

## Part A (AB): Graphing calculator required

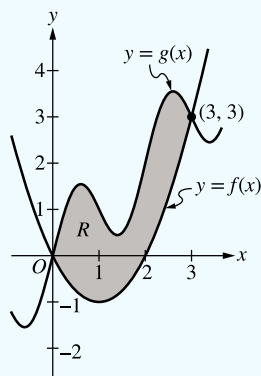
## Question 2

9 points

## General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The shaded region  $R$  is bounded by the graphs of the functions  $f$  and  $g$ , where  $f(x) = x^2 - 2x$  and  $g(x) = x + \sin(\pi x)$ , as shown in the figure.



(Note: Your calculator should be in radian mode.)

| Model Solution |                                                              | Scoring                               |
|----------------|--------------------------------------------------------------|---------------------------------------|
| A              | Find the area of $R$ . Show the setup for your calculations. |                                       |
|                | $\int_0^3 (g(x) - f(x)) \, dx$                               | Form of integrand <b>Point 1 (P1)</b> |
|                | $= 5.136620$                                                 | Answer <b>Point 2 (P2)</b>            |
|                | The area is 5.137 (or 5.136).                                |                                       |

## Scoring Notes for Part A

- P1** is earned for a response that presents an integrand of  $g(x) - f(x)$ ,  $|g(x) - f(x)|$ ,  $f(x) - g(x)$ , or  $|f(x) - g(x)|$  in a definite integral, with or without the differential  $dx$ .
- P1** could also be earned for a difference of definite integrals with integrands  $g(x)$  and  $f(x)$ .
- P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect communication between the integral and the correct answer is treated as scratch work and is not considered in scoring.
  - $\int_0^3 (f(x) - g(x)) \, dx = -5.137$  so the area is 5.137.  
Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer.
  - $\int_0^3 (f(x) - g(x)) \, dx = 5.137$   
Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- The exact answer is  $\frac{4 + 9\pi}{2\pi}$ .

- B** Region  $R$  is the base of a solid. For this solid, at each  $x$  the cross section perpendicular to the  $x$ -axis is a rectangle with height  $x$  and base in region  $R$ . Find the volume of the solid. Show the setup for your calculations.

|                                 |                   |                     |
|---------------------------------|-------------------|---------------------|
| $\int_0^3 x(g(x) - f(x)) \, dx$ | Form of integrand | <b>Point 3 (P3)</b> |
| $= 7.704930$                    | Answer            | <b>Point 4 (P4)</b> |

The volume of the solid is 7.705 (or 7.704).

#### Scoring Notes for Part B

- P3** is earned for a definite integral with an integrand presented as a product of two nonconstant factors, with one of the factors equal to  $x$ ,  $g(x) - f(x)$ , or  $f(x) - g(x)$ .
- The presence or absence of the differential  $dx$  will not be considered in scoring **P3** or **P4**.
- P4** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- Incorrect or unclear communication between the integral and the correct answer is treated as scratch work and is not considered in scoring. For example:
  - $\int_0^3 x(f(x) - g(x)) \, dx = -7.705$  so the volume is 7.705.  
Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer.
  - $\int_0^3 x(f(x) - g(x)) \, dx = 7.705$   
Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- The exact answer is  $\frac{12 + 27\pi}{4\pi}$ .

- C** Write, but do not evaluate, an integral expression for the volume of the solid generated when the region  $R$  is rotated about the horizontal line  $y = -2$ .

|                                                                          |                                    |                     |
|--------------------------------------------------------------------------|------------------------------------|---------------------|
| $\text{Volume} = \pi \int_0^3 ((g(x) - (-2))^2 - (f(x) - (-2))^2) \, dx$ | Form of integrand                  | <b>Point 5 (P5)</b> |
|                                                                          | Integrand                          | <b>Point 6 (P6)</b> |
|                                                                          | Limits, constant, and differential | <b>Point 7 (P7)</b> |

