

Part B (AB): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Two particles, H and J , are moving along the x -axis. For $0 \leq t \leq 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2-4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t(t^2 - 1)^3$.

	Model Solution	Scoring
A	Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.	
	$x_H'(t) = v_H(t) = (2t - 4)e^{t^2-4t}$	Considers x_H' Point 1 (P1)
	$x_H'(1) = v_H(1) = -2e^{-3}$	Answer Point 2 (P2)

Scoring Notes for Part A

- P1** can be earned by presenting $x_H'(t)$, $x_H'(1)$, $x'(t)$, $x'(1)$, $(2t - 4)e^{t^2-4t}$, or $(2 \cdot 1 - 4)e^{1^2-4 \cdot 1}$.
- An unsupported answer of $-2e^{-3}$ earns **P2** but not **P1**.

- B** During what open intervals of time t , for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

From part A, $x_H'(t) = v_H(t) = (2t - 4)e^{t^2 - 4t}$.

$$x_H'(t) = (2t - 4)e^{t^2 - 4t} = 0 \Rightarrow t = 2$$

$x_H'(t) < 0$ for $0 < t < 2$, and $x_H'(t) > 0$ for $2 < t < 5$.

Thus, particle H is moving to the left for $0 < t < 2$ and moving to the right for $2 < t < 5$.

$$v_J(t) = 2t(t^2 - 1)^3 = 0 \text{ for } 0 < t < 5 \Rightarrow t = 1$$

$v_J(t) < 0$ for $0 < t < 1$, and $v_J(t) > 0$ for $1 < t < 5$.

Thus, particle J is moving to the left for $0 < t < 1$ and moving to the right for $1 < t < 5$.

Therefore, particles H and J are moving in opposite directions for $1 < t < 2$.

Considers sign of $x_H'(t)$ or $v_J(t)$ **Point 3 (P3)**

Analysis for one particle **Point 4 (P4)**

Answer with reason **Point 5 (P5)**

Scoring Notes for Part B

- To earn **P3**, a response can do one of the following:
 - Set $x_H'(t) = 0$, $v_H(t) = 0$, or $(2t - 4)e^{t^2 - 4t} = 0$
 - Set $v_J(t) = 0$ or $2t(t^2 - 1)^3 = 0$
 - Identify $t = 2$ for particle H and no other values in the interval $0 < t < 5$
 - Identify $t = 1$ for particle J and no other values in the interval $0 < t < 5$
 - Identify the interval $1 < t < 2$
- To earn **P4**, a response can provide an analysis of signs of velocity or direction of motion on the interval $0 < t < 5$ for either particle H or particle J .
- To be eligible for **P5**, a response must provide correct analyses of signs of velocity or direction of motion on the interval $0 < t < 5$ for both particles.
- Only analysis within the interval $0 < t < 5$ will be considered in scoring.

- C** It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

$$v_J(2) > 0 \text{ and } v_J'(2) > 0.$$

Answer with reason **Point 6 (P6)**

Because $v_J(2)$ and $v_J'(2)$ have the same sign, the speed of particle J is increasing at $t = 2$.

Scoring Notes for Part C

- An evaluation of $v_J(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J(2) = 108$.
- An evaluation of $v_J'(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J'(2) = 486$.
- A response can either import the analysis for the sign of $v_J(2)$ from part B or restart.
- A response that stated “ $v_J(t) > 0$ for $1 < t < 5$ ” in part B does not need to restate $v_J(2) > 0$ and earns **P6** for “ $v_J(2)$ and $v_J'(2)$ have the same sign, so the speed is increasing.”

