

Part B (AB): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve defined by the equation $x^2 + 3y + 2y^2 = 48$. It can be shown that $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$.

Model Solution	Scoring
(a) There is a point on the curve near $(2, 4)$ with x -coordinate 3. Use the line tangent to the curve at $(2, 4)$ to approximate the y -coordinate of this point.	
$\frac{dy}{dx}\bigg _{(x,y)=(2,4)} = \frac{-2(2)}{3 + 4(4)} = -\frac{4}{19}$	Slope of tangent line 1 point
$y \approx 4 - \frac{4}{19}(3 - 2) = \frac{72}{19}$	Approximation 1 point

Scoring notes:

- A response earns the first point for finding $\frac{dy}{dx}\bigg|_{(x,y)=(2,4)} = -\frac{4}{19}$, even if this is not labeled or used as the slope of a tangent line.
- A response that does not explicitly find the value of $\frac{dy}{dx}\bigg|_{(x,y)=(2,4)}$ but uses a slope of $-\frac{4}{19}$ in a linear approximation also earns the first point.
- A response that declares $\frac{dy}{dx}\bigg|_{(x,y)=(2,4)}$ equal to any nonzero value other than $-\frac{4}{19}$ does not earn the first point but is eligible for the second point for a linear approximation at $x = 3$ through the point $(2, 4)$ with a slope of the declared value.
 - The second point cannot be earned with a linear approximation using a slope other than $-\frac{4}{19}$ if that slope has not been declared to be the value of $\frac{dy}{dx}\bigg|_{(x,y)=(2,4)}$.
- The second point cannot be earned for an approximation at any value of x other than 3.
- A response does not have to write the tangent line equation but must clearly demonstrate its use at $x = 3$ in finding the requested approximation in order to earn both points.
- The minimal work required to earn both points is $4 - \frac{4}{19}(3 - 2)$.

Total for part (a) 2 points

- (b) Is the horizontal line $y = 1$ tangent to the curve? Give a reason for your answer.

$\frac{dy}{dx} = \frac{-2x}{3+4y} = 0 \Rightarrow x = 0$	Considers $\frac{dy}{dx} = 0$	1 point
And so, if the horizontal line $y = 1$ is tangent to the curve, the point of tangency must be $(0, 1)$. However, the point $(0, 1)$ is not on the curve, because $0^2 + 3 \cdot 1 + 2 \cdot 1^2 = 5 \neq 48$. Therefore, the horizontal line $y = 1$ is not tangent to the curve.	Answer with reason	1 point

Scoring notes:

- The first point can be earned with a response of $\frac{dy}{dx} = 0$, $-2x = 0$, or $x = 0$, OR by identifying the point $(0, 1)$.
- To earn the second point a response must provide a reason that the line $y = 1$ is not tangent to the curve. Merely stating “ $(0, 1)$ does not lie on the curve” is not sufficient to earn the second point.
- Alternate solution:

If $y = 1$, then $x^2 + 3 \cdot 1 + 2 \cdot 1^2 = 48 \Rightarrow x = \pm\sqrt{43}$.

$$\left. \frac{dy}{dx} \right|_{(x,y)=(\pm\sqrt{43},1)} = \frac{\pm 2\sqrt{43}}{7} \neq 0$$

Therefore, the horizontal line $y = 1$ is not tangent to the curve.

- A response that uses this method earns the first point by using $y = 1$ in $x^2 + 3y + 2y^2 = 48$.
- A response that fails to consider both $x = +\sqrt{43}$ and $x = -\sqrt{43}$ does not earn the second point.

Total for part (b) 2 points

- (c) The curve intersects the positive x -axis at the point $(\sqrt{48}, 0)$. Is the line tangent to the curve at this point vertical? Give a reason for your answer.

At the point $(\sqrt{48}, 0)$, the slope of the line tangent to the curve is $\frac{dy}{dx} = \frac{-2\sqrt{48}}{3 + 4(0)}$.

The denominator of $\frac{dy}{dx}$ is $3 + 4(0)$, which does not equal 0.

Therefore, the line tangent to the curve at this point is not vertical.

Answer with reason

1 point

Scoring notes:

- A response does not need to consider the numerator of $\frac{dy}{dx}\bigg|_{(x,y)=(\sqrt{48},0)}$ in order to earn this point; considering the denominator is sufficient.
- To earn this point a response must clearly demonstrate that the slope of the tangent line at the point $(\sqrt{48}, 0)$ is defined and answer “no.”
 - Such demonstrations include, but are not limited to, the following:
 - $3 + 4(0) \neq 0$
 - At $(\sqrt{48}, 0)$, $\frac{dy}{dx} = \frac{-2\sqrt{48}}{3}$.
 - $\frac{-2\sqrt{48}}{3}$
 - When $3 + 4y = 0$, $y \neq 0$.

Total for part (c)

1 point

- (d) For time $t \geq 0$, a particle is moving along another curve defined by the equation $y^3 + 2xy = 24$. At the instant the particle is at the point $(4, 2)$, the y -coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the x -coordinate of the particle's position with respect to time?

$3y^2 \frac{dy}{dt} + 2x \frac{dy}{dt} + 2y \frac{dx}{dt} = 0$	Attempts implicit differentiation	1 point
	$3y^2 \frac{dy}{dt} + 2x \frac{dy}{dt} + 2y \frac{dx}{dt} = 0$	1 point
$\frac{dy}{dt} = -2$ $3(2)^2(-2) + 2(4)(-2) + 2(2) \frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = \frac{40}{4} = 10$ The rate of change with respect to time in the x -coordinate is 10 units per second.	Uses $\frac{dy}{dt} = \pm 2$	1 point
	Answer	1 point

Scoring notes:

- The first point is earned for implicitly differentiating $y^3 + 2xy = 24$ with respect to t with at most one error.
 - The first point can also be earned by correctly differentiating with respect to x :

$$3y^2 \frac{dy}{dx} + 2x \frac{dy}{dx} + 2y = 0.$$
- The second point is earned for an equation equivalent to $3y^2 \frac{dy}{dt} + 2x \frac{dy}{dt} + 2y \frac{dx}{dt} = 0$.
- A response does not need to explicitly declare $\frac{dy}{dt} = -2$ or $\frac{dy}{dt} = 2$ in order to earn the third point; this point may be earned by correctly substituting $\frac{dy}{dt} = -2$ or $\frac{dy}{dt} = 2$ in the implicitly differentiated equation. However, a response that uses both $\frac{dy}{dt} = -2$ and $\frac{dy}{dt} = 2$ in the implicitly differentiated equation does not earn the third point.
- The fourth point cannot be earned without the first 3 points. The fourth point is earned only for the value of 10 with no mistakes in supporting work.
 - Note that a response that uses $\frac{dy}{dt} = 2$ and then mishandles subtracting 40 from both sides of the equation, e.g., $3(2)^2(2) + 2(4)(2) + 2(2) \frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = \frac{40}{4}$, does not earn the fourth point.

Total for part (d) 4 points

Total for question 5 9 points