

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f , where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

Model Solution**Scoring**

- (a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t) dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t) dt$.

$\int_{60}^{135} f(t) dt$ represents the total number of gallons of gasoline pumped into the gas tank from time $t = 60$ seconds to time $t = 135$ seconds.

Interpretation with units **1 point**

$\int_{60}^{135} f(t) dt$
 $\approx f(90)(90 - 60) + f(120)(120 - 90) + f(135)(135 - 120)$
 $= (0.15)(30) + (0.1)(30) + (0.05)(15) = 8.25$

Form of Riemann sum **1 point**

Answer **1 point**

Scoring notes:

- To earn the first point the response must reference gallons of gasoline added/pumped and the time interval $t = 60$ to $t = 135$.
- To earn the second point at least five of the six factors in the Riemann sum must be correct.
- If there is any error in the Riemann sum, the response does not earn the third point.
- A response of $(0.15)(30) + (0.1)(30) + (0.05)(15)$ earns both the second and third points, unless there is a subsequent error in simplification, in which case the response would earn only the second point.

- A response that presents a correct value with accompanying work that shows the three products in the Riemann sum but does not show all six of the factors and/or the sum process, does not earn the second point but does earn the third point. For example, responses of either $4.5 + 3.0 + 0.75$ or $(0.15)(30)$, $0.1(30)$, $0.05(15) \rightarrow 8.25$ earn the third point but not the second.
- A response of $f(90)(90 - 60) + f(120)(120 - 90) + f(135)(135 - 120) = 8.25$ earns both the second and the third points.
- A response that presents an answer of only 8.25 does not earn either the second or third point.
- A response that provides a completely correct left Riemann sum with accompanying work, $f(60)(30) + f(90)(30) + f(120)(15) = 9$, or $(0.1)(30) + (0.15)(30) + (0.1)(15)$ earns 1 of the last 2 points. A response with any errors or missing factors in a left Riemann sum earns neither of the last 2 points.

Total for part (a) 3 points

(b) Must there exist a value of c , for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.

f is differentiable. $\Rightarrow f$ is continuous on $[60, 120]$.

$f(120) - f(60) = 0$ **1 point**

$$\frac{f(120) - f(60)}{120 - 60} = \frac{0.1 - 0.1}{60} = 0$$

Answer with justification **1 point**

By the Mean Value Theorem, there must exist a c , for $60 < c < 120$, such that $f'(c) = 0$.

Scoring notes:

- To earn the first point a response must present either $f(120) - f(60) = 0$, $0.1 - 0.1 = 0$ (perhaps as the numerator of a quotient), or $f(60) = f(120)$.
- To earn the second point a response must:
 - have earned the first point,
 - state that f is continuous because f is differentiable (or equivalent), and
 - answer “yes” in some way.
- A response may reference either the Mean Value Theorem or Rolle’s Theorem.
- A response that references the Intermediate Value Theorem cannot earn the second point.

Total for part (b) 2 points

(c) The rate of flow of gasoline, in gallons per second, can also be modeled by

$$g(t) = \left(\frac{t}{500}\right) \cos\left(\left(\frac{t}{120}\right)^2\right) \text{ for } 0 \leq t \leq 150. \text{ Using this model, find the average rate of flow of}$$

gasoline over the time interval $0 \leq t \leq 150$. Show the setup for your calculations.

$$\frac{1}{150 - 0} \int_0^{150} g(t) dt$$

Average value formula **1 point**

$$= 0.0959967$$

Answer **1 point**

The average rate of flow of gasoline, in gallons per second, is 0.096 (or 0.095).

