivalent with $k \neq 0$ in any definite integral. If $k \neq 1$, then the response is not eligible for the second point.
- A response that does not earn the first point is ineligible to earn the second point, with the following exceptions:
 - A response which has a presentation error in the integrand (for example, mismatched or missing parentheses, misplaced exponent) does not earn the first point but would earn the second point for the correct answer. A response which has a presentation error in the integrand and which reports an incorrect answer earns no points.
 - A response that presents an integrand of $(\ln(x+3) - x^4 + 2x^3)^2$. Scoring instructions for this case are provided in the “Global Special Case” after part (d).
- A response that uses incorrect limits is only eligible for the second point, provided the limits are imported from part (a) in Case 1 or Case 2. In both of these situations, the second point is earned only for answers consistent with the imported limits.

Total for part (c) 2 points

- (d) A vertical line in the xy -plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

The cross section has area $A(x) = (f(x) - g(x))^2$.	$\frac{dA}{dx} \cdot \frac{dx}{dt}$	1 point
$\frac{d}{dt}[A(x)] = \frac{dA}{dx} \cdot \frac{dx}{dt}$		
$\left. \frac{d}{dt}[A(x)] \right _{x=-0.5} = A'(-0.5) \cdot 7 = -9.271842$	Answer	1 point
At $x = -0.5$, the area of the cross section above the line is changing at a rate of -9.272 (or -9.271) square units per second.		

Scoring notes:

- The first point may be earned by presenting $\frac{dA}{dx} \cdot \frac{dx}{dt}$, $A' \cdot \frac{dx}{dt}$, $A'(x) \cdot \frac{dx}{dt}$, $A'(x) \cdot x'$, $A'(-0.5) \cdot 7$, or $k \cdot 7$, where k is a declared value of $A'(-0.5)$, or any equivalent expression, including $2(f(x) - g(x))(f'(x) - g'(x)) \frac{dx}{dt}$.
- If a response defines $f(x) - g(x)$ as a function in parts (b) or (c) (for example, $h(x) = f(x) - g(x)$), then a correct expression for $\frac{dA}{dt}$ (for example, $2h \frac{dh}{dt}$) earns the first point.
- A response that imports a function $A(x)$ declared in part (c) is eligible for both points (the answer must be consistent with the imported function $A(x)$).
- A response that presents an incorrect function for $A(x)$ that is not imported from part (c) is eligible only for the first point.
- Except when $A(x)$ is imported from part (c), the second point is earned only for the correct answer.
- A response that does not earn the first point is ineligible to earn the second point except in the special case noted below.

Total for part (d) 2 points**Total for question 2 9 points**

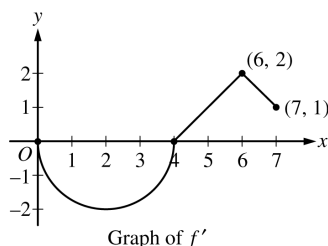
Global Special Case: A response may incorrectly simplify $f(x) - g(x)$ to $j(x) = \ln(x+3) - x^4 + 2x^3$ instead of $\ln(x+3) - x^4 - 2x^3$. Because this question is calculator active, a response with this incorrect simplification may nevertheless present correct answers.

- In any part of the question, a response that starts correctly by using $f(x) - g(x)$, then presents $j(x)$, is eligible for all points in that part.
- The first time a response implicitly presents $f(x) - g(x)$ as $j(x) = \ln(x+3) - x^4 + 2x^3$ (with no explicit connection) in any part of this question, the response loses a point. The response is then eligible for all remaining points for a correct or consistent answer.
- In part (a) the consistent answer using $j(x)$ is negative and will not earn the third point.
- In part (b) the consistent answer using $j(x)$ is that $j'(-0.5) = 2.4 > 0$, so h is increasing at $x = -0.5$.
- In part (c) the consistent answer using $j(x)$ is 252.187 (or 252.188).
- In part (d) the consistent answer using $j(x)$ is 20.287.

Part B (AB or BC): Graphing calculator not allowed**Question 3****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f' , the derivative of f , consists of a semicircle and two line segments, as shown in the figure above.

Model Solution**Scoring****(a)** Find $f(0)$ and $f(5)$.

$$f(0) = f(4) + \int_4^0 f'(x) dx = 3 - \int_0^4 f'(x) dx = 3 + 2\pi$$

Area of either region **1 point**

$$\text{— OR — } \int_0^4 f'(x) dx$$

$$f(5) = f(4) + \int_4^5 f'(x) dx = 3 + \frac{1}{2} = \frac{7}{2}$$

$$\text{— OR — } \int_4^5 f'(x) dx$$

 $f(0)$ **1 point** $f(5)$ **1 point****Scoring notes:**

- A response with answers of only $f(0) = \pm 2\pi$, or only $f(5) = \frac{1}{2}$, or both earns 1 of the 3 points.
- A response displaying $f(5) = \frac{7}{2}$ and a missing or incorrect value for $f(0)$ earns 2 of the 3 points.
- The second and third points can be earned in either order.
- Read unlabeled values from left to right and from top to bottom as $f(0)$ and $f(5)$. A single value must be labeled as either $f(0)$ or $f(5)$ in order to earn any points.

