

Part B (AB): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Two particles, H and J , are moving along the x -axis. For $0 \leq t \leq 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2-4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2 - 1)^3$.

	Model Solution	Scoring
A	Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.	
	$x_H'(t) = v_H(t) = (2t - 4)e^{t^2-4t}$	Considers x_H' Point 1 (P1)
	$x_H'(1) = v_H(1) = -2e^{-3}$	Answer Point 2 (P2)
Scoring Notes for Part A		
P1 can be earned by presenting $x_H'(t)$, $x_H'(1)$, $x'(t)$, $x'(1)$, $(2t - 4)e^{t^2-4t}$, or $(2 \cdot 1 - 4)e^{1^2-4 \cdot 1}$. - An unsupported answer of $-2e^{-3}$ earns P2 but not P1.		

- B** During what open intervals of time t , for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

From part A, $x_H'(t) = v_H(t) = (2t - 4)e^{t^2-4t}$.	Considers sign of $x_H'(t)$ or $v_J(t)$	Point 3 (P3)
$x_H'(t) = (2t - 4)e^{t^2-4t} = 0 \Rightarrow t = 2$		
$x_H'(t) < 0$ for $0 < t < 2$, and $x_H'(t) > 0$ for $2 < t < 5$.	Analysis for one particle	Point 4 (P4)
Thus, particle H is moving to the left for $0 < t < 2$ and moving to the right for $2 < t < 5$.	Answer with reason	Point 5 (P5)
$v_J(t) = 2t(t^2 - 1)^3 = 0$ for $0 < t < 5 \Rightarrow t = 1$		
$v_J(t) < 0$ for $0 < t < 1$, and $v_J(t) > 0$ for $1 < t < 5$.		
Thus, particle J is moving to the left for $0 < t < 1$ and moving to the right for $1 < t < 5$.		
Therefore, particles H and J are moving in opposite directions for $1 < t < 2$.		

Scoring Notes for Part B

- To earn **P3**, a response can do one of the following:
 - Set $x_H'(t) = 0$, $v_H(t) = 0$, or $(2t - 4)e^{t^2-4t} = 0$
 - Set $v_J(t) = 0$ or $2t(t^2 - 1)^3 = 0$
 - Identify $t = 2$ for particle H and no other values in the interval $0 < t < 5$
 - Identify $t = 1$ for particle J and no other values in the interval $0 < t < 5$
 - Identify the interval $1 < t < 2$
- To earn **P4**, a response can provide an analysis of signs of velocity or direction of motion on the interval $0 < t < 5$ for either particle H or particle J .
- To be eligible for **P5**, a response must provide correct analyses of signs of velocity or direction of motion on the interval $0 < t < 5$ for both particles.
- Only analysis within the interval $0 < t < 5$ will be considered in scoring.

- C** It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

$$v_J(2) > 0 \text{ and } v_J'(2) > 0.$$

Answer with reason **Point 6 (P6)**

Because $v_J(2)$ and $v_J'(2)$ have the same sign, the speed of particle J is increasing at $t = 2$.

Scoring Notes for Part C

- An evaluation of $v_J(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J(2) = 108$.
- An evaluation of $v_J'(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J'(2) = 486$.
- A response can either import the analysis for the sign of $v_J(2)$ from part B or restart.
- A response that stated “ $v_J(t) > 0$ for $1 < t < 5$ ” in part B does not need to restate $v_J(2) > 0$ and earns **P6** for “ $v_J(2)$ and $v_J'(2)$ have the same sign, so the speed is increasing.”

- D** Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

$$\begin{aligned} x_J(2) &= x_J(0) + \int_0^2 v_J(t) dt = 7 + \int_0^2 2t(t^2 - 1)^3 dt \\ &= 7 + \left[\frac{1}{4}(t^2 - 1)^4 \right]_0^2 \\ &= 7 + \frac{1}{4}((3)^4 - (-1)^4) = 7 + \frac{1}{4}(80) = 27 \end{aligned}$$

Integrand **Point 7 (P7)**
Antiderivative **Point 8 (P8)**
Answer **Point 9 (P9)**

Scoring Notes for Part D

- To earn **P7**, a response must present an indefinite or definite integral with an integrand of $v_J(t)$ or $2t(t^2 - 1)^3$. (See below for notes on how to handle a missing differential dt .)
- P8** is earned for an antiderivative of the form $k(t^2 - 1)^4$ or equivalent, for $k > 0$. If $k \neq \frac{1}{4}$, then the response is not eligible to earn **P9**.
- A response of $7 + \frac{1}{4}((3)^4 - (-1)^4)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

Note: An ambiguous response, such as $7 + \frac{1}{4}((3)^4 - (-1)^4)$, does not bank **P9** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., $7 + \frac{1}{4}(80)$ or 27) to earn **P9**.

