

6. The function f is twice differentiable. The table gives values of f and its derivative f' at selected values of x .

x	0	2	3	6
$f(x)$	-1	3	8	5
$f'(x)$	-5	4	9	-2

Part A

Find $\lim_{x \rightarrow 2} \frac{f(x)}{x}$, or state that the limit does not exist.

Part B

Let $g(x) = f(f(x))$. Find $g'(2)$. Show the work that leads to your answer.

Part C

Let h be a differentiable function such that $h(0) = 10$ and $h'(x) = f'(3x)$. Find $h(2)$. Show the work that leads to your answer.

Part D

Let k be the function defined by $k(x) = \int_0^x t^2 f(t) dt$.

- i. Find $k'(x)$.
- ii. Find $k''(3)$. Show the work that leads to your answer.

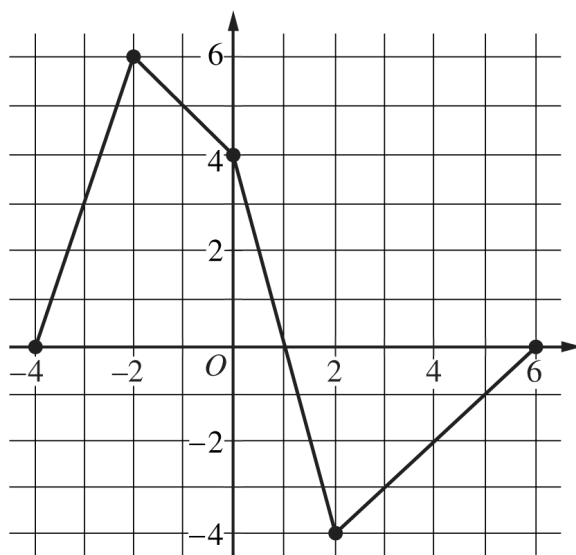
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x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x .

- (a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.
- (b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.
- (c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t) dt$. Find $m(2)$. Show the work that leads to your answer.
- (d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Graph of f

Let f be a continuous function defined on the closed interval $-4 \leq x \leq 6$. The graph of f , consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t) \, dt$.

(a) On what open intervals is the graph of G concave up? Give a reason for your answer.

(b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.

(c) Find $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$.

(d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c , $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

6. Functions f , g , and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{\pi}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.
- (a) Find $h'(2)$.
- (b) Let a be the function given by $a(x) = 3x^3h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.
- (c) The function h satisfies $h(x) = \frac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x \rightarrow 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \rightarrow 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.
- (d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.
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5. Let f be the function defined by $f(x) = e^x \cos x$.

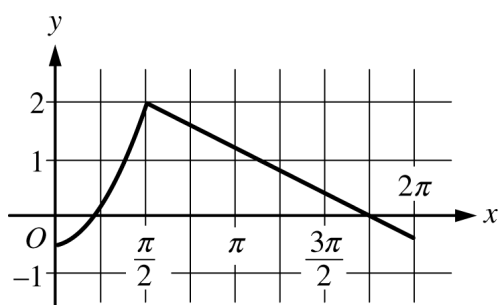
(a) Find the average rate of change of f on the interval $0 \leq x \leq \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \leq x \leq 2\pi$. Justify your answer.

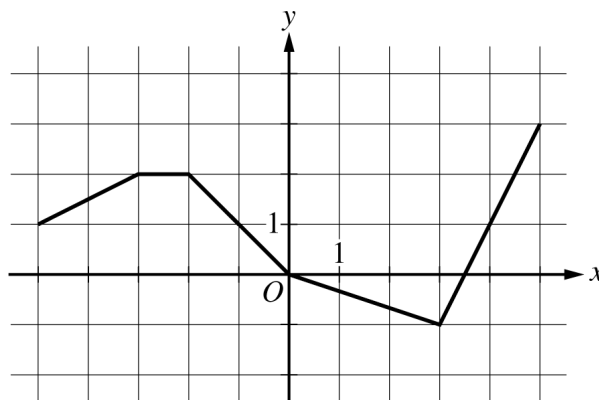
(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g' , the derivative of g , is shown

below. Find the value of $\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

x	$g(x)$	$g'(x)$
-5	10	-3
-4	5	-1
-3	2	4
-2	3	1
-1	1	-2
0	0	-3

Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x .

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

- (a) Find the slope of the line tangent to the graph of f at $x = \pi$.
- (b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.
- (c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.
- (d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

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