

Part B (AB): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x .

Model Solution	Scoring
(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer. $h'(x) = f'(g(x)) \cdot g'(x)$ $h'(7) = f'(g(7)) \cdot g'(7)$ $= f'(0) \cdot 8 = \frac{3}{2} \cdot 8 = 12$	Chain rule 1 point Answer 1 point
Scoring notes: - The first point is earned for either $h'(x) = f'(g(x)) \cdot g'(x)$ or $h'(7) = f'(g(7)) \cdot g'(7)$. - If the first point is earned, the second point is earned only for an answer of 12 (or equivalent). - If the first point is not earned, the second point can be earned only for a response of either $f'(0) \cdot 8 = 12$ or $\frac{3}{2} \cdot 8$. - A response of 12 with no supporting work does not earn either point.	
Total for part (a) 2 points	

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer. $k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$	Product or chain rule 1 point
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$k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$ $= 2 \cdot 4 \cdot 3 \cdot (-3) + 4^2 \cdot 2 = -72 + 32 = -40$	$k''(4)$ 1 point
The graph of k is concave down at the point where $x = 4$ because $k''(4) < 0$ and k'' is continuous.	Answer with reason 1 point

Scoring notes:

- The first point is earned for either $k''(x) = 2f(x) \cdot f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $k''(4) = 2f(4) \cdot f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$.
- The first point is also earned by any of the following incorrect expressions, each of which has a single error in the application of the product rule or the chain rule:
 - $2f(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
 - $2f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $2f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
 - $f'(x) \cdot g(x) + (f(x))^2 \cdot g'(x)$ or $f'(4) \cdot g(4) + (f(4))^2 \cdot g'(4)$
 - $2f(x) \cdot f'(x) \cdot g'(x)$ or $2f(4) \cdot f'(4) \cdot g'(4)$
 - Note: A response that presents one of these expressions cannot earn the second point.
- To earn the second point a response must correctly find $k''(4) = -40$ (or equivalent) with supporting work.
- The third point is earned for an answer and reason that are consistent with any declared nonzero value of $k''(4)$.

Total for part (b) 3 points

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t) dt$. Find $m(2)$. Show the work that leads to your answer. $m(2) = 5 \cdot 8 + \int_0^2 f'(t) dt = 40 + (f(2) - f(0))$ $= 40 + (7 - 10) = 37$	Answer with supporting work 1 point
Scoring notes: - The point is earned only for an answer of 37 (or equivalent) with supporting work equivalent to $5 \cdot 8 + (f(2) - f(0))$, $40 + (f(2) - f(0))$, $5 \cdot 8 + (7 - 10)$, or $40 + (7 - 10)$. - An answer of 37 with no supporting work does not earn the point.	
Total for part (c) 1 point	

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer. $m'(x) = 15x^2 + f'(x)$ $m'(2) = 15 \cdot 4 + f'(2) = 60 + (-8) = 52$	Considers $m'(x)$ 1 point $m'(2)$ with supporting work 1 point
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The graph of m is increasing at $x = 2$ because $m'(2) > 0$.

Answer with
justification

1 point

Scoring notes:

- The first point is earned for considering $m'(x)$, $m'(2)$, or m' . This consideration may appear in a justification statement.
- The second point is earned for $m'(2) = 15 \cdot 2^2 + f'(2)$, $m'(2) = 60 + f'(2)$, or $m'(2) = 60 - 8$ but is not earned for an unsupported response of $m'(2) = 52$.
- The third point is earned for an answer and justification consistent with any declared value of $m'(2)$.

Total for part (d) 3 points

Total for question 5 9 points