Total for part (a) **3 points****(b)** Find the x -coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

The graph of f has a point of inflection at each of $x = 2$ and $x = 6$, because $f'(x)$ changes from decreasing to increasing at $x = 2$ and from increasing to decreasing at $x = 6$.

Answer **1 point**Justification **1 point****Scoring notes:**

- A response that gives only one of $x = 2$ or $x = 6$, along with a correct justification, earns 1 of the 2 points.
- A response that claims that there is a point of inflection at any value other than $x = 2$ or $x = 6$ earns neither point.
- To earn the second point a response must use correct reasoning based on the graph of f' . Examples of correct reasoning include:
 - Correctly discussing the signs of the slopes of the graph of f'
 - Citing $x = 2$ and $x = 6$ as the locations of local extrema on the graph of f'
- Examples of reasoning not (sufficiently) connected to the graph of f' include:
 - Reasoning based on sign changes in f'' unless the connection is made between the sign of f'' and the slopes of the graph of f'
 - Reasoning based only on the concavity of the graph of f
- The second point cannot be earned by use of vague or undefined terms such as “it” or “the function” or “the derivative.”
- Responses that report inflection points as ordered pairs must report the points $(2, 3 + \pi)$ and $(6, 5)$ in order to earn the first point. If the y -coordinates are reported incorrectly, the response remains eligible for the second point.

Total for part (b) **2 points****(c)** Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.

$$g'(x) = f'(x) - 1$$

$$g'(x) = f'(x) - 1$$
 1 point

$$f'(x) - 1 \leq 0 \Rightarrow f'(x) \leq 1$$

Interval with reason **1 point**

The graph of g is decreasing on the interval $0 \leq x \leq 5$ because $g'(x) \leq 0$ on this interval.

Scoring notes:

- The first point can be earned for $f'(x) \leq 1$ or the equivalent, in words or symbols.
- Endpoints do not need to be included in the interval to be eligible for the second point.

Total for part (c) **2 points**

- (d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

g is continuous, $g'(x) < 0$ for $0 < x < 5$, and $g'(x) > 0$ for $5 < x < 7$.

Therefore, the absolute minimum occurs at $x = 5$, and

$g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$ is the absolute minimum value of g .

Considers $g'(x) = 0$ **1 point**

Answer with justification **1 point**

Scoring notes:

- A justification that uses a local argument, such as “ g' changes from negative to positive (or g changes from decreasing to increasing) at $x = 5$ ” must also state that $x = 5$ is the only critical point.
- If $g'(x) = 0$ (or equivalent) is not declared explicitly, a response that isolates $x = 5$ as the only critical number belonging to $(0, 7)$ earns the first point.
- A response that imports $g'(x) = f'(x)$ from part (c) is eligible for the first point but not the second.
 - In this case, consideration of $x = 4$ as the only critical number on $(0, 7)$ earns the first point.
- Solution using Candidates Test:

$$g'(x) = f'(x) - 1 = 0 \Rightarrow x = 5, x = 7$$

x	$g(x)$
0	$3 + 2\pi$
5	$-\frac{3}{2}$
7	$-\frac{1}{2}$

The absolute minimum value of g on the interval $0 \leq x \leq 7$ is $-\frac{3}{2}$.

- When using a Candidates Test, a response may import an incorrect value of $f(0) = g(0) > -\frac{3}{2}$ from part (a). The second point can only be earned for an answer of $-\frac{3}{2}$.

Total for part (d) 2 points

Total for question 3 9 points

2018

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AP Calculus AB

Scoring Guidelines

2018 SCORING GUIDELINES

Question 1

(a) $\int_0^{300} r(t) dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$.

(b) $20 + \int_0^{300} (r(t) - 0.7) dt = 20 + \int_0^{300} r(t) dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or } 414.285 \text{) seconds.}$$

(d) The total number of people in line at time t , $0 \leq t \leq 300$, is modeled by

$$20 + \int_0^t r(x) dx - 0.7t.$$

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

1 : answer

4 : $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

2018 SCORING GUIDELINES

Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118 .

(b) $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760 .

(c) $\int_0^{3.5} v(t) dt = 2.844$ (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral $\int_0^{3.5} v(t) dt$ is the displacement of the particle over the time interval $0 \leq t \leq 3.5$.

