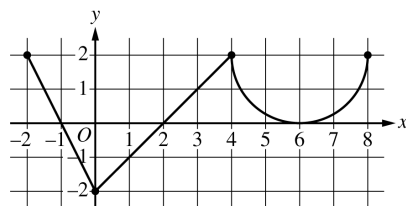


Part B (AB or BC): Graphing calculator not allowed**Question 4****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Graph of f'

The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f' , the derivative of f , consists of two line segments and a semicircle, as shown in the figure.

Model Solution**Scoring**

- (a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

$$f'(x) > 0 \text{ on } (2, 6) \text{ and } f'(x) > 0 \text{ on } (6, 8).$$

$f'(x)$ does not change sign at $x = 6$, so there is neither a relative maximum nor a relative minimum at this location.

Answer with reason

1 point**Scoring notes:**

- A response that declares $f'(x)$ does not change sign at $x = 6$, so neither, is sufficient to earn the point.
- A response does not have to present intervals on which $f'(x)$ is positive or negative, but if any are given, they must be correct. Any presented intervals may include none, one, or both endpoints.
- A response that declares $f'(x) > 0$ before and after $x = 6$ does not earn the point.

Total for part (a)**1 point**

- (b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

The graph of f is concave down on $(-2, 0)$ and $(4, 6)$ because f' is decreasing on these intervals.

Intervals

1 point

Reason

1 point**Scoring notes:**

- The first point is earned only by an answer of $(-2, 0)$ and $(4, 6)$, or these intervals including one or both endpoints.
- A response must earn the first point to be eligible for second point.
- To earn the second point a response must correctly discuss the behavior of f' or the slopes of f' .
- Special case: A response that presents exactly one of the two correct intervals with a correct reason earns 1 out of 2 points.

Total for part (b)**2 points**

- (c) Find the value of $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

Because f is differentiable at $x = 2$, f is continuous at $x = 2$, so $\lim_{x \rightarrow 2} f(x) = f(2) = 1$.

$$\lim_{x \rightarrow 2} (6f(x) - 3x) = 6 \cdot 1 - 3 \cdot 2 = 0$$

$$\lim_{x \rightarrow 2} (x^2 - 5x + 6) = 0$$

Limits of numerator and denominator

1 point

Because $\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$ is of indeterminate form $\frac{0}{0}$,

L'Hospital's Rule can be applied.

Uses L'Hospital's Rule

1 point

Answer

1 point

Using L'Hospital's Rule,

$$\lim_{x \rightarrow 2} \frac{6f(x) - 3x}{x^2 - 5x + 6} = \lim_{x \rightarrow 2} \frac{6f'(x) - 3}{2x - 5} = \frac{6 \cdot 0 - 3}{2 \cdot 2 - 5} = 3.$$

Scoring notes:

- The first point is earned by the presentation of two separate limits for the numerator and denominator.
- A response that presents a limit explicitly equal to $\frac{0}{0}$ does not earn the first point.
- The second point is earned by applying L'Hospital's Rule, that is, by presenting at least one correct derivative in the limit of a ratio of derivatives.
- The third point is earned for the correct answer with supporting work.

Total for part (c)**3 points**

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

$f'(x) = 0 \Rightarrow x = -1, x = 2, x = 6$	Considers $f'(x) = 0$	1 point
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The function f is continuous on $[-2, 8]$, so the candidates for the location of an absolute minimum for f are $x = -2$, $x = -1$, $x = 2$, $x = 6$, and $x = 8$.	Justification	1 point
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	Answer	1 point
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x	$f(x)$
-2	3
-1	4
2	1
6	$7 - \pi$
8	$11 - 2\pi$

The absolute minimum value of f is $f(2) = 1$.

Scoring notes:

- To earn the first point a response must state $f' = 0$ or equivalent. Listing the zeros of f' is not sufficient.
- A response that presents any error in evaluating f at any critical point or endpoint will not earn the justification point.
- A response need not present the value of $f(-1)$ provided $x = -1$ is eliminated because it is the location of a local maximum. A response need not present the value of $f(6)$ provided $x = 6$ is eliminated by reference to part (a) or eliminated through analysis.
- A response need not present the value of $f(8)$ provided there is a presentation that argues $f'(x) \geq 0$ for $x > 2$ and, therefore, $f(8) > f(2)$.
- A response that does not consider both endpoints does not earn the justification point.
- The answer point is earned only for indicating that the minimum value is 1. It is not earned for noting that the minimum occurs at $x = 2$.

Total for part (d) 3 points

Total for question 4 9 points