Scoring notes:

- The exact value of $\frac{1}{150} \int_0^{150} g(t) dt$ is $\frac{12}{125} \sin\left(\frac{25}{16}\right)$.
- A response may present the average value formula in single or multiple steps. For example, the following response earns both points: $\int_0^{150} g(t) dt = 14.399504$ so the average rate is 0.0959967.
- A response that presents the average value formula in multiple steps but provides incorrect or incomplete communication (e.g., $\int_0^{150} g(t) dt = \frac{14.399504}{150} = 0.0959967$) earns 1 out of 2 points.
- A response of $\int_0^{150} g(t) dt = 0.0959967$ does not earn either point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{150} \int_0^{150} g(t) dt = 0.149981$ or 0.002618.

Total for part (c) 2 points

- (d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

$g'(140) \approx -0.004908$ $g'(140) = -0.005$ (or -0.004)	$g'(140)$ 1 point
The rate at which gasoline is flowing into the tank is decreasing at a rate of 0.005 (or 0.004) gallon per second per second at time $t = 140$ seconds.	Interpretation 1 point

Scoring notes:

- The exact value of $g'(140)$ is $\frac{1}{500} \cos\left(\frac{49}{36}\right) - \frac{49}{9000} \sin\left(\frac{49}{36}\right)$.
- The value of $g'(140)$ may appear only in the interpretation.
- To be eligible for the second point a response must present some numerical value for $g'(140)$.
- To earn the second point the interpretation must include “the rate of flow of gasoline is changing at a rate of [the declared value of $g'(140)$]” and “at $t = 140$ ” (or equivalent).
- An interpretation of “decreasing at a rate of -0.005 ” or “increasing at a rate of 0.005” does not earn the second point.
- Degree mode: In degree mode, $g'(140) = 0.001997$ or 0.00187.

Total for part (d) 2 points**Total for question 1 9 points**

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f , where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

Model Solution**Scoring**

- (a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{0.5} = 8$$

Estimate **1 point**

At a distance of $r = 2.25$ centimeters from the center of the petri dish, the density of the bacteria population is increasing at a rate of 8 milligrams per square centimeter per centimeter.

Interpretation with units **1 point**

Scoring notes:

- To earn the first point the response must provide both a difference and a quotient and must explicitly use values of f from the table.
- Simplification of the numerical value is not required to earn the first point. If the numerical value is simplified, it must be correct.
- The interpretation requires all of the following: distance $r = 2.25$, density of bacteria (population) is increasing or changing, at a rate of 8, and units of milligrams per square centimeter per centimeter.
- The second point (interpretation) cannot be earned without a nonzero presented value for $f'(2.25)$.
- To earn the second point the interpretation must be consistent with the presented nonzero value for $f'(2.25)$. In particular, if the presented value for $f'(2.25)$ is negative, the interpretation must include “decreasing at a rate of $|f'(2.25)|$ ” or “changing at a rate of $f'(2.25)$.” The second point cannot be earned for an incorrect statement such as “the bacteria density is decreasing at a rate of $-8 \dots$ ” even for a presented $f'(2.25) = -8$.
- The units ($\text{mg}/\text{cm}^2/\text{cm}$) may be attached to the estimate of $f'(2.25)$ and, if so, do not need to be repeated in the interpretation.
- If units attached to the estimate do not agree with units in the interpretation, read the units in the interpretation.

Total for part (a) 2 points

- (b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression

$2\pi \int_0^4 r f(r) dr$. Approximate the value of $2\pi \int_0^4 r f(r) dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

$2\pi \int_0^4 r f(r) dr \approx 2\pi(1 \cdot f(1) \cdot (1 - 0) + 2 \cdot f(2) \cdot (2 - 1) + 2.5 \cdot f(2.5) \cdot (2.5 - 2) + 4 \cdot f(4) \cdot (4 - 2.5))$	Right Riemann sum setup	1 point
$= 2\pi(1 \cdot 2 \cdot 1 + 2 \cdot 6 \cdot 1 + 2.5 \cdot 10 \cdot 0.5 + 4 \cdot 18 \cdot 1.5)$ $= 269\pi = 845.088$	Approximation	1 point

Scoring notes:

