

**Part A (AB or BC): Graphing calculator required****Question 1****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

An invasive species of plant appears in a fruit grove at time  $t = 0$  and begins to spread. The function  $C$  defined by  $C(t) = 7.6 \arctan(0.2t)$  models the number of acres in the fruit grove affected by the species  $t$  weeks after the species appears. It can be shown that  $C'(t) = \frac{38}{25 + t^2}$ .

(Note: Your calculator should be in radian mode.)

|          | Model Solution                                                                                                                                   | Scoring                                   |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------|
| <b>A</b> | Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations. |                                           |
|          | $\frac{1}{4 - 0} \int_0^4 C(t) dt$                                                                                                               | Average value formula <b>Point 1 (P1)</b> |
|          | $= \frac{1}{4}(11.112896) = 2.778224$                                                                                                            | Answer <b>Point 2 (P2)</b>                |
|          | From time $t = 0$ to $t = 4$ weeks, the average number of acres affected by the invasive species was 2.778 acres.                                |                                           |

**Scoring Notes for Part A**

- P1** is earned for the correct integral, with or without the differential, along with evidence of division by 4. In the presence of the correct integral, the correct answer will suffice as evidence of division by 4. These may appear all in one step, as in the model solution, or in multiple steps.
- P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect or unclear communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring. For example:
  - $\int_0^4 C(t) dt = 11.112896$  so the average velocity is 2.778224.  
Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer.
  - $\int_0^4 C(t) dt = \frac{11.112896}{4} = 2.778224$   
Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
  - $\int_0^4 C(t) dt = 2.778224$   
Note: This response earns **P1** for the correct integral with the correct answer as evidence of division by 4. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- Note that the values  $\frac{1}{4}(11.112)$  and  $\frac{1}{4}(11.113)$  are accurate to three digits after the decimal and therefore earn **P2**.

- B** Find the time  $t$  when the instantaneous rate of change of  $C$  equals the average rate of change of  $C$  over the time interval  $0 \leq t \leq 4$ . Show the setup for your calculations.

$$\frac{C(4) - C(0)}{4 - 0} = 1.282008$$

Uses average rate of change **Point 3 (P3)**

$$C'(t) = \frac{38}{25 + t^2} = 1.282008 \Rightarrow t = 2.154298$$

Answer with supporting work **Point 4 (P4)**

The instantaneous rate of change of  $C$  equals the average rate of change of  $C$  over the interval  $0 \leq t \leq 4$  at time  $t = 2.154$ .

#### Scoring Notes for Part B

- P3** may be earned by presenting the expression or value for the average rate of change. Note that because  $C(0) = 0$  and the interval is  $0 \leq t \leq 4$ , any of the following will earn **P3**:  $\frac{\int_0^4 C'(t) dt}{4}$ ,  $\frac{C(4) - C(0)}{4 - 0}$ ,  $\frac{C(4)}{4}$ ,  $\frac{5.128 - 0}{4 - 0}$ ,  $\frac{5.128}{4}$ , or 1.282. However, neither **P3** nor **P4** is earned by just presenting  $t = 1.282$ .
  - P4** is earned for the correct answer supported by the appropriate equation. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- The following response, for example, earns both **P3** and **P4**:  $C'(t) = \frac{C(4) - C(0)}{4}$  when  $t = 2.154$ .

- C** Assume that the invasive species continues to spread according to the given model for all times  $t > 0$ . Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

$$\lim_{t \rightarrow \infty} C'(t) = \lim_{t \rightarrow \infty} \frac{38}{25 + t^2}$$

Limit expression **Point 5 (P5)**

$$= 0$$

Value **Point 6 (P6)**

#### Scoring Notes for Part C

- P5** can be earned for either  $\lim_{t \rightarrow \infty} C'(t)$  or  $\lim_{t \rightarrow \infty} C(t)$ .
- A response that includes  $\lim_{t \rightarrow \infty} C(t)$  is not eligible to earn **P6**.
- For **P6**, arithmetic with infinity, e.g.,  $\frac{38}{25 + \infty^2} = 0$ , will be considered as scratch work and will not be considered in scoring.

- D** At time  $t = 4$  weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function  $A$ , defined by  $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x) dx$ , models the number of acres affected by the species over the time interval  $4 \leq t \leq 36$ . At what time  $t$ , for  $4 \leq t \leq 36$ , does  $A$  attain its maximum value? Justify your answer.

$$A'(t) = C'(t) - 0.1 \cdot \ln t$$

Considers  $A'(t) = 0$  **Point 7 (P7)**

For  $4 \leq t \leq 36$ , the maximum value of  $A(t)$  occurs when  $A'(t) = 0$  or at an endpoint.

$$A'(t) = C'(t) - 0.1 \cdot \ln t = 0 \Rightarrow C'(t) = 0.1 \cdot \ln t$$

$$\Rightarrow t = 11.441700$$

Justification **Point 8 (P8)**

| $t$       | $A(t)$   |
|-----------|----------|
| 4         | 5.128031 |
| 11.441700 | 7.316978 |
| 36        | 1.743056 |

Therefore, the number of acres affected by the species is a maximum at time  $t = 11.442$  (or 11.441) weeks.

Answer with supporting work **Point 9 (P9)**

#### Scoring Notes for Part D

- P7** is earned for considering  $A'(t) = 0$ ,  $C'(t) - 0.1 \cdot \ln t = 0$ , or  $C'(t) = 0.1 \cdot \ln t$ . **P7** is not earned by just presenting  $t = 11.441700$ . A response that discusses the sign of  $A'(t)$  changing or uses the phrase “critical points of  $A$ ” also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by correctly evaluating  $A(t)$  at  $t = 4$ ,  $t = 11.441700$ , and  $t = 36$ . The evaluations must be correct to the first digit after the decimal, rounded or truncated.
- Alternate justifications:
  - $A'(t) > 0$  for  $4 < t < 11.442$ , and  $A'(t) < 0$  for  $11.442 < t < 36$ . Therefore,  $t = 11.442$  is the location of the absolute maximum for  $A$  on the interval  $4 \leq t \leq 36$ .
  - Because  $A'(t)$  changes sign from positive to negative at  $t = 11.442$  (this might be presented as “ $A'(t) > 0$  for  $t < 11.442$ , and  $A'(t) < 0$  for  $t > 11.442$ ”), it is the location of a relative maximum for  $A$ . And because  $t = 11.442$  is the only critical point of  $A$  in the interval  $4 \leq t \leq 36$ , it is the location of the absolute maximum for  $A$  on the interval.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.