- If the differential dt is missing:
 - Writing $\int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
 - Writing $7 + \int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
 - Writing $\int_0^2 v_J(t) + 7$ introduces an ambiguity for the intended integrand.
 - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}(t^2 - 1)^4 \right]_0^2 + 7$ resolves the ambiguity. Therefore, this earns **P7** and **P8** and is eligible for **P9**.
 - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}(t^2 - 1)^4 + 7t \right]_0^2$ confirms that an incorrect integrand was used. Therefore, this does not earn **P7**, earns **P8**, and is not eligible for **P9**.
 - If the ambiguity is not resolved, this does not earn **P7**, **P8**, or **P9**.
- Alternate solution using u -substitution:

Let $u = t^2 - 1$, then $du = 2t dt$.

$$t = 0 \Rightarrow u = -1$$

$$t = 2 \Rightarrow u = 3$$

$$\begin{aligned} x_J(2) &= x_J(0) + \int_0^2 v_J(t) dt = 7 + \int_0^2 2t(t^2 - 1)^3 dt \\ &= 7 + \int_{-1}^3 u^3 du = 7 + \left[\frac{1}{4}u^4 \right]_{-1}^3 \\ &= 7 + \frac{1}{4}((3)^4 - (-1)^4) = 7 + \frac{1}{4}(80) = 27 \end{aligned}$$

- Alternate solution using indefinite integral:

$$\int 2t(t^2 - 1)^3 dt = \frac{1}{4}(t^2 - 1)^4 + C$$

$$x_J(0) = 7 = \frac{1}{4}(0^2 - 1)^4 + C \Rightarrow C = \frac{27}{4}$$

$$x_J(t) = \frac{1}{4}(t^2 - 1)^4 + \frac{27}{4}$$

$$x_J(2) = \frac{1}{4}(2^2 - 1)^4 + \frac{27}{4} = \frac{108}{4} = 27$$

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Question 4

Train A runs back and forth on an east-west section of railroad track. Train A 's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

- (a) Find the average acceleration of train A over the interval $2 \leq t \leq 8$.
- (b) Do the data in the table support the conclusion that train A 's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.
- (c) At time $t = 2$, train A 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A , in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.
- (d) A second train, train B , travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

(a) average accel = $\frac{v_A(8) - v_A(2)}{8 - 2} = \frac{-120 - 100}{6} = -\frac{110}{3}$ m/min²

1 : average acceleration

(b) v_A is differentiable $\Rightarrow v_A$ is continuous
 $v_A(8) = -120 < -100 < 40 = v_A(5)$

2 : $\begin{cases} 1 : v_A(8) < -100 < v_A(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

Therefore, by the Intermediate Value Theorem, there is a time t , $5 < t < 8$, such that $v_A(t) = -100$.

(c) $s_A(12) = s_A(2) + \int_2^{12} v_A(t) dt = 300 + \int_2^{12} v_A(t) dt$
 $\int_2^{12} v_A(t) dt \approx 3 \cdot \frac{100 + 40}{2} + 3 \cdot \frac{40 - 120}{2} + 4 \cdot \frac{-120 - 150}{2}$
 $= -450$

3 : $\begin{cases} 1 : \text{position expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{position at time } t = 12 \end{cases}$

$s_A(12) \approx 300 - 450 = -150$

The position of Train A at time $t = 12$ minutes is approximately 150 meters west of Origin Station.

- (d) Let x be train A 's position, y train B 's position, and z the distance between train A and train B .

$z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$

$x = 300, y = 400 \Rightarrow z = 500$

$v_B(2) = -20 + 120 + 25 = 125$

$500 \frac{dz}{dt} = (300)(100) + (400)(125)$

$\frac{dz}{dt} = \frac{80000}{500} = 160$ meters per minute

2 : implicit differentiation of distance relationship
 3 : $\begin{cases} 2 : \text{implicit differentiation of distance relationship} \\ 1 : \text{answer} \end{cases}$