- The presence or absence of 2π has no bearing on earning the first point.
- The first point is earned for a sum of four products with at most one error in any single value among the four products. Multiplication by 1 in any term does not need to be shown, but all other products must be explicitly shown.
- A response of $1 \cdot f(1) \cdot (1 - 0) + 2 \cdot f(2) \cdot (2 - 1) + 2.5 \cdot f(2.5) \cdot (2.5 - 2) + 4 \cdot f(4) \cdot (4 - 2.5)$ earns the first point but not the second.
- A response with any error in the Riemann sum is not eligible for the second point.
- A response that provides a completely correct left Riemann sum for $2\pi \int_0^4 r f(r) dr$ and approximation (91π) earns one of the two points. A response that has any error in a left Riemann sum or evaluation for $2\pi \int_0^4 r f(r) dr$ earns no points.
- A response that provides a completely correct right Riemann sum for $2\pi \int_0^4 r f(r) dr$ and approximation (80π) earns one of the two points. A response that has any error in a right Riemann sum or evaluation for $2\pi \int_0^4 r f(r) dr$ earns no points.
- Simplification of the numerical value is not required to earn the second point. If a numerical value is given, it must be correct to three decimal places.

Total for part (b) 2 points

- (c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

$\frac{d}{dr}(rf(r)) = f(r) + rf'(r)$	Product rule expression for $\frac{d}{dr}(rf(r))$	1 point
Because f is nonnegative and increasing, $\frac{d}{dr}(rf(r)) > 0$ on the interval $0 \leq r \leq 4$. Thus, the integrand $rf(r)$ is strictly increasing. Therefore, the right Riemann sum approximation of $2\pi \int_0^4 rf(r) dr$ is an overestimate.	Answer with explanation	1 point

Scoring notes:

- To earn the second point a response must explain that $rf(r)$ is increasing and, therefore, the right Riemann sum is an overestimate. The second point can be earned without having earned the first point.
- A response that attempts to explain based on a left Riemann sum for $2\pi \int_0^4 rf(r) dr$ from part (b) earns no points.
- A response that attempts to explain based on a right Riemann sum for $2\pi \int_0^4 f(r) dr$ from part (b) earns no points.

Total for part (c) 2 points

- (d) The density of bacteria in the petri dish, for $1 \leq r \leq 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k , $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \leq r \leq 4$?

Average value = $g_{\text{avg}} = \frac{1}{4-1} \int_1^4 g(r) dr$	Definite integral	1 point
$\frac{1}{4-1} \int_1^4 g(r) dr = 9.875795$	Average value	1 point
$g(k) = g_{\text{avg}} \Rightarrow k = 2.497$	Answer	1 point

Scoring notes:

- The first point is earned for a definite integral, with or without $\frac{1}{4-1}$ or $\frac{1}{3}$.
- A response that presents a definite integral with incorrect limits but a correct integrand earns the first point.
- Presentation of the numerical value 9.875795 is not required to earn the second point. This point can be earned by the average value setup: $\frac{1}{3} \int_1^4 g(r) dr$.
- Once earned for the average value setup, the second point cannot be lost. Subsequent errors will result in not earning the third point.
- The third point is earned only for the value $k = 2.497$.
- The third point cannot be earned without the second.
- Special case: A response that does not provide the average value setup but presents an average value of -13.955 is using degree mode on their calculator. This response would not earn the second point but could earn the third point for an answer of $k = 2.5$ (or 2.499).