- D** Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

$$\begin{aligned} x_J(2) &= x_J(0) + \int_0^2 v_J(t) dt = 7 + \int_0^2 2t(t^2 - 1)^3 dt \\ &= 7 + \left[\frac{1}{4}(t^2 - 1)^4 \right]_0^2 \\ &= 7 + \frac{1}{4}((3)^4 - (-1)^4) = 7 + \frac{1}{4}(80) = 27 \end{aligned}$$

Integrand **Point 7 (P7)**

Antiderivative **Point 8 (P8)**

Answer **Point 9 (P9)**

Scoring Notes for Part D

- To earn **P7**, a response must present an indefinite or definite integral with an integrand of $v_J(t)$ or $2t(t^2 - 1)^3$. (See below for notes on how to handle a missing differential dt .)
- P8** is earned for an antiderivative of the form $k(t^2 - 1)^4$ or equivalent, for $k > 0$. If $k \neq \frac{1}{4}$, then the response is not eligible to earn **P9**.
- A response of $7 + \frac{1}{4}((3)^4 - (-1)^4)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

Note: An ambiguous response, such as $7 + \frac{1}{4}((3)^4 - (-1)^4)$, does not bank **P9** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., $7 + \frac{1}{4}(80)$ or 27) to earn **P9**.

- If the differential dt is missing:
 - Writing $\int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
 - Writing $7 + \int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
 - Writing $\int_0^2 v_J(t) + 7$ introduces an ambiguity for the intended integrand.
 - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}(t^2 - 1)^4 \right]_0^2 + 7$ resolves the ambiguity.
Therefore, this earns **P7** and **P8** and is eligible for **P9**.
 - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}(t^2 - 1)^4 + 7t \right]_0^2$ confirms that an incorrect integrand was used.
Therefore, this does not earn **P7**, earns **P8**, and is not eligible for **P9**.
 - If the ambiguity is not resolved, this does not earn **P7**, **P8**, or **P9**.
- Alternate solution using u -substitution:

Let $u = t^2 - 1$, then $du = 2t \, dt$.

$$t = 0 \Rightarrow u = -1$$

$$t = 2 \Rightarrow u = 3$$

$$\begin{aligned} x_J(2) &= x_J(0) + \int_0^2 v_J(t) \, dt = 7 + \int_0^2 2t(t^2 - 1)^3 \, dt \\ &= 7 + \int_{-1}^3 u^3 \, du = 7 + \left[\frac{1}{4}u^4 \right]_{-1}^3 \\ &= 7 + \frac{1}{4}((3)^4 - (-1)^4) = 7 + \frac{1}{4}(80) = 27 \end{aligned}$$

- Alternate solution using indefinite integral:

$$\int 2t(t^2 - 1)^3 \, dt = \frac{1}{4}(t^2 - 1)^4 + C$$

$$x_J(0) = 7 = \frac{1}{4}(0^2 - 1)^4 + C \Rightarrow C = \frac{27}{4}$$

$$x_J(t) = \frac{1}{4}(t^2 - 1)^4 + \frac{27}{4}$$

$$x_J(2) = \frac{1}{4}(2^2 - 1)^4 + \frac{27}{4} = \frac{108}{4} = 27$$

AP[®] Calculus AB

2014 Scoring Guidelines

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Question 1

Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

- Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.
- Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.
- Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.
- For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$$(a) \frac{A(30) - A(0)}{30 - 0} = -0.197 \text{ (or } -0.196) \text{ lbs/day}$$

1 : answer with units

$$(b) A'(15) = -0.164 \text{ (or } -0.163)$$

The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.

2 : $\begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$

$$(c) A(t) = \frac{1}{30} \int_0^{30} A(t) dt \Rightarrow t = 12.415 \text{ (or } 12.414)$$

2 : $\begin{cases} 1 : \frac{1}{30} \int_0^{30} A(t) dt \\ 1 : \text{answer} \end{cases}$

$$(d) L(t) = A(30) + A'(30) \cdot (t - 30)$$

$$A'(30) = -0.055976$$

$$A(30) = 0.782928$$

$$L(t) = 0.5 \Rightarrow t = 35.054$$

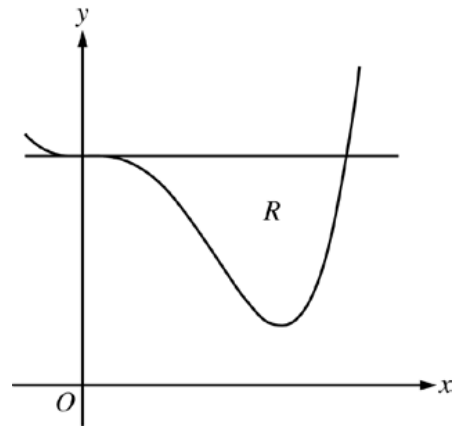
4 : $\begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

2014 SCORING GUIDELINES

Question 2

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

- (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
- (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in R . Find the volume of the solid.
- (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k .



