Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

	Model Solution	Scoring	
A	Show that $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$.		
	$\frac{d}{dx}\left(y^3 - y^2 - y + \frac{1}{4}x^2\right) = \frac{d}{dx}(0)$ $\Rightarrow 3y^2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} - \frac{dy}{dx} + \frac{x}{2} = 0$	Implicit differentiation	**Point 1 (P1)**
	$\Rightarrow (3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}$ $\Rightarrow \frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$	Verification	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned only for the correct implicit differentiation of $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y'.
- To be eligible for **P2**, a response must have earned **P1**.
- It is sufficient to present $(3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}$ to earn **P2**, provided there are no subsequent errors.

B	There is a point P on the curve G near $(2, -1)$ with x-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y-coordinate of point P.		
	$\left.\frac{dy}{dx}\right\vert_{(x,\,y)=(2,\,-1)} = \frac{-2}{2(3+2-1)} = -\frac{1}{4}$	Slope of tangent line	**Point 3 (P3)**
	$y \approx -1 - \frac{1}{4}(1.6 - 2) = -0.9$	Tangent line approximation	**Point 4 (P4)**

Scoring Notes for Part B

- A response can earn **P3** with $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,-1)} = -\frac{1}{4}$, $\frac{dy}{dx} = -\frac{1}{4}$, "slope is $-\frac{1}{4}$," or equivalent.
- A response that presents a linear approximation with a slope of $-\frac{1}{4}$ also earns **P3**.
- A response that declares $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,-1)}$ (or the slope) equal to any nonzero value $k \neq -\frac{1}{4}$ does not earn **P3**. Such a response earns **P4** for a presented approximation mathematically equivalent to $-1 + k(-0.4)$.
- **P4** cannot be earned with a linear approximation using a slope other than $-\frac{1}{4}$ if that slope has not been declared to be the value of $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,-1)}$.
- A response does not have to present the tangent line equation but must clearly demonstrate its use at $x = 1.6$ in finding the requested approximation to be eligible for **P4**.
- A response of $-1 - \frac{1}{4}(-0.4)$ earns both **P3** and **P4**.
- A response of $-1 - \frac{1}{4}(1.6 - 2)$ or equivalent banks **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4)**.

  Note: An ambiguous response, such as $-1 - \frac{1}{4}(1.6 - 2$, does not bank **P4** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., -0.9) to earn **P4**.

C	For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y-coordinate of point S. Show the work that leads to your answer.		
	For $x > 0$, the curve G has a vertical tangent line when $2(3y^2 - 2y - 1) = 0$.	Sets denominator equal to 0	**Point 5 (P5)**
	$2(3y^2 - 2y - 1) = 0 \Rightarrow 2(3y+1)(y-1) = 0$ Because $y > 0$, it follows that $y = 1$. The line tangent to the curve is vertical at the point on the curve where $y = 1$.	Answer	**Point 6 (P6)**

Scoring Notes for Part C

- **P5** is earned with any of $2(3y^2 - 2y - 1) = 0$, $3y^2 - 2y - 1 = 0$, $2(3y \pm 1)(y \pm 1) = 0$, or $(3y \pm 1)(y \pm 1) = 0$.
- To be eligible for **P6**, a response must have earned **P5**.
- A response does not need to consider the numerator of $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$ to earn **P5** or **P6**; considering the denominator is sufficient.
- A response that states solutions of $y = -\frac{1}{3}$ and $y = 1$, but does not clearly identify that $y = 1$ is the only solution to the prompt, does not earn **P6**.
- In the presence of algebraic work to find the value of y, **P6** is earned only if the algebraic work is correct.
- A response of "$2(3y^2 - 2y - 1) = 0$, $y = 1$" earns both **P5** and **P6**.

D	A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.		
	$\frac{d}{dt}(2xy + \ln y) = \frac{d}{dt}(8)$ $2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$	Attempts implicit differentiation with respect to t	**Point 7 (P7)**
		$2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$	**Point 8 (P8)**
	$2(3)(1) + 2(4)\frac{dy}{dt} + \frac{1}{1}\frac{dy}{dt} = 0$ $\Rightarrow 6 + 9\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{2}{3}$	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for implicitly differentiating $2xy + \ln y = 8$ with respect to t with at most one error.
- **P8** is earned for an equation equivalent to $2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$.
- To be eligible for **P9**, a response must have earned **P7** and **P8**, with no errors in implicit differentiation.
- **P9** is earned only for the value of $-\frac{2}{3}$.
- Alternate solution:

$$\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$$

$$\frac{d}{dx}(2xy + \ln y) = \frac{d}{dx}(8) \Rightarrow 2y + 2x\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-2y}{2x + \frac{1}{y}}$$

$$\left.\frac{dy}{dx}\right|_{(x,\,y)=(4,\,1)} = \frac{-2(1)}{2(4) + \frac{1}{1}} = -\frac{2}{9}$$

$$\left.\frac{dy}{dt}\right|_{(x,\,y)=(4,\,1)} = \left.\frac{dy}{dx}\cdot\frac{dx}{dt}\right|_{(x,\,y)=(4,\,1)} = -\frac{2}{9}\cdot 3 = -\frac{2}{3}$$

  - **P7** is earned for an implicit differentiation with respect to x with at most one error, as long as $\frac{dy}{dx}$ is eventually correctly linked to $\frac{dx}{dt}$.
  - **P8** is earned for finding $\frac{dy}{dx}$ and multiplying the result by $\frac{dx}{dt} = 3$.
  - **P8** can be earned for a stated incorrect $\frac{dy}{dx}$, as long as it is multiplied by 3.
  - **P9** is only earned for a correct value of $-\frac{2}{3}$ or equivalent.
  - Stating $\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$ alone does not earn any points.

Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3+4y}$.

Model Solution	Scoring	

(a) There is a point on the curve near $(2, 4)$ with x-coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y-coordinate of this point.

$\left.\frac{dy}{dx}\right\|_{(x,\,y)=(2,\,4)} = \frac{-2(2)}{3+4(4)} = -\frac{4}{19}$	Slope of tangent line	**1 point**
$y \approx 4 - \frac{4}{19}(3-2) = \frac{72}{19}$	Approximation	**1 point**

Scoring notes:

- A response earns the first point for finding $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,4)} = -\frac{4}{19}$, even if this is not labeled or used as the slope of a tangent line.
- A response that does not explicitly find the value of $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,4)}$ but uses a slope of $-\frac{4}{19}$ in a linear approximation also earns the first point.
- A response that declares $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,4)}$ equal to any nonzero value other than $-\frac{4}{19}$ does not earn the first point but is eligible for the second point for a linear approximation at $x = 3$ through the point $(2, 4)$ with a slope of the declared value.
  - The second point cannot be earned with a linear approximation using a slope other than $-\frac{4}{19}$ if that slope has not been declared to be the value of $\left.\frac{dy}{dx}\right|_{(x,\,y)=(2,\,4)}$.
- The second point cannot be earned for an approximation at any value of x other than 3.
- A response does not have to write the tangent line equation but must clearly demonstrate its use at $x = 3$ in finding the requested approximation in order to earn both points.
- The minimal work required to earn both points is $4 - \frac{4}{19}(3-2)$.

Total for part (a) 2 points

(b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

$\frac{dy}{dx} = \frac{-2x}{3+4y} = 0 \Rightarrow x = 0$	Considers $\frac{dy}{dx} = 0$	**1 point**
And so, if the horizontal line $y = 1$ is tangent to the curve, the point of tangency must be $(0, 1)$. However, the point $(0, 1)$ is not on the curve, because $0^2 + 3 \cdot 1 + 2 \cdot 1^2 = 5 \neq 48$. Therefore, the horizontal line $y = 1$ is not tangent to the curve.	Answer with reason	**1 point**

Scoring notes:

- The first point can be earned with a response of $\frac{dy}{dx} = 0$, $-2x = 0$, or $x = 0$, OR by identifying the point $(0, 1)$.
- To earn the second point a response must provide a reason that the line $y = 1$ is not tangent to the curve. Merely stating "$(0, 1)$ does not lie on the curve" is not sufficient to earn the second point.
- Alternate solution:

  If $y = 1$, then $x^2 + 3 \cdot 1 + 2 \cdot 1^2 = 48 \Rightarrow x = \pm\sqrt{43}$.

  $\left.\frac{dy}{dx}\right|_{(x,\,y)=(\pm\sqrt{43},\,1)} = \frac{\pm 2\sqrt{43}}{7} \neq 0$

  Therefore, the horizontal line $y = 1$ is not tangent to the curve.

  - A response that uses this method earns the first point by using $y = 1$ in $x^2 + 3y + 2y^2 = 48$.
  - A response that fails to consider both $x = +\sqrt{43}$ and $x = -\sqrt{43}$ does not earn the second point.

Total for part (b) 2 points

(c) The curve intersects the positive x-axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

At the point $(\sqrt{48}, 0)$, the slope of the line tangent to the curve is $\frac{dy}{dx} = \frac{-2\sqrt{48}}{3 + 4(0)}$. The denominator of $\frac{dy}{dx}$ is $3 + 4(0)$, which does not equal 0. Therefore, the line tangent to the curve at this point is not vertical.	Answer with reason	**1 point**

Scoring notes:

- A response does not need to consider the numerator of $\left.\frac{dy}{dx}\right|_{(x,\, y)=(\sqrt{48},\, 0)}$ in order to earn this point; considering the denominator is sufficient.
- To earn this point a response must clearly demonstrate that the slope of the tangent line at the point $(\sqrt{48}, 0)$ is defined and answer "no."
  - Such demonstrations include, but are not limited to, the following:
    - $3 + 4(0) \neq 0$
    - At $(\sqrt{48}, 0)$, $\frac{dy}{dx} = \frac{-2\sqrt{48}}{3}$.
    - $\frac{-2\sqrt{48}}{3}$
    - When $3 + 4y = 0$, $y \neq 0$.

Total for part (c) 1 point

(d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y-coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x-coordinate of the particle's position with respect to time?

$3y^2\frac{dy}{dt} + 2x\frac{dy}{dt} + 2y\frac{dx}{dt} = 0$	Attempts implicit differentiation	**1 point**
	$3y^2\frac{dy}{dt} + 2x\frac{dy}{dt} + 2y\frac{dx}{dt} = 0$	**1 point**
$\frac{dy}{dt} = -2$	Uses $\frac{dy}{dt} = \pm 2$	**1 point**
$3(2)^2(-2) + 2(4)(-2) + 2(2)\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = \frac{40}{4} = 10$ The rate of change with respect to time in the x-coordinate is 10 units per second.	Answer	**1 point**

Scoring notes:

- The first point is earned for implicitly differentiating $y^3 + 2xy = 24$ with respect to t with at most one error.
  - The first point can also be earned by correctly differentiating with respect to x: $3y^2\frac{dy}{dx} + 2x\frac{dy}{dx} + 2y = 0$.
- The second point is earned for an equation equivalent to $3y^2\frac{dy}{dt} + 2x\frac{dy}{dt} + 2y\frac{dx}{dt} = 0$.
- A response does not need to explicitly declare $\frac{dy}{dt} = -2$ or $\frac{dy}{dt} = 2$ in order to earn the third point; this point may be earned by correctly substituting $\frac{dy}{dt} = -2$ or $\frac{dy}{dt} = 2$ in the implicitly differentiated equation. However, a response that uses both $\frac{dy}{dt} = -2$ and $\frac{dy}{dt} = 2$ in the implicitly differentiated equation does not earn the third point.
- The fourth point cannot be earned without the first 3 points. The fourth point is earned only for the value of 10 with no mistakes in supporting work.
  - Note that a response that uses $\frac{dy}{dt} = 2$ and then mishandles subtracting 40 from both sides of the equation, e.g., $3(2)^2(2) + 2(4)(2) + 2(2)\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = \frac{40}{4}$, does not earn the fourth point.

Total for part (d) 4 points

Total for question 5 9 points

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Question 6 — 9 points

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Consider the curve given by the equation $6xy = 2 + y^3$.

Model Solution	Scoring	
(a) Show that $\frac{dy}{dx} = \frac{2y}{y^2 - 2x}$.		
$\frac{d}{dx}(6xy) = \frac{d}{dx}\left(2 + y^3\right) \Rightarrow 6y + 6x\frac{dy}{dx} = 3y^2\frac{dy}{dx}$	Implicit differentiation	**1 point**
$\Rightarrow 2y = \frac{dy}{dx}\left(y^2 - 2x\right) \Rightarrow \frac{dy}{dx} = \frac{2y}{y^2 - 2x}$	Verification	**1 point**

Scoring notes:

- The first point is earned only for the correct implicit differentiation of $6xy = 2 + y^3$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y'.
- The second point cannot be earned without the first point.
- It is sufficient to present $2y = \frac{dy}{dx}\left(y^2 - 2x\right)$ to earn the second point, provided there are no subsequent errors.