Total for part (d) 3 points

Total for question 1 9 points

Part B (AB): Graphing calculator not allowed**Question 6****9 points****General Scoring Notes**

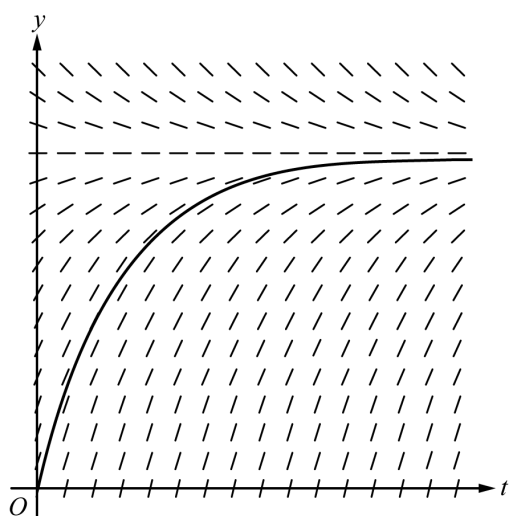
Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

Model Solution**Scoring**

- (a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



Solution curve

1 point**Scoring notes:**

- To earn the point the solution curve must pass through the point $(0, 0)$, be generally increasing and concave down, and approach the horizontal asymptote from below as t increases. The point is not earned if two or more solution curves are presented.

Total for part (a) 1 point

- (b) Using correct units, interpret the statement $\lim_{t \rightarrow \infty} A(t) = 12$ in the context of this problem.

Over time the amount of medication in the patient approaches 12 milligrams.	Interpretation	1 point
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Scoring notes:

- To earn the point the interpretation must include “medication in the patient,” “approaches 12,” and units (milligrams), or their equivalents.

Total for part (b) 1 point

- (c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation

$$\frac{dy}{dt} = \frac{12 - y}{3} \text{ with initial condition } A(0) = 0.$$

$\frac{dy}{dt} = \frac{12 - y}{3} \Rightarrow \frac{dy}{12 - y} = \frac{dt}{3}$	Separation of variables	1 point
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$\int \frac{dy}{12 - y} = \int \frac{dt}{3} \Rightarrow -\ln 12 - y = \frac{t}{3} + C$	Antiderivatives	1 point
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$\ln 12 - y = -\frac{t}{3} - C \Rightarrow 12 - y = e^{-t/3 - C}$ $\Rightarrow y = 12 + Ke^{-t/3}$	Constant of integration and uses initial condition	1 point
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$$0 = 12 + K \Rightarrow K = -12$$

$y = A(t) = 12 - 12e^{-t/3}$	Solves for y	1 point
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Scoring notes:

- A response of $\frac{dy}{12 - y} = 3 dt$ is a bad separation and does not earn the first point. However, this response is eligible for the second and third points. It cannot earn the fourth point.
- Absolute value bars are not required in this part.
- A response that correctly separates to $\frac{3 dy}{12 - y} = dt$ but then incorrectly simplifies to $\frac{dy}{4 - y} = dt$ earns the first point (for the initial correct separation), is eligible for the second point (for $-\ln|4 - y| = t$, with or without $+C$), but is not eligible for the third or fourth points.
- $+\ln|12 - y| = \frac{t}{3}$ (with or without $+C$) does not earn the second point and is not eligible for the fourth point; $+\ln|12 - y| = \frac{t}{3} + C$ is eligible for the third point.
- In all other cases, the points are earned consecutively—the second point cannot be earned without the first, the third without the second, etc.

Total for part (c) 4 points

- (d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

$\frac{dy}{dt} = 3 - \frac{y}{t+2} \Rightarrow \frac{d^2y}{dt^2} = (-1) \frac{\frac{dy}{dt}(t+2) - y}{(t+2)^2}$	Quotient rule	1 point
$B'(1) = 3 - \frac{B(1)}{3} = 3 - \frac{2.5}{3} = \frac{6.5}{3}$ $B''(1) = -\frac{B'(1) \cdot 3 - B(1)}{3^2} = -\frac{6.5 - 2.5}{9} = -\frac{4}{9} < 0$	$B''(1) < 0$	1 point
The rate of change of the amount of medication is decreasing at time $t = 1$ because $B''(1) < 0$ and $\frac{d^2y}{dt^2}$ is continuous in an interval containing $t = 1$.	Answer with reason	1 point

Scoring notes:

- The first point is for correctly applying the quotient rule to $\frac{y}{t+2}$ or applying the product rule to $y(t+2)^{-1}$. Errors in differentiating the constant, 3, or handling the sign of the second term of $\frac{dy}{dt}$ will result in not earning the second point.
- The second point cannot be earned unless the second derivative $\frac{d^2y}{dt^2}$ is correct.
- For the second point it is sufficient to state the sign of $B''(1)$ is negative with supporting work. If a value is declared for $B''(1)$, it must be correct in order to earn the second point.
- Eligibility for the third point: An attempt at using the quotient rule (or product rule) to find $B''(1)$. In this case the third point will be earned for a consistent conclusion based on the declared value (or sign) of $B''(1)$.

Total for part (d) 3 points

Total for question 6 9 points

2018**AP[®]** **CollegeBoard**

AP Calculus AB

Scoring Guidelines

2018 SCORING GUIDELINES

Question 1

(a) $\int_0^{300} r(t) dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $20 + \int_0^{300} (r(t) - 0.7) dt = 20 + \int_0^{300} r(t) dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

2 : $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or 414.285) seconds.}$$

1 : answer

(d) The total number of people in line at time t , $0 \leq t \leq 300$, is modeled by

$$20 + \int_0^t r(x) dx - 0.7t.$$

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

4 : $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

2018 SCORING GUIDELINES

Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118 .

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760 .

3 : $\begin{cases} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{3.5} v(t) dt = 2.844$ (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral $\int_0^{3.5} v(t) dt$ is the displacement of the particle over the time interval $0 \leq t \leq 3.5$.

The integral $\int_0^{3.5} |v(t)| dt$ is the total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

3 : $\begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{cases}$

(d) $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

2 : $\begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$

2018 SCORING GUIDELINES

Question 3

$$\begin{aligned} \text{(a)} \quad f(-5) &= f(1) + \int_1^{-5} g(x) \, dx = f(1) - \int_{-5}^1 g(x) \, dx \\ &= 3 - \left(-9 - \frac{3}{2} + 1 \right) = 3 - \left(-\frac{19}{2} \right) = \frac{25}{2} \end{aligned}$$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b)} \quad \int_1^6 g(x) \, dx &= \int_1^3 g(x) \, dx + \int_3^6 g(x) \, dx \\ &= \int_1^3 2 \, dx + \int_3^6 2(x-4)^2 \, dx \\ &= 4 + \left[\frac{2}{3}(x-4)^3 \right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3} \right) = 10 \end{aligned}$$

3 : $\begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f''(x) = g'(x)$ is increasing on those intervals.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

2018 SCORING GUIDELINES

Question 4

$$(a) \quad H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$$(b) \quad \frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$$

Because H is differentiable on $3 \leq t \leq 5$, H is continuous on $3 \leq t \leq 5$.

By the Mean Value Theorem, there exists a value c , $3 < c < 5$, such that $H'(c) = 2$.

$$(c) \quad \text{The average height of the tree over the time interval } 2 \leq t \leq 10 \text{ is given by } \frac{1}{10 - 2} \int_2^{10} H(t) \, dt.$$

$$\begin{aligned} \frac{1}{8} \int_2^{10} H(t) \, dt &\approx \frac{1}{8} \left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3 \right) \\ &= \frac{1}{8} (65.75) = \frac{263}{32} \end{aligned}$$

The average height of the tree over the time interval $2 \leq t \leq 10$ is $\frac{263}{32}$ meters.

$$(d) \quad G(x) = 50 \Rightarrow x = 1$$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$$

$$\left. \frac{d}{dt}(G(x)) \right|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\frac{3}{4}$ meter per year.

$$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$$

$$2 : \begin{cases} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$$

$$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$$

$$3 : \begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$$

Note: max 1/3 [1-0] if no chain rule

2018 SCORING GUIDELINES

Question 5

- (a) The average rate of change of
- f
- on the interval
- $0 \leq x \leq \pi$
- is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

1 : answer

- (b)
- $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

2 : $\begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

- (c)
- $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

3 : $\begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$

The absolute minimum value of f on $0 \leq x \leq 2\pi$ is $-\frac{1}{\sqrt{2}}e^{5\pi/4}$.