The integral $\int_0^{3.5} |v(t)| dt$ is the total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

(d) $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

1 : answer

3 : $\begin{cases} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{cases}$

2 : $\begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$

2018 SCORING GUIDELINES

Question 3

$$\begin{aligned} \text{(a)} \quad f(-5) &= f(1) + \int_1^{-5} g(x) \, dx = f(1) - \int_{-5}^1 g(x) \, dx \\ &= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2} \end{aligned}$$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b)} \quad \int_1^6 g(x) \, dx &= \int_1^3 g(x) \, dx + \int_3^6 g(x) \, dx \\ &= \int_1^3 2 \, dx + \int_3^6 2(x-4)^2 \, dx \\ &= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10 \end{aligned}$$

3 : $\begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

2018 SCORING GUIDELINES

Question 4

$$\text{(a)} \quad H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

$$\text{(b)} \quad \frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$$

Because H is differentiable on $3 \leq t \leq 5$, H is continuous on $3 \leq t \leq 5$.

By the Mean Value Theorem, there exists a value c , $3 < c < 5$, such that $H'(c) = 2$.

2 : $\begin{cases} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

(c) The average height of the tree over the time interval $2 \leq t \leq 10$ is given by $\frac{1}{10 - 2} \int_2^{10} H(t) \, dt$.

2 : $\begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned} \frac{1}{8} \int_2^{10} H(t) \, dt &\approx \frac{1}{8} \left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3 \right) \\ &= \frac{1}{8} (65.75) = \frac{263}{32} \end{aligned}$$

The average height of the tree over the time interval $2 \leq t \leq 10$ is $\frac{263}{32}$ meters.

$$\text{(d)} \quad G(x) = 50 \Rightarrow x = 1$$

3 : $\begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$$

$$\left. \frac{d}{dt}(G(x)) \right|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\frac{3}{4}$ meter per year.

Note: max 1/3 [1-0] if no chain rule

2018 SCORING GUIDELINES

Question 5

- (a) The average rate of change of
- f
- on the interval
- $0 \leq x \leq \pi$
- is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

- (b)
- $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

- (c)
- $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \leq x \leq 2\pi$ is $-\frac{1}{\sqrt{2}}e^{5\pi/4}$.

- (d)
- $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

1 : answer

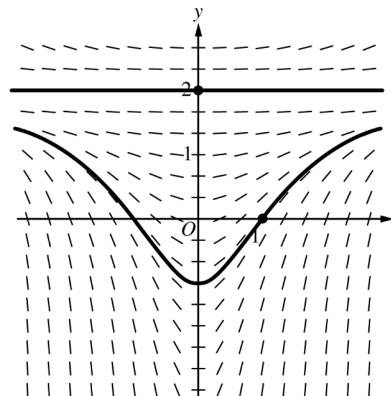
2 : $\begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$ 3 : $\begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$ 3 : $\begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \text{and limits equal 0} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

2018 SCORING GUIDELINES

Question 6

- (a)



- 2 :
- $\begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

- (b)
- $\left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = \frac{4}{3}$

An equation for the line tangent to the graph of $y = f(x)$ at

$$x = 1 \text{ is } y = \frac{4}{3}(x - 1).$$

$$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$$

- 2 :
- $\begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

- (c)
- $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$

$$\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$$

$$y = 2 - \frac{6}{x^2 + 2}$$

Note: this solution is valid for $-\infty < x < \infty$.

- 5 :
- $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

AP[®] Calculus AB

2015 Scoring Guidelines

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2015 SCORING GUIDELINES

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

- How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
- Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
- At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t) dt = 76.570$

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$

Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

2 : $\begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t , $0 \leq t \leq 8$, is $30 + \int_0^t [R(x) - D(x)] dx$.

3 : $\begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

$$R(t) - D(t) = 0 \Rightarrow t = 0, 3.271658$$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

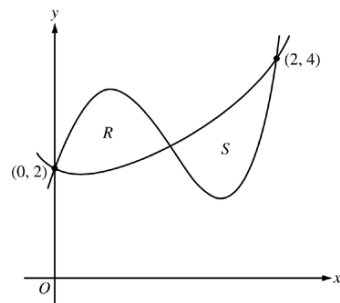
(d) $30 + \int_0^w [R(t) - D(t)] dt = 50$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

2015 SCORING GUIDELINES

Question 2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.



- (a) Find the sum of the areas of regions R and S .
- (b) Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.
- (c) Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.

- (a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$.

$$\begin{aligned} \text{Area} &= \int_0^A [g(x) - f(x)] dx + \int_A^2 [f(x) - g(x)] dx \\ &= 0.997427 + 1.006919 = 2.004 \end{aligned}$$

- (b) Volume $= \int_A^2 [f(x) - g(x)]^2 dx = 1.283$

- (c) $h(x) = f(x) - g(x)$
 $h'(x) = f'(x) - g'(x)$
 $h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

4 : $\begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

- (a) Use the data in the table to estimate the value of $v'(16)$.
- (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.