Total for part (a) — 2 points

(b) Find the coordinates of a point on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

For the line tangent to the curve to be horizontal, it is necessary that $2y = 0$ (so $y = 0$) and that $y^2 - 2x \neq 0$.	Sets $2y = 0$	**1 point**
Substituting $y = 0$ into $6xy = 2 + y^3$ yields the equation $6x \cdot 0 = 2$, which has no solution. Therefore, there is no point on the curve at which the line tangent to the curve is horizontal.	Answer with reason	**1 point**

Scoring notes:

- The first point is earned with any of $2y = 0$, $y = 0$, $\frac{dy}{dx} = 0$, $dy = 0$, $y' = 0$, or $\frac{2y}{y^2 - 2x} = 0$.
- A response need not state that at a horizontal tangent, $y^2 - 2x \neq 0$.

Total for part (b) — 2 points

(c) Find the coordinates of a point on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.

For a line tangent to this curve to be vertical, it is necessary that $2y \neq 0$ and that $y^2 - 2x = 0$ (so $x = \frac{y^2}{2}$).	Sets $y^2 - 2x = 0$	**1 point**
Substituting $x = \frac{y^2}{2}$ into $6xy = 2 + y^3$ yields the equation $3y^2 \cdot y = 2 + y^3 \Rightarrow 2y^3 = 2 \Rightarrow y = 1$.	Substitutes $x = \frac{y^2}{2}$ into $6xy = 2 + y^3$	**1 point**
Substituting $y = 1$ in $6xy = 2 + y^3$ yields $6x = 2 + 1$, or $x = \frac{1}{2}$. The tangent line to the curve is vertical at the point $\left(\frac{1}{2}, 1\right)$.	Answer	**1 point**

Scoring notes:

- The first point can be earned by presenting $y^2 = 2x$ or $y = \sqrt{2x}$.
- The second point can be earned for the substitution of $y = \sqrt{2x}$ into $6xy = 2 + y^3$, or for substituting $x = \frac{2 + y^3}{6y}$ into $y^2 - 2x = 0$.
- A response earns all three points by setting $y^2 - 2x = 0$, declaring the point $\left(\frac{1}{2}, 1\right)$, and verifying that this point is on the curve $6xy = 2 + y^3$.
- A response that identifies the point $\left(\frac{1}{2}, 1\right)$ but does not verify that the point is on the curve, does not earn the second or the third point.
- To earn the third point the response must present both coordinates of the point $\left(\frac{1}{2}, 1\right)$. The coordinates need not appear as an ordered pair as long as they are labeled.

Total for part (c) — 3 points

(d) A particle is moving along the curve. At the instant when the particle is at the point $\left(\frac{1}{2}, -2\right)$, its horizontal position is increasing at a rate of $\frac{dx}{dt} = \frac{2}{3}$ unit per second. What is the value of $\frac{dy}{dt}$, the rate of change of the particle's vertical position, at that instant?

$6y\frac{dx}{dt} + 6x\frac{dy}{dt} = 0 + 3y^2\frac{dy}{dt}$	Uses implicit differentiation with respect to t	**1 point**
At the point $(x, y) = \left(\frac{1}{2}, -2\right)$, $6(-2)\left(\frac{2}{3}\right) + 6\left(\frac{1}{2}\right)\frac{dy}{dt} = 3(-2)^2\frac{dy}{dt}$ $\Rightarrow -8 + 3\frac{dy}{dt} = 12\frac{dy}{dt}$ $\Rightarrow \frac{dy}{dt} = -\frac{8}{9}$ unit per second	Answer	**1 point**

Scoring notes:

- The first point is earned by presenting one or more of the terms $6y\frac{dx}{dt}$, $6x\frac{dy}{dt}$, or $3y^2\frac{dy}{dt}$.
- Units will not affect scoring in this part.
- An unsupported response of $-\frac{8}{9}$ earns no points.
- Alternate solution:

$$\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$$

$$\left.\frac{dy}{dx}\right|_{(x,\,y)=(1/2,\,-2)} = \frac{2(-2)}{(-2)^2 - 2(1/2)} = -\frac{4}{3}$$

$$\left.\frac{dy}{dt}\right|_{(x,\,y)=(1/2,\,-2)} = \frac{dy}{dx}\cdot\left.\frac{dx}{dt}\right|_{(x,\,y)=(1/2,\,-2)} = -\frac{4}{3}\cdot\frac{2}{3} = -\frac{8}{9} \text{ unit per second}$$

  - The first point is earned for the statement $\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt}$ or equivalent.
  - A numerical expression, such as $-\frac{4}{3}\cdot\frac{2}{3}$ or $\frac{2(-2)}{(-2)^2 - 2\left(\frac{1}{2}\right)}\cdot\frac{2}{3}$, earns both points.