- (d)
- $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

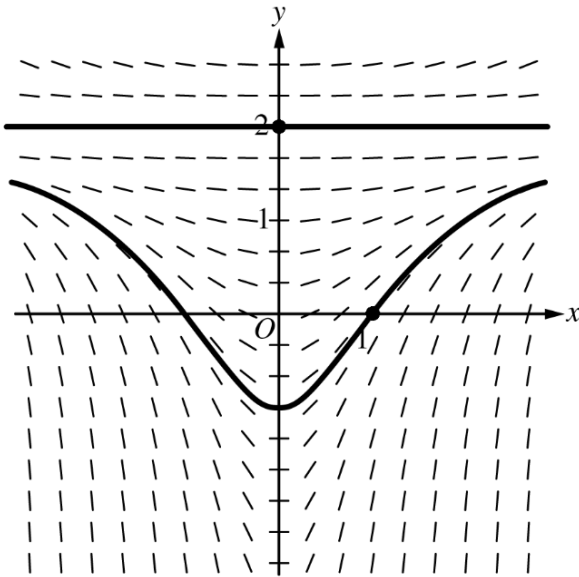
3 : $\begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \text{and limits equal 0} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

2018 SCORING GUIDELINES

Question 6

(a)



$$2 : \begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

$$(b) \left. \frac{dy}{dx} \right|_{(x, y)=(1, 0)} = \frac{4}{3}$$

An equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ is $y = \frac{4}{3}(x - 1)$.

$$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$$

$$2 : \begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$$

$$(c) \frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$$

$$\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$$

$$y = 2 - \frac{6}{x^2 + 2}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: this solution is valid for $-\infty < x < \infty$.

AP[®] Calculus AB

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2014 SCORING GUIDELINES

Question 1

Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

- Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.
- Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.
- Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.
- For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$$(a) \frac{A(30) - A(0)}{30 - 0} = -0.197 \text{ (or } -0.196) \text{ lbs/day}$$

1 : answer with units

$$(b) A'(15) = -0.164 \text{ (or } -0.163)$$

The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.

2 : $\begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$

$$(c) A(t) = \frac{1}{30} \int_0^{30} A(t) dt \Rightarrow t = 12.415 \text{ (or } 12.414)$$

2 : $\begin{cases} 1 : \frac{1}{30} \int_0^{30} A(t) dt \\ 1 : \text{answer} \end{cases}$

$$(d) L(t) = A(30) + A'(30) \cdot (t - 30)$$

$$A'(30) = -0.055976$$

$$A(30) = 0.782928$$

$$L(t) = 0.5 \Rightarrow t = 35.054$$

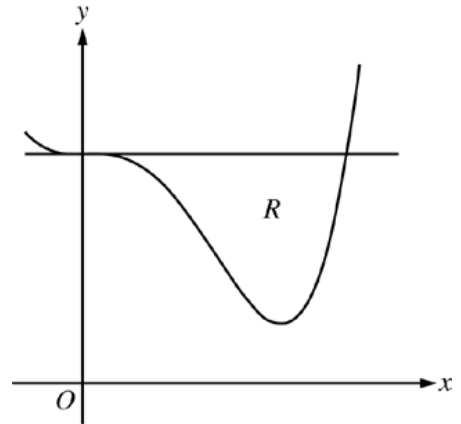
4 : $\begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

2014 SCORING GUIDELINES

Question 2

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

- (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
- (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in R . Find the volume of the solid.
- (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k .



