Part A (AB or BC): Graphing calculator required

Question 1 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring	
A	Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.		
	$\frac{1}{4-0}\int_0^4 C(t)\,dt$	Average value formula	**Point 1 (P1)**
	$= \frac{1}{4}(11.112896) = 2.778224$ From time $t = 0$ to $t = 4$ weeks, the average number of acres affected by the invasive species was 2.778 acres.	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned for the correct integral, with or without the differential, along with evidence of division by 4. In the presence of the correct integral, the correct answer will suffice as evidence of division by 4. These may appear all in one step, as in the model solution, or in multiple steps.
- **P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect or unclear communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring. For example:
  - $\int_0^4 C(t)\,dt = 11.112896$ so the average velocity is 2.778224.

    Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer.
  - $\int_0^4 C(t)\,dt = \frac{11.112896}{4} = 2.778224$

    Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
  - $\int_0^4 C(t)\,dt = 2.778224$

    Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- Note that the values $\frac{1}{4}(11.112)$ and $\frac{1}{4}(11.113)$ are accurate to three digits after the decimal and therefore earn **P2**.

B	Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.		
	$\dfrac{C(4) - C(0)}{4 - 0} = 1.282008$	Uses average rate of change	**Point 3 (P3)**
	$C'(t) = \dfrac{38}{25 + t^2} = 1.282008 \Rightarrow t = 2.154298$ The instantaneous rate of change of C equals the average rate of change of C over the interval $0 \le t \le 4$ at time $t = 2.154$.	Answer with supporting work	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** may be earned by presenting the expression or value for the average rate of change. Note that because $C(0) = 0$ and the interval is $0 \le t \le 4$, any of the following will earn **P3**: $\dfrac{\int_0^4 C'(t)\,dt}{4}$, $\dfrac{C(4) - C(0)}{4 - 0}$, $\dfrac{C(4)}{4}$, $\dfrac{5.128 - 0}{4 - 0}$, $\dfrac{5.128}{4}$, or 1.282. However, neither **P3** nor **P4** is earned by just presenting $t = 1.282$.
- **P4** is earned for the correct answer supported by the appropriate equation. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

  The following response, for example, earns both **P3** and **P4**: $C'(t) = \dfrac{C(4) - C(0)}{4}$ when $t = 2.154$.

C	Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.		
	$\lim\limits_{t \to \infty} C'(t) = \lim\limits_{t \to \infty} \dfrac{38}{25 + t^2}$	Limit expression	**Point 5 (P5)**
	$= 0$	Value	**Point 6 (P6)**

Scoring Notes for Part C

- **P5** can be earned for either $\lim\limits_{t \to \infty} C'(t)$ or $\lim\limits_{t \to \infty} C(t)$.
- A response that includes $\lim\limits_{t \to \infty} C(t)$ is not eligible to earn **P6**.
- For **P6**, arithmetic with infinity, e.g., $\dfrac{38}{25 + \infty^2} = 0$, will be considered as scratch work and will not be considered in scoring.

D	At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.		
	$A'(t) = C'(t) - 0.1 \cdot \ln t$ For $4 \le t \le 36$, the maximum value of $A(t)$ occurs when $A'(t) = 0$ or at an endpoint. $A'(t) = C'(t) - 0.1 \cdot \ln t = 0 \Rightarrow C'(t) = 0.1 \cdot \ln t$	Considers $A'(t) = 0$	**Point 7 (P7)**
	$\Rightarrow t = 11.441700$ t : 4, 11.441700, 36 $A(t)$: 5.128031, 7.316978, 1.743056	Justification	**Point 8 (P8)**
	Therefore, the number of acres affected by the species is a maximum at time $t = 11.442$ (or 11.441) weeks.	Answer with supporting work	**Point 9 (P9)**

t	$A(t)$
4	5.128031
11.441700	7.316978
36	1.743056

Scoring Notes for Part D

- **P7** is earned for considering $A'(t) = 0$, $C'(t) - 0.1 \cdot \ln t = 0$, or $C'(t) = 0.1 \cdot \ln t$. **P7** is not earned by just presenting $t = 11.441700$.