- (c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.
- (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

(a) $v'(16) \approx \frac{240 - 200}{20 - 12} = 5$ meters/min²

1 : approximation

- (b) $\int_0^{40} |v(t)| dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \leq t \leq 40$ minutes.

3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned} \int_0^{40} |v(t)| dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters} \end{aligned}$$

- (c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.
 $B'(5) = 3(25) - 12(5) = 15$ meters/min²

2 : $\begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{Avg vel} &= \frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300) dt \\ &= \frac{1}{10} \left[\frac{t^4}{4} - 2t^3 + 300t \right]_0^{10} \\ &= \frac{1}{10} \left[\frac{10000}{4} - 2000 + 3000 \right] = 350 \text{ meters/min} \end{aligned}$$

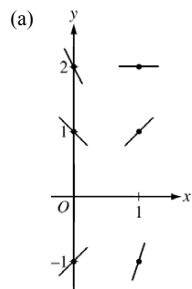
3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.
- (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.
- (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.



- 2 : $\begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$

(b) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$

In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$.
Therefore, all solution curves are concave up in Quadrant II.

- 2 : $\begin{cases} 1 : \frac{d^2y}{dx^2} \\ 1 : \text{concave up with reason} \end{cases}$

(c) $\left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} = 2(2) - 3 = 1 \neq 0$

Therefore, f has neither a relative minimum nor a relative maximum at $x = 2$.

- 2 : $\begin{cases} 1 : \text{considers } \left. \frac{dy}{dx} \right|_{(x,y)=(2,3)} \\ 1 : \text{conclusion with justification} \end{cases}$

(d) $y = mx + b \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(mx + b) = m$

$$2x - y = m$$

$$2x - (mx + b) = m$$

$$(2 - m)x - (m + b) = 0$$

$$2 - m = 0 \Rightarrow m = 2$$

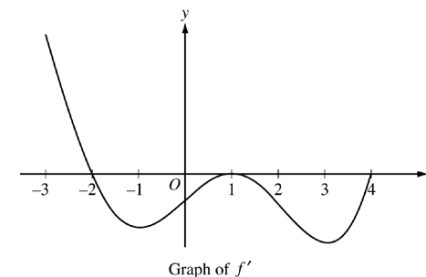
$$b = -m \Rightarrow b = -2$$

- 3 : $\begin{cases} 1 : \frac{d}{dx}(mx + b) = m \\ 1 : 2x - y = m \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 5

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



- (a) Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- (c) Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

- (a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$.
 $f'(x)$ changes from positive to negative at $x = -2$.
Therefore, f has a relative maximum at $x = -2$.

- 2 : $\begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$

- (b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.

- 2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

- (c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points.

- 2 : $\begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$

The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.

(d) $f(x) = 3 + \int_1^x f'(t) dt$

$$f(4) = 3 + \int_1^4 f'(t) dt = 3 + (-12) = -9$$

$$f(-2) = 3 + \int_1^{-2} f'(t) dt = 3 - \int_{-2}^1 f'(t) dt = 3 - (-9) = 12$$

- 3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$

2015 SCORING GUIDELINES

Question 6

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

- (a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.
- (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.
- (c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

$$(a) \quad \left. \frac{dy}{dx} \right|_{(x,y)=(-1,1)} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$$

An equation for the tangent line is $y = \frac{1}{4}(x + 1) + 1$.

$$(b) \quad 3y^2 - x = 0 \Rightarrow x = 3y^2$$

$$\text{So, } y^3 - xy = 2 \Rightarrow y^3 - (3y^2)(y) = 2 \Rightarrow y = -1$$

$$(-1)^3 - x(-1) = 2 \Rightarrow x = 3$$

The tangent line to the curve is vertical at the point $(3, -1)$.

$$(c) \quad \frac{d^2y}{dx^2} = \frac{(3y^2 - x)\frac{dy}{dx} - y\left(6y\frac{dy}{dx} - 1\right)}{(3y^2 - x)^2}$$

$$\left. \frac{d^2y}{dx^2} \right|_{(x,y)=(-1,1)} = \frac{(3 \cdot 1^2 - (-1)) \cdot \frac{1}{4} - 1 \cdot \left(6 \cdot 1 \cdot \frac{1}{4} - 1\right)}{(3 \cdot 1^2 - (-1))^2}$$

$$= \frac{1 - \frac{1}{2}}{16} = \frac{1}{32}$$

$$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{equation for tangent line} \end{cases}$$

$$3 : \begin{cases} 1 : \text{sets } 3y^2 - x = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{coordinates} \end{cases}$$

$$4 : \begin{cases} 2 : \text{implicit differentiation} \\ 1 : \text{substitution for } \frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$$