(a) $f(x) = 4 \Rightarrow x = 0, 2.3$

$$\begin{aligned} \text{Volume} &= \pi \int_0^{2.3} \left[(4 + 2)^2 - (f(x) + 2)^2 \right] dx \\ &= 98.868 \text{ (or } 98.867) \end{aligned}$$

$$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$$

(b) $\text{Volume} = \int_0^{2.3} \frac{1}{2} (4 - f(x))^2 dx$
 $= 3.574 \text{ (or } 3.573)$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(c) $\int_0^k (4 - f(x)) dx = \int_k^{2.3} (4 - f(x)) dx$

$$2 : \begin{cases} 1 : \text{area of one region} \\ 1 : \text{equation} \end{cases}$$

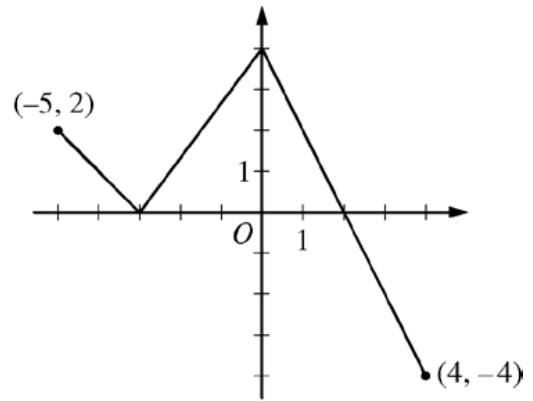
2014 SCORING GUIDELINES

Question 3

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above.

Let g be the function defined by $g(x) = \int_{-3}^x f(t) dt$.

- (a) Find $g(3)$.
- (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
- (c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.
- (d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

Graph of f

(a) $g(3) = \int_{-3}^3 f(t) dt = 6 + 4 - 1 = 9$

1 : answer

(b) $g'(x) = f(x)$

The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $h'(x) = \frac{5xg'(x) - g(x)5}{(5x)^2} = \frac{5xg'(x) - 5g(x)}{25x^2}$

3 : $\begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} h'(3) &= \frac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2} \\ &= \frac{15(-2) - 5(9)}{225} = \frac{-75}{225} = -\frac{1}{3} \end{aligned}$$

(d) $p'(x) = f'(x^2 - x)(2x - 1)$

3 : $\begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$

$$p'(-1) = f'(2)(-3) = (-2)(-3) = 6$$

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Question 4

Train A runs back and forth on an east-west section of railroad track. Train A 's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

- (a) Find the average acceleration of train A over the interval $2 \leq t \leq 8$.
- (b) Do the data in the table support the conclusion that train A 's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.
- (c) At time $t = 2$, train A 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A , in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.
- (d) A second train, train B , travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

(a) average accel = $\frac{v_A(8) - v_A(2)}{8 - 2} = \frac{-120 - 100}{6} = -\frac{110}{3} \text{ m/min}^2$

(b) v_A is differentiable $\Rightarrow v_A$ is continuous
 $v_A(8) = -120 < -100 < 40 = v_A(5)$

Therefore, by the Intermediate Value Theorem, there is a time t , $5 < t < 8$, such that $v_A(t) = -100$.

(c) $s_A(12) = s_A(2) + \int_2^{12} v_A(t) dt = 300 + \int_2^{12} v_A(t) dt$
 $\int_2^{12} v_A(t) dt \approx 3 \cdot \frac{100 + 40}{2} + 3 \cdot \frac{40 - 120}{2} + 4 \cdot \frac{-120 - 150}{2}$
 $= -450$

$$s_A(12) \approx 300 - 450 = -150$$

The position of Train A at time $t = 12$ minutes is approximately 150 meters west of Origin Station.