  A response that discusses the sign of $A'(t)$ changing or uses the phrase "critical points of A" also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by correctly evaluating $A(t)$ at $t = 4$, $t = 11.441700$, and $t = 36$. The evaluations must be correct to the first digit after the decimal, rounded or truncated.
- Alternate justifications:
  - $A'(t) > 0$ for $4 < t < 11.442$, and $A'(t) < 0$ for $11.442 < t < 36$. Therefore, $t = 11.442$ is the location of the absolute maximum for A on the interval $4 \le t \le 36$.
  - Because $A'(t)$ changes sign from positive to negative at $t = 11.442$ (this might be presented as "$A'(t) > 0$ for $t < 11.442$, and $A'(t) < 0$ for $t > 11.442$"), it is the location of a relative maximum for A. And because $t = 11.442$ is the only critical point of A in the interval $4 \le t \le 36$, it is the location of the absolute maximum for A on the interval.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Particle P moves along the x-axis such that, for time $t > 0$, its position is given by $x_P(t) = 6 - 4e^{-t}$.

Particle Q moves along the y-axis such that, for time $t > 0$, its velocity is given by $v_Q(t) = \frac{1}{t^2}$. At time $t = 1$, the position of particle Q is $y_Q(1) = 2$.

Model Solution	Scoring	

(a) Find $v_P(t)$, the velocity of particle P at time t.

$v_P(t) = x_P'(t) = 4e^{-t}$	Answer	**1 point**

Scoring notes:

- A response that equates $x_P(t)$ with $v_P(t)$ does not earn the point.
- An unlabeled response earns the point.

Total for part (a) **1 point**

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

$a_Q(t) = v_Q'(t) = \frac{-2}{t^3}$	$a_Q(t)$	**1 point**
For $t > 0$, $a_Q(t) < 0$ and $v_Q(t) > 0$.	Considers signs of $a_Q(t)$ and $v_Q(t)$	**1 point**
Because the velocity and acceleration have opposite signs, the speed of particle Q is decreasing for all $t > 0$.	Answer with justification	**1 point**

Scoring notes:

- Earning the first point is not necessary for a response to be eligible to earn the second or third points; however, the response must present an expression for $a_Q(t)$ to be eligible for third point.
- A response earns the second point with either of the following statements: "$v_Q(t)$ and $a_Q(t)$ have opposite signs" or "$v_Q(t)$ and $a_Q(t)$ have the same sign." This statement, however, must be consistent with $v_Q(t)$ and the presented expression for $a_Q(t)$.
- A response must earn the second point to be eligible for the third point. The answer must be consistent with the presented justification. Furthermore, responses for which $a_Q(t) > 0$ for $t > 0$ must conclude that there is no time at which the speed of the particle is decreasing.
- A response that indicates $v_Q(t) < 0$ does not earn the third point, even if the answer and justification are consistent with a reported sign of $a_Q(t)$.

Total for part (b) **3 points**

(c) Find $y_Q(t)$, the position of particle Q at time t.

$y_Q(t) = y_Q(1) + \int_1^t \frac{1}{s^2}\,ds$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 2 - \left(\frac{1}{s}\Big\vert_1^t\right) = 2 - \frac{1}{t} + 1 = 3 - \frac{1}{t}$	Answer	**1 point**

Scoring notes:

- A response that presents $\int_1^t \frac{1}{t^2}\,dt$ (using the same variable as a limit and integrand function) does not earn the first point unless it is followed by an attempt at integration.
- A response that presents either $\int \frac{1}{t^2}\,dt$ or $-\frac{1}{t}$ (with no integral) earns the first point. If the response continues and presents $2 = -1 + C$, then the response earns the second point.
- A response that presents only $y_Q(t) = -\frac{1}{t} + 3$ will earn all 3 points. Note that the right side of this equation suffices to earn all points. A response of $y_Q(t) = -\frac{1}{t} + C$, where $C \neq 3$, (with no additional supporting work) earns only the first point.

Total for part (c) 3 points

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

For particle P, $\lim_{t\to\infty}\left(6 - 4e^{-t}\right) = 6$. For particle Q, $\lim_{t\to\infty}\left(3 - \frac{1}{t}\right) = 3$.	One correct limit	**1 point**
Because $6 > 3$, particle P will eventually be farther from the origin.	Answer with reason	**1 point**

Scoring notes:

- A response with an incorrect $y_Q(t)$ from part (c) is eligible for both points in part (d) provided $y_Q(t)$ is a non-constant function. The second point is earned for a consistent answer with reason, and limits correct for particle P and the presented $y_Q(t)$.
- Responses that present statements such as "$6 - 4e^{-t}$ approaches 6" or "Q goes to 3" earn the first point and are eligible for the second point.
- A response that treats ∞ as an input for $x_P(t)$ or $y_Q(t)$, such as "$6 - 4e^{-\infty}$" or "$3 - \frac{1}{\infty}$" is not eligible for the second point.

Total for part (d) 2 points

Total for question 6 9 points