(a) $f(x) = 4 \Rightarrow x = 0, 2.3$

$$\begin{aligned} \text{Volume} &= \pi \int_0^{2.3} \left[(4 + 2)^2 - (f(x) + 2)^2 \right] dx \\ &= 98.868 \text{ (or } 98.867) \end{aligned}$$

$$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$$

(b) $\text{Volume} = \int_0^{2.3} \frac{1}{2} (4 - f(x))^2 dx$
 $= 3.574 \text{ (or } 3.573)$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(c) $\int_0^k (4 - f(x)) dx = \int_k^{2.3} (4 - f(x)) dx$

$$2 : \begin{cases} 1 : \text{area of one region} \\ 1 : \text{equation} \end{cases}$$

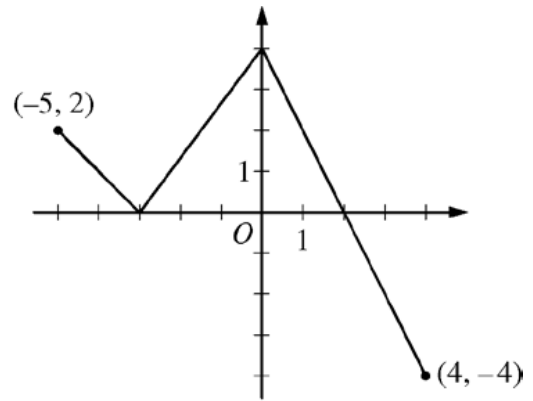
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Question 3

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above.

Let g be the function defined by $g(x) = \int_{-3}^x f(t) dt$.

- (a) Find $g(3)$.
- (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.
- (c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.
- (d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

Graph of f

(a) $g(3) = \int_{-3}^3 f(t) dt = 6 + 4 - 1 = 9$

1 : answer

(b) $g'(x) = f(x)$

The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $h'(x) = \frac{5xg'(x) - g(x)5}{(5x)^2} = \frac{5xg'(x) - 5g(x)}{25x^2}$

3 : $\begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

$$\begin{aligned} h'(3) &= \frac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2} \\ &= \frac{15(-2) - 5(9)}{225} = \frac{-75}{225} = -\frac{1}{3} \end{aligned}$$

(d) $p'(x) = f'(x^2 - x)(2x - 1)$

3 : $\begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$

$$p'(-1) = f'(2)(-3) = (-2)(-3) = 6$$

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Question 4

Train A runs back and forth on an east-west section of railroad track. Train A 's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

- (a) Find the average acceleration of train A over the interval $2 \leq t \leq 8$.
- (b) Do the data in the table support the conclusion that train A 's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.
- (c) At time $t = 2$, train A 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A , in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.
- (d) A second train, train B , travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

(a) average accel = $\frac{v_A(8) - v_A(2)}{8 - 2} = \frac{-120 - 100}{6} = -\frac{110}{3} \text{ m/min}^2$

(b) v_A is differentiable $\Rightarrow v_A$ is continuous
 $v_A(8) = -120 < -100 < 40 = v_A(5)$

Therefore, by the Intermediate Value Theorem, there is a time t , $5 < t < 8$, such that $v_A(t) = -100$.

(c) $s_A(12) = s_A(2) + \int_2^{12} v_A(t) dt = 300 + \int_2^{12} v_A(t) dt$
 $\int_2^{12} v_A(t) dt \approx 3 \cdot \frac{100 + 40}{2} + 3 \cdot \frac{40 - 120}{2} + 4 \cdot \frac{-120 - 150}{2}$
 $= -450$

$$s_A(12) \approx 300 - 450 = -150$$

The position of Train A at time $t = 12$ minutes is approximately 150 meters west of Origin Station.

