

Part B (AB): Graphing calculator not allowed**Question 6****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

	Model Solution	Scoring
A	Show that $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$.	
	$\frac{d}{dx}\left(y^3 - y^2 - y + \frac{1}{4}x^2\right) = \frac{d}{dx}(0)$ $\Rightarrow 3y^2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} - \frac{dy}{dx} + \frac{x}{2} = 0$	Implicit differentiation Point 1 (P1)
	$\Rightarrow (3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}$ $\Rightarrow \frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$	Verification Point 2 (P2)
Scoring Notes for Part A		

- P1** is earned only for the correct implicit differentiation of $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y' .
- To be eligible for **P2**, a response must have earned **P1**.
- It is sufficient to present $(3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}$ to earn **P2**, provided there are no subsequent errors.

- B** There is a point P on the curve G near $(2, -1)$ with x -coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y -coordinate of point P .

$$\left. \frac{dy}{dx} \right|_{(x, y)=(2, -1)} = \frac{-2}{2(3 + 2 - 1)} = -\frac{1}{4}$$

Slope of tangent line **Point 3 (P3)**

$$y \approx -1 - \frac{1}{4}(1.6 - 2) = -0.9$$

Tangent line approximation **Point 4 (P4)**

Scoring Notes for Part B

- A response can earn **P3** with $\left. \frac{dy}{dx} \right|_{(x, y)=(2, -1)} = -\frac{1}{4}$, $\frac{dy}{dx} = -\frac{1}{4}$, “slope is $-\frac{1}{4}$,” or equivalent.
- A response that presents a linear approximation with a slope of $-\frac{1}{4}$ also earns **P3**.
- A response that declares $\left. \frac{dy}{dx} \right|_{(x, y)=(2, -1)}$ (or the slope) equal to any nonzero value $k \neq -\frac{1}{4}$ does not earn **P3**. Such a response earns **P4** for a presented approximation mathematically equivalent to $-1 + k(-0.4)$.
- P4** cannot be earned with a linear approximation using a slope other than $-\frac{1}{4}$ if that slope has not been declared to be the value of $\left. \frac{dy}{dx} \right|_{(x, y)=(2, -1)}$.
- A response does not have to present the tangent line equation but must clearly demonstrate its use at $x = 1.6$ in finding the requested approximation to be eligible for **P4**.
- A response of $-1 - \frac{1}{4}(-0.4)$ earns both **P3** and **P4**.
- A response of $-1 - \frac{1}{4}(1.6 - 2)$ or equivalent earns **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4**).

Note: An ambiguous response, such as $-1 - \frac{1}{4}(1.6 - 2)$, does not earn **P4** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., -0.9) to earn **P4**.

- C** For $x > 0$ and $y > 0$, there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y -coordinate of point S . Show the work that leads to your answer.

For $x > 0$, the curve G has a vertical tangent line when
 $2(3y^2 - 2y - 1) = 0$.

Sets denominator equal to 0 **Point 5 (P5)**

$$2(3y^2 - 2y - 1) = 0 \Rightarrow 2(3y + 1)(y - 1) = 0$$

Because $y > 0$, it follows that $y = 1$.

Answer **Point 6 (P6)**

The line tangent to the curve is vertical at the point on the curve where $y = 1$.

Scoring Notes for Part C

- **P5** is earned with any of $2(3y^2 - 2y - 1) = 0$, $3y^2 - 2y - 1 = 0$, $2(3y \pm 1)(y \pm 1) = 0$, or $(3y \pm 1)(y \pm 1) = 0$.
- To be eligible for **P6**, a response must have earned **P5**.
- A response does not need to consider the numerator of $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$ to earn **P5** or **P6**; considering the denominator is sufficient.
- A response that states solutions of $y = -\frac{1}{3}$ and $y = 1$, but does not clearly identify that $y = 1$ is the only solution to the prompt, does not earn **P6**.
- In the presence of algebraic work to find the value of y , **P6** is earned only if the algebraic work is correct.
- A response of “ $2(3y^2 - 2y - 1) = 0$, $y = 1$ ” earns both **P5** and **P6**.

- D** A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

$$\frac{d}{dt}(2xy + \ln y) = \frac{d}{dt}(8)$$

$$2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$$

Attempts implicit differentiation with respect to t **Point 7 (P7)**

$$2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$$
 Point 8 (P8)

$$2(3)(1) + 2(4)\frac{dy}{dt} + \frac{1}{1}\frac{dy}{dt} = 0$$

$$\Rightarrow 6 + 9\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{2}{3}$$

Answer **Point 9 (P9)**

Scoring Notes for Part D

- P7** is earned for implicitly differentiating $2xy + \ln y = 8$ with respect to t with at most one error.
- P8** is earned for an equation equivalent to $2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$.
- To be eligible for **P9**, a response must have earned **P7** and **P8**, with no errors in implicit differentiation.
- P9** is earned only for the value of $-\frac{2}{3}$.
- Alternate solution:

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\frac{d}{dx}(2xy + \ln y) = \frac{d}{dx}(8) \Rightarrow 2y + 2x\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-2y}{2x + \frac{1}{y}}$$

$$\left. \frac{dy}{dx} \right|_{(x,y)=(4,1)} = \frac{-2(1)}{2(4) + \frac{1}{1}} = -\frac{2}{9}$$

$$\left. \frac{dy}{dt} \right|_{(x,y)=(4,1)} = \left. \frac{dy}{dx} \cdot \frac{dx}{dt} \right|_{(x,y)=(4,1)} = -\frac{2}{9} \cdot 3 = -\frac{2}{3}$$

- P7** is earned for an implicit differentiation with respect to x with at most one error, as long as $\frac{dy}{dx}$ is eventually correctly linked to $\frac{dx}{dt}$.
- P8** is earned for finding $\frac{dy}{dx}$ and multiplying the result by $\frac{dx}{dt} = 3$.
- P8** can be earned for a stated incorrect $\frac{dy}{dx}$, as long as it is multiplied by 3.
- P9** is only earned for a correct value of $-\frac{2}{3}$ or equivalent.
- Stating $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ alone does not earn any points.