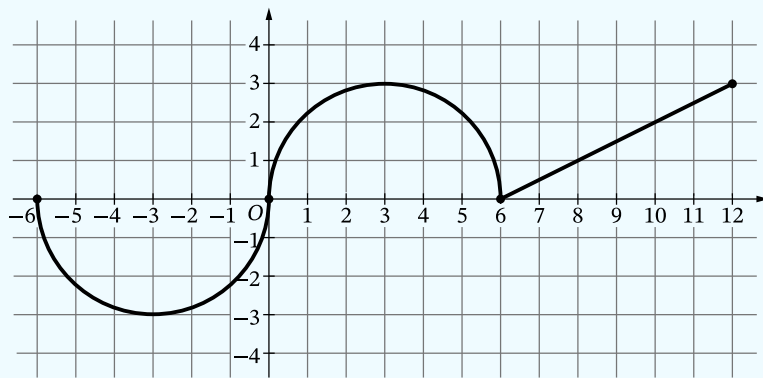


Part A (AB or BC): Graphing calculator not allowed**Question 4****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f , consisting of two semicircles and one line segment, is shown in the figure.

Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t) dt$.

Model Solution**Scoring**

- A** Find $g'(8)$. Give a reason for your answer.

$$g'(x) = f(x)$$

Considers
 $g'(x) = f(x)$

Point 1 (P1)

$$g'(8) = f(8) = 1$$

Answer

Point 2 (P2)**Scoring Notes for Part A**

- P1** is earned for $g' = f$, $g'(x) = f(x)$, or $g'(8) = f(8)$ in part A.
- A response of $g'(8) = f(8) = 1$ earns both **P1** and **P2**.
- A response that does not earn **P1** can earn **P2** with an implied application of the Fundamental Theorem of Calculus (e.g., $g'(8) = 1$ or $f(8) = 1$).
- A response of $g'(8) = f(8) - f(6) = 1$ earns **P2** but not **P1**.

- B** Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

The graph of g has a point of inflection where $g'' = f'$ changes sign, which is where $g' = f$ changes from decreasing to increasing or vice versa.

Answer

Point 3 (P3)

Reason

Point 4 (P4)

The graph of g has points of inflection at $x = -3$ and $x = 6$ because f changes from decreasing to increasing there.

The graph of g also has a point of inflection at $x = 3$ because f changes from increasing to decreasing there.

Scoring Notes for Part B

- P3** is earned only for an answer of $x = -3$, $x = 3$, and $x = 6$. If any other/additional values of x in $-6 < x < 12$ are declared to be points of inflection, the response does not earn either **P3** or **P4**. Consideration of $x = -6$ or of $x = 12$ does not impact scoring.
- To earn **P4**, a response must tie the reason to the given graph of f .
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f changes from increasing to decreasing or decreasing to increasing there” earns both **P3** and **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because the slope of f changes sign there” earns both **P3** and **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f attains relative extrema there” earns both **P3** and **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because g changes concavity there” earns **P3** but not **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $g'' = f'$ changes sign there” earns **P3** but not **P4**.
 - A response that relies upon an ambiguous term such as “the function” or “the graph” does not earn **P4**.
- Special case:** A response with two of the three correct x -values with correct reasoning and no other/additional values of x declared to be points of inflection earns **P4** but not **P3**.

C Find $g(12)$ and $g(0)$. Label your answers.

$$g(12) = \int_6^{12} f(t) \, dt = \frac{1}{2} \cdot 6 \cdot 3 = 9$$

$g(12)$

Point 5 (P5)

$$g(0) = \int_6^0 f(x) \, dx = -\int_0^6 f(x) \, dx = -\frac{\pi}{2} 3^2 = -\frac{9\pi}{2}$$

$g(0)$

Point 6 (P6)

Scoring Notes for Part C

- Unlabeled values do not earn either **P5** or **P6**.
- P5** is earned for a response of $g(12) = 9$, with or without supporting work.
- P6** is earned for a response of $g(0) = -\frac{9\pi}{2}$, with or without supporting work.

Note: Incorrect communication between the label “ $g(0)$ ” and the answer will be treated as scratch work and will not impact scoring. For example, $g(0) = \int_0^6 f(x) \, dx = -\frac{9\pi}{2}$ earns **P6**.

D Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

For $-6 \leq x \leq 12$, g attains a minimum either when $g'(x) = f(x) = 0$ or at an endpoint.

Considers $g'(x) = 0$ **Point 7 (P7)**

$$g'(x) = f(x) = 0$$

$$\Rightarrow x = 0, x = 6$$

Justification **Point 8 (P8)**

x	$g(x)$
-6	0
0	$-\frac{9\pi}{2}$
6	0
12	9

Therefore, on the closed interval $-6 \leq x \leq 12$, g attains an absolute minimum value at $x = 0$.

Answer **Point 9 (P9)**

Scoring Notes for Part D

- P7** is earned for considering $g'(x) = 0$ or $f(x) = 0$. **P7** is not earned by just presenting $x = 0$ and $x = 6$.
A response that discusses the sign of $g'(x)$ or $f(x)$ changing OR uses the phrase “critical points of g ” also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by providing evaluations or reasoning for each of $g(-6)$, $g(0)$, $g(6)$, and $g(12)$ (and no other x -values).
- Alternate justification and answer:
Because $g'(x) \leq 0$ (or $f(x) \leq 0$) for $-6 \leq x < 0$ and $g'(x) \geq 0$ (or $f(x) \geq 0$) for $0 < x \leq 12$, the absolute minimum of g occurs at $x = 0$.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer of $x = 0$.
- For **P8**, values of $g(0)$ and $g(12)$ can be imported from part C. A response can earn **P9** with an answer that is consistent with the imported values.