- (d) Let x be train A 's position, y train B 's position, and z the distance between train A and train B .

$$z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$x = 300, y = 400 \Rightarrow z = 500$$

$$v_B(2) = -20 + 120 + 25 = 125$$

$$500 \frac{dz}{dt} = (300)(100) + (400)(125)$$

$$\frac{dz}{dt} = \frac{80000}{500} = 160 \text{ meters per minute}$$

1 : average acceleration

2 : $\begin{cases} 1 : v_A(8) < -100 < v_A(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

3 : $\begin{cases} 1 : \text{position expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{position at time } t = 12 \end{cases}$

3 : $\begin{cases} 2 : \text{implicit differentiation of} \\ \text{distance relationship} \\ 1 : \text{answer} \end{cases}$

2014 SCORING GUIDELINES

Question 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.

- (a) Find the x -coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.
- (b) Explain why there must be a value c , for $-1 < c < 1$, such that $f''(c) = 0$.
- (c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.
- (d) Evaluate $\int_{-2}^3 f'(g(x))g'(x) dx$.

- (a) $x = 1$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a relative minimum at $x = 1$.

- (b) f' is differentiable $\Rightarrow f'$ is continuous on the interval $-1 \leq x \leq 1$

$$\frac{f'(1) - f'(-1)}{1 - (-1)} = \frac{0 - 0}{2} = 0$$

Therefore, by the Mean Value Theorem, there is at least one value c , $-1 < c < 1$, such that $f''(c) = 0$.

- (c) $h'(x) = \frac{1}{f(x)} \cdot f'(x)$

$$h'(3) = \frac{1}{f(3)} \cdot f'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}$$

- (d) $\int_{-2}^3 f'(g(x))g'(x) dx = \left[f(g(x)) \right]_{x=-2}^{x=3}$
 $= f(g(3)) - f(g(-2))$
 $= f(1) - f(-1)$
 $= 2 - 8 = -6$

1 : answer with justification

$$2 : \begin{cases} 1 : f'(1) - f'(-1) = 0 \\ 1 : \text{explanation, using Mean Value Theorem} \end{cases}$$

$$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 2 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$$

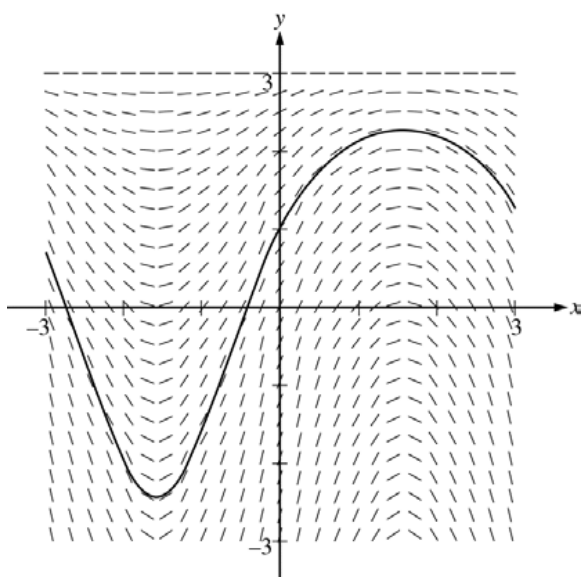
2014 SCORING GUIDELINES

Question 6

Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

- (a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.
- (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.
- (c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

(a) 1 : solution curve



(b) $\left. \frac{dy}{dx} \right|_{(x,y)=(0,1)} = 2 \cos 0 = 2$

An equation for the tangent line is $y = 2x + 1$.

$$f(0.2) \approx 2(0.2) + 1 = 1.4$$

2 : $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{dy}{dx} = (3 - y)\cos x$

$$\int \frac{dy}{3 - y} = \int \cos x \, dx$$

$$-\ln|3 - y| = \sin x + C$$

$$-\ln 2 = \sin 0 + C \Rightarrow C = -\ln 2$$

$$-\ln|3 - y| = \sin x - \ln 2$$

Because $y(0) = 1$, $y < 3$, so $|3 - y| = 3 - y$

$$3 - y = 2e^{-\sin x}$$

$$y = 3 - 2e^{-\sin x}$$

6 : $\begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables