Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Two particles, *H* and *J*, are moving along the *x*-axis. For $0 \le t \le 5$, the position of particle *H* at time *t* is given by $x_H(t) = e^{t^2-4t}$ and the velocity of particle *J* at time *t* is given by $v_J(t) = 2t\left(t^2-1\right)^3$.

	Model Solution	Scoring	
A	Find the velocity of particle *H* at time $t = 1$. Show the work that leads to your answer.		
	$x_H{}'(t) = v_H(t) = (2t-4)e^{t^2-4t}$	Considers $x_H{}'$	**Point 1 (P1)**
	$x_H{}'(1) = v_H(1) = -2e^{-3}$	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** can be earned by presenting $x_H{}'(t)$, $x_H{}'(1)$, $x'(t)$, $x'(1)$, $(2t-4)e^{t^2-4t}$, or $(2\cdot1-4)e^{1^2-4\cdot1}$.
- An unsupported answer of $-2e^{-3}$ earns **P2** but not **P1**.

	Model Solution	Scoring	
B	During what open intervals of time *t*, for $0 < t < 5$, are particles *H* and *J* moving in opposite directions? Give a reason for your answer.		
	From part A, $x_H{}'(t) = v_H(t) = (2t-4)e^{t^2-4t}$. $x_H{}'(t) = (2t-4)e^{t^2-4t} = 0 \Rightarrow t = 2$ $x_H{}'(t) < 0$ for $0 < t < 2$, and $x_H{}'(t) > 0$ for $2 < t < 5$. Thus, particle *H* is moving to the left for $0 < t < 2$ and moving to the right for $2 < t < 5$.	Considers sign of $x_H{}'(t)$ or $v_J(t)$	**Point 3 (P3)**
		Analysis for one particle	**Point 4 (P4)**
	$v_J(t) = 2t\left(t^2-1\right)^3 = 0$ for $0 < t < 5 \Rightarrow t = 1$ $v_J(t) < 0$ for $0 < t < 1$, and $v_J(t) > 0$ for $1 < t < 5$. Thus, particle *J* is moving to the left for $0 < t < 1$ and moving to the right for $1 < t < 5$. Therefore, particles *H* and *J* are moving in opposite directions for $1 < t < 2$.	Answer with reason	**Point 5 (P5)**

Scoring Notes for Part B

- To earn **P3**, a response can do one of the following:
  - Set $x_H{}'(t) = 0$, $v_H(t) = 0$, or $(2t-4)e^{t^2-4t} = 0$
  - Set $v_J(t) = 0$ or $2t\left(t^2-1\right)^3 = 0$
  - Identify $t = 2$ for particle *H* and no other values in the interval $0 < t < 5$
  - Identify $t = 1$ for particle *J* and no other values in the interval $0 < t < 5$
  - Identify the interval $1 < t < 2$
- To earn **P4**, a response can provide an analysis of signs of velocity or direction of motion on the interval $0 < t < 5$ for either particle *H* or particle *J*.
- To be eligible for **P5**, a response must provide correct analyses of signs of velocity or direction of motion on the interval $0 < t < 5$ for both particles.
- Only analysis within the interval $0 < t < 5$ will be considered in scoring.

C	It can be shown that $v_J'(2) > 0$. Is the speed of particle *J* increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.		
	$v_J(2) > 0$ and $v_J'(2) > 0$. Because $v_J(2)$ and $v_J'(2)$ have the same sign, the speed of particle *J* is increasing at $t = 2$.	Answer with reason	**Point 6 (P6)**

Scoring Notes for Part C

- An evaluation of $v_J(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J(2) = 108$.
- An evaluation of $v_J'(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J'(2) = 486$.
- A response can either import the analysis for the sign of $v_J(2)$ from part B or restart.
- A response that stated "$v_J(t) > 0$ for $1 < t < 5$" in part B does not need to restate $v_J(2) > 0$ and earns **P6** for "$v_J(2)$ and $v_J'(2)$ have the same sign, so the speed is increasing."

D	Particle *J* is at position $x = 7$ at time $t = 0$. Find the position of particle *J* at time $t = 2$. Show the work that leads to your answer.		
	$x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t\left(t^2-1\right)^3 dt$	Integrand	**Point 7 (P7)**
	$= 7 + \left[\frac{1}{4}\left(t^2-1\right)^4\right]_0^2$	Antiderivative	**Point 8 (P8)**
	$= 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 27$	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- To earn **P7**, a response must present an indefinite or definite integral with an integrand of $v_J(t)$ or $2t\left(t^2-1\right)^3$. (See below for notes on how to handle a missing differential dt.)
- **P8** is earned for an antiderivative of the form $k\left(t^2-1\right)^4$ or equivalent, for $k > 0$. If $k \neq \frac{1}{4}$, then the response is not eligible to earn **P9**.
- A response of $7 + \frac{1}{4}\left((3)^4 - (-1)^4\right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

  Note: An ambiguous response, such as $7 + \frac{1}{4}\left((3)^4 - (-1)^4\right.$, does not bank **P9** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., $7 + \frac{1}{4}(80)$ or 27) to earn **P9**.

- If the differential dt is missing:
  - Writing $\int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
  - Writing $7 + \int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
  - Writing $\int_0^2 v_J(t) + 7$ introduces an ambiguity for the intended integrand.
    - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}\left(t^2-1\right)^4\right]_0^2 + 7$ resolves the ambiguity.

      Therefore, this earns **P7** and **P8** and is eligible for **P9**.
    - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}\left(t^2-1\right)^4 + 7t\right]_0^2$ confirms that an incorrect integrand was used.

      Therefore, this does not earn **P7**, earns **P8**, and is not eligible for **P9**.
    - If the ambiguity is not resolved, this does not earn **P7**, **P8**, or **P9**.
- Alternate solution using *u*-substitution:

  Let $u = t^2 - 1$, then $du = 2t\,dt$.

  $t = 0 \Rightarrow u = -1$

  $t = 2 \Rightarrow u = 3$

  $x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t\left(t^2-1\right)^3 dt$

  $= 7 + \int_{-1}^3 u^3\,du = 7 + \left[\frac{1}{4}u^4\right]_{-1}^3$

  $= 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 27$
- Alternate solution using indefinite integral:

  $\int 2t\left(t^2-1\right)^3 dt = \frac{1}{4}\left(t^2-1\right)^4 + C$

  $x_J(0) = 7 = \frac{1}{4}\left(0^2-1\right)^4 + C \Rightarrow C = \frac{27}{4}$

  $x_J(t) = \frac{1}{4}\left(t^2-1\right)^4 + \frac{27}{4}$

  $x_J(2) = \frac{1}{4}\left(2^2-1\right)^4 + \frac{27}{4} = \frac{108}{4} = 27$