

6. Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

- A. Show that $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$.
- B. There is a point P on the curve G near $(2, -1)$ with x -coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y -coordinate of point P .
- C. For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y -coordinate of point S . Show the work that leads to your answer.
- D. A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

STOP

END OF EXAM

5. Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

- (a) There is a point on the curve near $(2, 4)$ with x -coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y -coordinate of this point.
- (b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.
- (c) The curve intersects the positive x -axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.
- (d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y -coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x -coordinate of the particle's position with respect to time?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

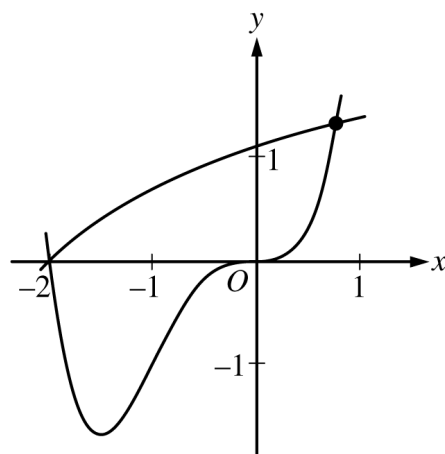
6. Consider the curve given by the equation $6xy = 2 + y^5$.

- (a) Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.
- (b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.
- (c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.
- (d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\frac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\frac{dx}{dt} = \frac{2}{3}$ unit per second. What is the value of $\frac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

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STOP

END OF EXAM



2. Let f and g be the functions defined by $f(x) = \ln(x + 3)$ and $g(x) = x^4 + 2x^3$. The graphs of f and g , shown in the figure above, intersect at $x = -2$ and $x = B$, where $B > 0$.
- (a) Find the area of the region enclosed by the graphs of f and g .
- (b) For $-2 \leq x \leq B$, let $h(x)$ be the vertical distance between the graphs of f and g . Is h increasing or decreasing at $x = -0.5$? Give a reason for your answer.
- (c) The region enclosed by the graphs of f and g is the base of a solid. Cross sections of the solid taken perpendicular to the x -axis are squares. Find the volume of the solid.
- (d) A vertical line in the xy -plane travels from left to right along the base of the solid described in part (c). The vertical line is moving at a constant rate of 7 units per second. Find the rate of change of the area of the cross section above the vertical line with respect to time when the vertical line is at position $x = -0.5$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Name:

AP Calculus AB: **2022**

END OF PART A

CALCULUS AB
SECTION II, Part B

Time—1 hour

4 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

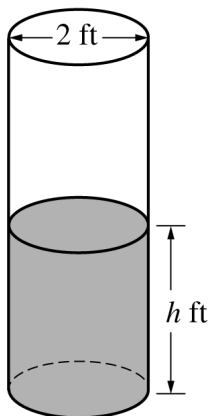
4. An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r , where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \leq t \leq 12$.

- (a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \leq t \leq 10$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $0 \leq t \leq 3$, for which $r'(t) = -6$? Justify your answer.
- (c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of

$$\int_0^{12} r'(t) dt.$$

- (d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)
- (a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.
- (b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.
- (c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t .
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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H , where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.
- (a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.
- (b) Explain why there must be at least one time t , for $2 < t < 10$, such that $H'(t) = 2$.
- (c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \leq t \leq 10$.
- (d) The height of the tree, in meters, can also be modeled by the function G , given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?
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CALCULUS AB
SECTION II, Part A

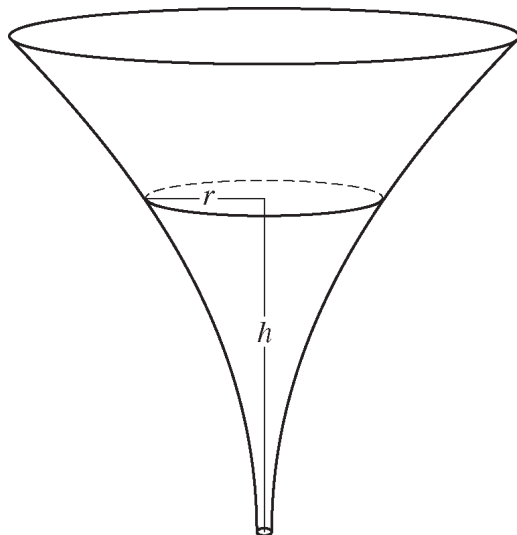
Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A , where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.
- (a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.
- (b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.
- (c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \frac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.
- (d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.
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5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.
- (a) Find the average value of the radius of the funnel.
 - (b) Find the volume of the funnel.
 - (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?
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t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

4. Train A runs back and forth on an east-west section of railroad track. Train A 's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.
- (a) Find the average acceleration of train A over the interval $2 \leq t \leq 8$.
- (b) Do the data in the table support the conclusion that train A 's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.
- (c) At time $t = 2$, train A 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A , in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.
- (d) A second train, train B , travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.
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