Total for part (d) **2 points**

Total for question 6 **9 points**

Part B (AB): Graphing calculator not allowed
Question 5 9 points

General Scoring Notes

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y\sin x$ for $y > 0$.

	Model Solution	Scoring	
(a)	Show that $\frac{dy}{dx} = \frac{y\cos x}{4y - \sin x}$.		
	$\frac{d}{dx}\left(2y^2 - 6\right) = \frac{d}{dx}(y\sin x) \Rightarrow 4y\frac{dy}{dx} = \frac{dy}{dx}\sin x + y\cos x$	Implicit differentiation	**1 point**
	$\Rightarrow 4y\frac{dy}{dx} - \frac{dy}{dx}\sin x = y\cos x \Rightarrow \frac{dy}{dx}(4y - \sin x) = y\cos x$ $\Rightarrow \frac{dy}{dx} = \frac{y\cos x}{4y - \sin x}$	Verification	**1 point**

Scoring notes:

- The first point is earned only for correctly implicitly differentiating $2y^2 - 6 = y\sin x$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y'.
- The second point may not be earned without the first point.
- It is sufficient to present $\frac{dy}{dx}(4y - \sin x) = y\cos x$ to earn the second point, provided that there are no subsequent errors.

Total for part (a) **2 points**

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

At the point $(0, \sqrt{3})$, $\frac{dy}{dx} = \frac{\sqrt{3}\cos 0}{4\sqrt{3} - \sin 0} = \frac{1}{4}$. An equation for the tangent line is $y = \sqrt{3} + \frac{1}{4}x$.	Answer	**1 point**

Scoring notes:

- Any correct tangent line equation will earn the point. No supporting work is required. Simplification of the slope value is not required.

Total for part (b) 1 point

(c) For $0 \le x \le \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

$\frac{dy}{dx} = \frac{y\cos x}{4y - \sin x} = 0 \Rightarrow y\cos x = 0$ and $4y - \sin x \neq 0$	Sets $\frac{dy}{dx} = 0$	**1 point**
$y\cos x = 0$ and $y > 0 \Rightarrow x = \frac{\pi}{2}$	$x = \frac{\pi}{2}$	**1 point**
When $x = \frac{\pi}{2}$, $y\sin x = 2y^2 - 6 \Rightarrow y\sin\frac{\pi}{2} = 2y^2 - 6$ $\Rightarrow y = 2y^2 - 6 \Rightarrow 2y^2 - y - 6 = 0$ $\Rightarrow (2y+3)(y-2) = 0 \Rightarrow y = 2$ When $x = \frac{\pi}{2}$ and $y = 2$, $4y - \sin x = 8 - 1 \neq 0$. Therefore, the line tangent to the curve is horizontal at the point $\left(\frac{\pi}{2}, 2\right)$.	$y = 2$	**1 point**

Scoring notes:

- The first point is earned by any of $\frac{dy}{dx} = 0$, $\frac{y\cos x}{4y - \sin x} = 0$, $y\cos x = 0$, or $\cos x = 0$.
- If additional "correct" x-values are considered outside of the given domain, the response must commit to only $x = \frac{\pi}{2}$ to earn the second point. Any presented y-values, correct or incorrect, are not considered for the second point.
- Entering with $x = \frac{\pi}{2}$ does not earn the first point, earns the second point, and is eligible for the third point. The third point is earned for finding $y = 2$. The coordinates do not have to be presented as an ordered pair.
- The third point is not earned with additional points present unless the response commits to the correct point.

Total for part (c) 3 points

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

$\frac{d^2y}{dx^2} = \frac{(4y - \sin x)\left(\frac{dy}{dx}\cos x - y\sin x\right) - (y\cos x)\left(4\frac{dy}{dx} - \cos x\right)}{(4y - \sin x)^2}$	Considers $\frac{d^2y}{dx^2}$	**1 point**
When $x = \frac{\pi}{2}$ and $y = 2$, $\frac{d^2y}{dx^2} = \frac{\left(4\cdot 2 - \sin\frac{\pi}{2}\right)\left(0\cdot\cos\frac{\pi}{2} - 2\cdot\sin\frac{\pi}{2}\right) - \left(2\cos\frac{\pi}{2}\right)\left(4\cdot 0 - \cos\frac{\pi}{2}\right)}{\left(4\cdot 2 - \sin\frac{\pi}{2}\right)^2}$ $= \frac{(7)(-2) - (0)(0)}{(7)^2} = \frac{-2}{7} < 0.$	$\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{2}, 2\right)$	**1 point**
f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$ because $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$.	Answer with justification	**1 point**

Scoring notes:

- The first point is earned for an attempt to use the quotient rule (or product rule) to find $\frac{d^2y}{dx^2}$.
- The second point is earned for correctly finding $\frac{d^2y}{dx^2}$ and evaluating to find that $\frac{d^2y}{dx^2} < 0$ at $\left(\frac{\pi}{2}, 2\right)$. The explicit value of $-\frac{2}{7}$ or the equivalent does not need to be reported, but any reported values must be correct in order to earn this point.
- The third point can be earned without the second point by reaching a consistent conclusion based on the reported sign of a nonzero value of $\frac{d^2y}{dx^2}$ obtained utilizing $\frac{dy}{dx} = 0$.
- Imports: A response is eligible to earn all 3 points in part (d) with a point of the form $\left(\frac{\pi}{2}, k\right)$ with $k > 0$, imported from part (c).

Alternate Solution for part (d)	Scoring for Alternate Solution	
For the function $y = f(x)$ near the point $\left(\frac{\pi}{2}, 2\right)$, $4y - \sin x > 0$ and $y > 0$.	Considers sign of $4y - \sin x$	**1 point**
Thus, $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$ changes from positive to negative at $x = \frac{\pi}{2}$.	$\frac{dy}{dx}$ changes from positive to negative at $x = \frac{\pi}{2}$	**1 point**
By the First Derivative Test, f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$.	Conclusion	**1 point**

Scoring notes:

- The first point for considering the sign of $4y - \sin x$ may also be earned by stating that $4y - \sin x$ is not equal to zero.
- The second and third points can be earned without the first point.
- To earn the second point a response must state that $\frac{dy}{dx}$ (or $\cos x$) changes from positive to negative at $x = \frac{\pi}{2}$.
- The third point cannot be earned without the second point.
- A response that concludes there is a minimum at this point does not earn the third point.

Total for part (d) 3 points

Total for question 5 9 points

AP® Calculus AB 2015 Scoring Guidelines

2015 SCORING GUIDELINES

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t)\,dt = 76.570$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$

Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

$2 : \begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t, $0 \le t \le 8$, is $30 + \int_0^t [R(x) - D(x)]\,dx$.

$R(t) - D(t) = 0 \Rightarrow t = 0,\ 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

$3 : \begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $30 + \int_0^w [R(t) - D(t)]\,dt = 50$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

2015 SCORING GUIDELINES

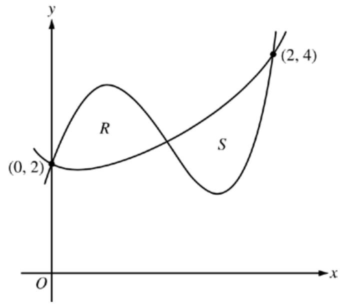
Question 2

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.



(a) The graphs of $y = f(x)$ and $y = g(x)$ intersect in the first quadrant at the points $(0, 2)$, $(2, 4)$, and $(A, B) = (1.032832, 2.401108)$.

Area $= \int_0^A [g(x) - f(x)]\,dx + \int_A^2 [f(x) - g(x)]\,dx$
$= 0.997427 + 1.006919 = 2.004$

$4 : \begin{cases} 1 : \text{limits} \\ 2 : \text{integrands} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \int_A^2 [f(x) - g(x)]^2\,dx = 1.283$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $h(x) = f(x) - g(x)$
$h'(x) = f'(x) - g'(x)$
$h'(1.8) = f'(1.8) - g'(1.8) = -3.812$ (or -3.811)

$2 : \begin{cases} 1 : \text{considers } h' \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

(a) $v'(16) \approx \dfrac{240 - 200}{20 - 12} = 5$ meters/min^2

1 : approximation

(b) $\int_0^{40} |v(t)|\, dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \le t \le 40$ minutes.

$$\begin{aligned}\int_0^{40} |v(t)|\, dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters}\end{aligned}$$

$3 : \begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.

$B'(5) = 3(25) - 12(5) = 15$ meters/min^2

$2 : \begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

(d) $$\begin{aligned}\text{Avg vel} &= \frac{1}{10}\int_0^{10} \left(t^3 - 6t^2 + 300\right) dt \\ &= \frac{1}{10}\left[\frac{t^4}{4} - 2t^3 + 300t\right]_0^{10} \\ &= \frac{1}{10}\left[\frac{10000}{4} - 2000 + 3000\right] = 350 \text{ meters/min}\end{aligned}$$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

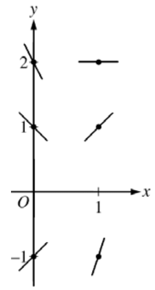
(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

(a)



$2 : \begin{cases} 1 : \text{slopes where } x = 0 \\ 1 : \text{slopes where } x = 1 \end{cases}$

(b) $\dfrac{d^2y}{dx^2} = 2 - \dfrac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$

In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$. Therefore, all solution curves are concave up in Quadrant II.

$2 : \begin{cases} 1 : \dfrac{d^2y}{dx^2} \\ 1 : \text{concave up with reason} \end{cases}$

(c) $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} = 2(2) - 3 = 1 \neq 0$

Therefore, f has neither a relative minimum nor a relative maximum at $x = 2$.

$2 : \begin{cases} 1 : \text{considers } \left.\dfrac{dy}{dx}\right|_{(x,\,y)=(2,\,3)} \\ 1 : \text{conclusion with justification} \end{cases}$

(d) $y = mx + b \Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}(mx + b) = m$

$2x - y = m$

$2x - (mx + b) = m$

$(2 - m)x - (m + b) = 0$

$2 - m = 0 \Rightarrow m = 2$

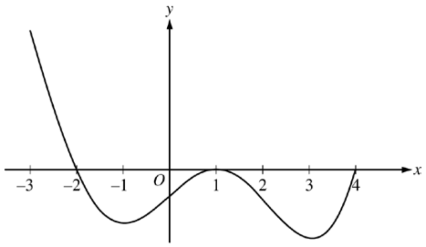
$b = -m \Rightarrow b = -2$

$3 : \begin{cases} 1 : \dfrac{d}{dx}(mx + b) = m \\ 1 : 2x - y = m \\ 1 : \text{answer} \end{cases}$

2015 SCORING GUIDELINES

Question 5

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



Graph of f'

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

(a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$.
$f'(x)$ changes from positive to negative at $x = -2$.
Therefore, f has a relative maximum at $x = -2$.

$2 : \begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$

(b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points.

The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.

$2 : \begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$

(d) $f(x) = 3 + \int_1^x f'(t)\,dt$

$f(4) = 3 + \int_1^4 f'(t)\,dt = 3 + (-12) = -9$

$$f(-2) = 3 + \int_1^{-2} f'(t)\,dt = 3 - \int_{-2}^{1} f'(t)\,dt = 3 - (-9) = 12$$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$

2015 SCORING GUIDELINES

Question 6

Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.

(a) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(-1,\,1)} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$

An equation for the tangent line is $y = \frac{1}{4}(x + 1) + 1$.

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{equation for tangent line} \end{cases}$

(b) $3y^2 - x = 0 \Rightarrow x = 3y^2$

So, $y^3 - xy = 2 \Rightarrow y^3 - \left(3y^2\right)(y) = 2 \Rightarrow y = -1$

$(-1)^3 - x(-1) = 2 \Rightarrow x = 3$

The tangent line to the curve is vertical at the point $(3, -1)$.

$3 : \begin{cases} 1 : \text{sets } 3y^2 - x = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{coordinates} \end{cases}$

(c) $$\frac{d^2y}{dx^2} = \frac{\left(3y^2 - x\right)\frac{dy}{dx} - y\left(6y\frac{dy}{dx} - 1\right)}{\left(3y^2 - x\right)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(x,\,y)=(-1,\,1)} = \frac{\left(3\cdot 1^2 - (-1)\right)\cdot\frac{1}{4} - 1\cdot\left(6\cdot 1\cdot\frac{1}{4} - 1\right)}{\left(3\cdot 1^2 - (-1)\right)^2} = \frac{1 - \frac{1}{2}}{16} = \frac{1}{32}$$

$4 : \begin{cases} 2 : \text{implicit differentiation} \\ 1 : \text{substitution for } \frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$