- (d) Let x be train A 's position, y train B 's position, and z the distance between train A and train B .

$$z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$x = 300, y = 400 \Rightarrow z = 500$$

$$v_B(2) = -20 + 120 + 25 = 125$$

$$500 \frac{dz}{dt} = (300)(100) + (400)(125)$$

$$\frac{dz}{dt} = \frac{80000}{500} = 160 \text{ meters per minute}$$

1 : average acceleration

2 : $\begin{cases} 1 : v_A(8) < -100 < v_A(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

3 : $\begin{cases} 1 : \text{position expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{position at time } t = 12 \end{cases}$

3 : $\begin{cases} 2 : \text{implicit differentiation of} \\ \quad \text{distance relationship} \\ 1 : \text{answer} \end{cases}$

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Question 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.

- (a) Find the x -coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.
- (b) Explain why there must be a value c , for $-1 < c < 1$, such that $f''(c) = 0$.
- (c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.
- (d) Evaluate $\int_{-2}^3 f'(g(x))g'(x) dx$.

- (a) $x = 1$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a relative minimum at $x = 1$.

- (b) f' is differentiable $\Rightarrow f'$ is continuous on the interval $-1 \leq x \leq 1$

$$\frac{f'(1) - f'(-1)}{1 - (-1)} = \frac{0 - 0}{2} = 0$$

Therefore, by the Mean Value Theorem, there is at least one value c , $-1 < c < 1$, such that $f''(c) = 0$.

- (c) $h'(x) = \frac{1}{f(x)} \cdot f'(x)$

$$h'(3) = \frac{1}{f(3)} \cdot f'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}$$

- (d) $\int_{-2}^3 f'(g(x))g'(x) dx = \left[f(g(x)) \right]_{x=-2}^{x=3}$
 $= f(g(3)) - f(g(-2))$
 $= f(1) - f(-1)$
 $= 2 - 8 = -6$

1 : answer with justification

$$2 : \begin{cases} 1 : f'(1) - f'(-1) = 0 \\ 1 : \text{explanation, using Mean Value Theorem} \end{cases}$$

$$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 2 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$$

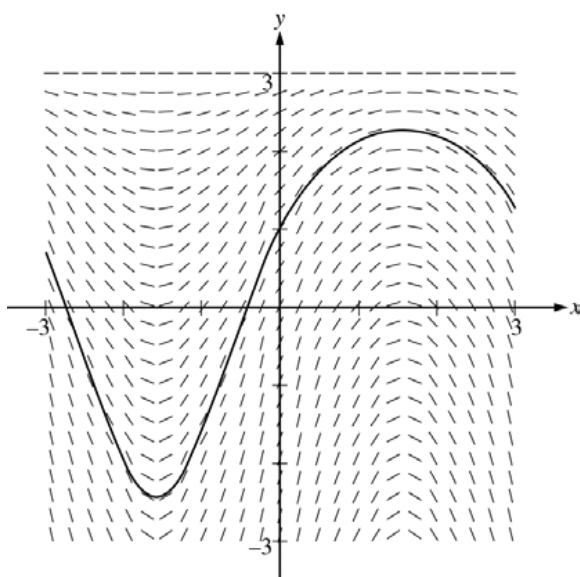
2014 SCORING GUIDELINES

Question 6

Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

- (a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.
- (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.
- (c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

(a) 1 : solution curve



(b) $\left. \frac{dy}{dx} \right|_{(x, y) = (0, 1)} = 2 \cos 0 = 2$

An equation for the tangent line is $y = 2x + 1$.

$$f(0.2) \approx 2(0.2) + 1 = 1.4$$

2 : $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{dy}{dx} = (3 - y)\cos x$

$$\int \frac{dy}{3 - y} = \int \cos x \, dx$$

$$-\ln|3 - y| = \sin x + C$$

$$-\ln 2 = \sin 0 + C \Rightarrow C = -\ln 2$$

$$-\ln|3 - y| = \sin x - \ln 2$$

Because $y(0) = 1$, $y < 3$, so $|3 - y| = 3 - y$

$$3 - y = 2e^{-\sin x}$$

$$y = 3 - 2e^{-\sin x}$$

6 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables