

6. The function f is twice differentiable. The table gives values of f and its derivative f' at selected values of x .

x	0	2	3	6
$f(x)$	-1	3	8	5
$f'(x)$	-5	4	9	-2

Part A

Find $\lim_{x \rightarrow 2} \frac{f(x)}{x}$, or state that the limit does not exist.

Part B

Let $g(x) = f(f(x))$. Find $g'(2)$. Show the work that leads to your answer.

Part C

Let h be a differentiable function such that $h(0) = 10$ and $h'(x) = f'(3x)$. Find $h(2)$. Show the work that leads to your answer.

Part D

Let k be the function defined by $k(x) = \int_0^x t^2 f(t) dt$.

- Find $k'(x)$.
- Find $k''(3)$. Show the work that leads to your answer.

STOP
END OF EXAM

5. Two particles, H and J , are moving along the x -axis. For $0 \leq t \leq 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2 - 4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2 - 1)^3$.

- Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.
- During what open intervals of time t , for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.
- It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.
- Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f , where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.
- (a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t) \, dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t) \, dt$.
- (b) Must there exist a value of c , for $60 < c < 120$, such that $f'(c) = 0$? Justify your answer.
- (c) The rate of flow of gasoline, in gallons per second, can also be modeled by $g(t) = \left(\frac{t}{500}\right) \cos\left(\left(\frac{t}{120}\right)^2\right)$ for $0 \leq t \leq 150$. Using this model, find the average rate of flow of gasoline over the time interval $0 \leq t \leq 150$.
- Show the setup for your calculations.
- (d) Using the model g defined in part (c), find the value of $g'(140)$. Interpret the meaning of your answer in the context of the problem.

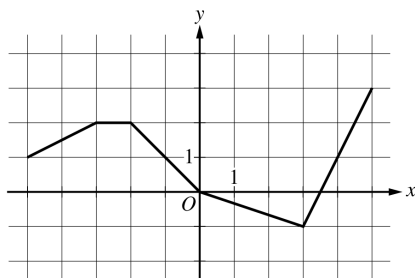
Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x .
- (a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.
- (b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.
- (c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t) \, dt$. Find $m(2)$. Show the work that leads to your answer.
- (d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

x	$g(x)$	$g'(x)$
-5	10	-3
-4	5	-1
-3	2	4
-2	3	1
-1	1	-2
0	0	-3

Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x .

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

- Find the slope of the line tangent to the graph of f at $x = \pi$.
- Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.
- Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.
- Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

STOP
END OF EXAM

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

- Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.
- Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.
- Evaluate $\int_1^3 f''(2x) dx$.

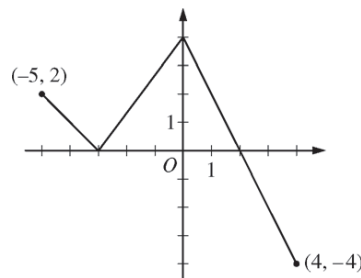
STOP
END OF EXAM

CALCULUS AB
SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^x f(t) dt$.
- Find $g(3)$.
 - On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
 - The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.
 - The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.
- Find the x -coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.
 - Explain why there must be a value c , for $-1 < c < 1$, such that $f''(c) = 0$.
 - The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.
 - Evaluate $\int_{-2}^3 f'(g(x))g'(x) dx$.