#### Scoring Notes for Part C

- P5** is earned for a definite integral with an integrand of  $R^2 - r^2$  or  $|R^2 - r^2|$ , where one of  $\{R, r\}$  is correct or a difference between  $g$  and a nonzero constant, and the other is correct or a difference between  $f$  and a nonzero constant.
- P6** is earned for the integral  $\int_0^3 ((g(x) + 2)^2 - (f(x) + 2)^2) \, dx$ ,  $\int_0^3 ((f(x) + 2)^2 - (g(x) + 2)^2) \, dx$ , or a mathematically equivalent expression.
- Note **P5** and **P6** could be earned for a difference of definite integrals.
- A response that presents an integral expression that does not include the constant  $\pi$  is eligible for **P5** and **P6** but does not earn **P7**.
- To be eligible for **P7**, a response must have earned **P5**.
- A response that reverses the difference of squares must resolve the reversal with either the constant or the limits of integration AND include the differential to earn **P7**. For example:
  - A response of  $-\pi \int_0^3 ((f(x) + 2)^2 - (g(x) + 2)^2) \, dx$  or  $\pi \int_3^0 ((f(x) + 2)^2 - (g(x) + 2)^2) \, dx$  earns **P5**, **P6**, and **P7**.
  - A response of  $\pi \int_0^3 ((f(x) + 2)^2 - (g(x) + 2)^2) \, dx$  earns **P5** and **P6** but does not earn **P7**.
- A response of only  $\pi \int_0^3 ((g(x) - (-2))^2 - (f(x) - (-2))^2) \, dx$  earns **P5**, **P6**, and **P7**.

- D** It can be shown that  $g'(x) = 1 + \pi \cos(\pi x)$ . Find the value of  $x$ , for  $0 < x < 1$ , at which the line tangent to the graph of  $f$  is parallel to the line tangent to the graph of  $g$ .

|                                                          |                 |                     |
|----------------------------------------------------------|-----------------|---------------------|
| $f'(x) = g'(x) \Rightarrow 2x - 2 = 1 + \pi \cos(\pi x)$ | $f'(x) = g'(x)$ | <b>Point 8 (P8)</b> |
| $\Rightarrow x = 0.675819$                               | Answer          | <b>Point 9 (P9)</b> |

The lines tangent to the graphs of  $f$  and  $g$  are parallel at  $x = 0.676$  (or 0.675).

#### Scoring Notes for Part D

- P8** is earned for a general statement, such as  $f'(x) = g'(x)$ , or for any correct equation formed by substituting  $2x - 2$  for  $f'(x)$ ,  $1 + \pi \cos(\pi x)$  for  $g'(x)$ , or both.
- P9** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

# AP Calculus AB

## Scoring Guidelines

### 2018 SCORING GUIDELINES

#### Question 1

(a)  $\int_0^{300} r(t) \, dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval  $0 \leq t \leq 300$ .

(b)  $20 + \int_0^{300} (r(t) - 0.7) \, dt = 20 + \int_0^{300} r(t) \, dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time  $t = 300$ .

(c) Based on part (b), the number of people in line at time  $t = 300$  is 80.

The first time  $t$  that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or 414.285) seconds.}$$

(d) The total number of people in line at time  $t$ ,  $0 \leq t \leq 300$ , is modeled by  $20 + \int_0^t r(x) \, dx - 0.7t$ .

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

| $t$   | People in line for escalator |
|-------|------------------------------|
| 0     | 20                           |
| $t_1$ | 3.803                        |
| $t_2$ | 158.070                      |
| 300   | 80                           |

The number of people in line is a minimum at time  $t = 33.013$  seconds, when there are 4 people in line.

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 :  $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

1 : answer

4 :  $\begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$

## 2018 SCORING GUIDELINES

## Question 2

(a)  $v'(3) = -2.118$

The acceleration of the particle at time  $t = 3$  is  $-2.118$ .

1 : answer

(b)  $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time  $t = 3$  is  $-1.760$ .

3 :  $\begin{cases} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(c)  $\int_0^{3.5} v(t) dt = 2.844$  (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral  $\int_0^{3.5} v(t) dt$  is the displacement of the particle over the time interval  $0 \leq t \leq 3.5$ .

The integral  $\int_0^{3.5} |v(t)| dt$  is the total distance traveled by the particle over the time interval  $0 \leq t \leq 3.5$ .

3 :  $\begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{cases}$

(d)  $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time  $t = 1.571$  (or 1.570).

2 :  $\begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$

## 2018 SCORING GUIDELINES

## Question 3

$$\begin{aligned} \text{(a)} \quad f(-5) &= f(1) + \int_1^{-5} g(x) dx = f(1) - \int_{-5}^1 g(x) dx \\ &= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2} \end{aligned}$$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} \text{(b)} \quad \int_1^6 g(x) dx &= \int_1^3 g(x) dx + \int_3^6 g(x) dx \\ &= \int_1^3 2 dx + \int_3^6 2(x-4)^2 dx \\ &= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10 \end{aligned}$$

3 :  $\begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of  $f$  is increasing and concave up on  $0 < x < 1$  and  $4 < x < 6$  because  $f'(x) = g(x) > 0$  and  $f''(x) = g(x)$  is increasing on those intervals.

2 :  $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of  $f$  has a point of inflection at  $x = 4$  because  $f''(x) = g(x)$  changes from decreasing to increasing at  $x = 4$ .

2 :  $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

## 2018 SCORING GUIDELINES

## Question 4

$$(a) \quad H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2}$$

$H'(6)$  is the rate at which the height of the tree is changing, in meters per year, at time  $t = 6$  years.

$$(b) \quad \frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2$$

Because  $H$  is differentiable on  $3 \leq t \leq 5$ ,  $H$  is continuous on  $3 \leq t \leq 5$ .

By the Mean Value Theorem, there exists a value  $c$ ,  $3 < c < 5$ , such that  $H'(c) = 2$ .

(c) The average height of the tree over the time interval  $2 \leq t \leq 10$  is given by  $\frac{1}{10 - 2} \int_2^{10} H(t) \, dt$ .

$$\begin{aligned} \frac{1}{8} \int_2^{10} H(t) \, dt &\approx \frac{1}{8} \left( \frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3 \right) \\ &= \frac{1}{8} (65.75) = \frac{263}{32} \end{aligned}$$

The average height of the tree over the time interval  $2 \leq t \leq 10$  is  $\frac{263}{32}$  meters.

$$(d) \quad G(x) = 50 \Rightarrow x = 1$$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$$

$$\left. \frac{d}{dt}(G(x)) \right|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is  $\frac{3}{4}$  meter per year.

2 :  $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

2 :  $\begin{cases} 1 : \frac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

2 :  $\begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

3 :  $\begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0] if no chain rule

## 2018 SCORING GUIDELINES

## Question 5

(a) The average rate of change of  $f$  on the interval  $0 \leq x \leq \pi$  is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

(b)  $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The slope of the line tangent to the graph of  $f$  at  $x = \frac{3\pi}{2}$  is  $e^{3\pi/2}$ .

(c)  $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

| $x$              | $f(x)$                           |
|------------------|----------------------------------|
| 0                | 1                                |
| $\frac{\pi}{4}$  | $\frac{1}{\sqrt{2}} e^{\pi/4}$   |
| $\frac{5\pi}{4}$ | $-\frac{1}{\sqrt{2}} e^{5\pi/4}$ |
| $2\pi$           | $e^{2\pi}$                       |

The absolute minimum value of  $f$  on  $0 \leq x \leq 2\pi$  is  $-\frac{1}{\sqrt{2}} e^{5\pi/4}$ .

(d)  $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because  $g$  is differentiable,  $g$  is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

1 : answer

2 :  $\begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$

3 :  $\begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$

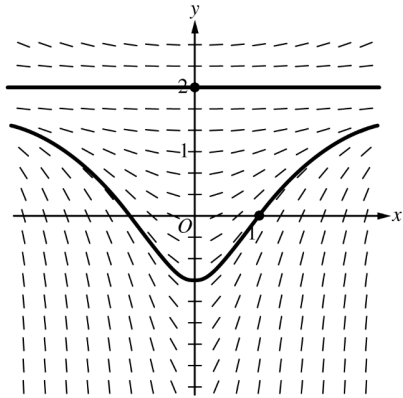
3 :  $\begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \text{and limits equal 0} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

## 2018 SCORING GUIDELINES

## Question 6

(a)



- 2 :  $\begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b)  $\left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = \frac{4}{3}$

An equation for the line tangent to the graph of  $y = f(x)$  at

$x = 1$  is  $y = \frac{4}{3}(x - 1)$ .

$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$

- 2 :  $\begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

(c)  $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$   
 $\int \frac{dy}{(y - 2)^2} = \int \frac{1}{3}x \, dx$

$\frac{-1}{y - 2} = \frac{1}{6}x^2 + C$

$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$

$\frac{-1}{y - 2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$

$y = 2 - \frac{6}{x^2 + 2}$

- 5 :  $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \text{and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: this solution is valid for  $-\infty < x < \infty$ .