

6. Consider the curve  $G$  defined by the equation  $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$ .

- A. Show that  $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$ .
- B. There is a point  $P$  on the curve  $G$  near  $(2, -1)$  with  $x$ -coordinate 1.6. Use the line tangent to the curve at  $(2, -1)$  to approximate the  $y$ -coordinate of point  $P$ .
- C. For  $x > 0$  and  $y > 0$ , there is a point  $S$  on the curve  $G$  at which the line tangent to the curve at that point is vertical. Find the  $y$ -coordinate of point  $S$ . Show the work that leads to your answer.
- D. A particle moves along the curve  $H$  defined by the equation  $2xy + \ln y = 8$ . At the instant when the particle is at the point  $(4, 1)$ ,  $\frac{dx}{dt} = 3$ . Find  $\frac{dy}{dt}$  at that instant. Show the work that leads to your answer.

**STOP**  
**END OF EXAM**

5. Consider the curve defined by the equation  $x^2 + 3y + 2y^2 = 48$ . It can be shown that  $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$ .

- (a) There is a point on the curve near  $(2, 4)$  with  $x$ -coordinate 3. Use the line tangent to the curve at  $(2, 4)$  to approximate the  $y$ -coordinate of this point.
- (b) Is the horizontal line  $y = 1$  tangent to the curve? Give a reason for your answer.
- (c) The curve intersects the positive  $x$ -axis at the point  $(\sqrt{48}, 0)$ . Is the line tangent to the curve at this point vertical? Give a reason for your answer.
- (d) For time  $t \geq 0$ , a particle is moving along another curve defined by the equation  $y^3 + 2xy = 24$ . At the instant the particle is at the point  $(4, 2)$ , the  $y$ -coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the  $x$ -coordinate of the particle's position with respect to time?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

6. Consider the curve given by the equation  $6xy = 2 + y^5$ .

- (a) Show that  $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$ .
- (b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.
- (c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.
- (d) A particle is moving along the curve. At the instant when the particle is at the point  $\left(\frac{1}{2}, -2\right)$ , its horizontal position is increasing at a rate of  $\frac{dx}{dt} = \frac{2}{3}$  unit per second. What is the value of  $\frac{dy}{dt}$ , the rate of change of the particle's vertical position, at that instant?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

**STOP**  
**END OF EXAM**

6. Consider the curve given by the equation  $y^3 - xy = 2$ . It can be shown that  $\frac{dy}{dx} = \frac{y}{3y^2 - x}$ .

- (a) Write an equation for the line tangent to the curve at the point  $(-1, 1)$ .
- (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.
- (c) Evaluate  $\frac{d^2y}{dx^2}$  at the point on the curve where  $x = -1$  and  $y = 1$ .

**STOP**

**END OF EXAM**