

## 7 Solving simple differential equations

### – Part 2

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

#### Recall

In Part A you met differential equations. Now solve simple ones:

- Solving means finding the function from its rate of change.
- One special kind describes things that grow by a fixed percentage – like money or populations.

Each idea has a worked example before the Practice.

#### New ideas

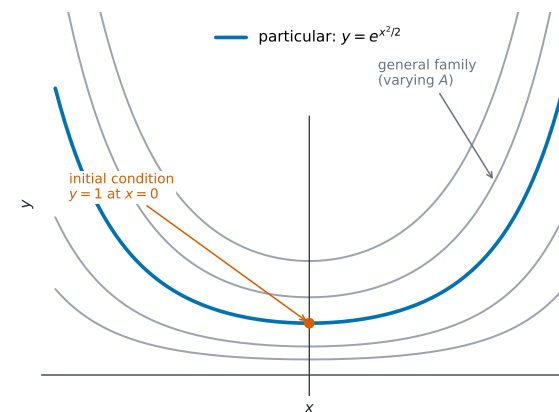
##### ■ 1 Solving by integrating

If  $\frac{dy}{dx}$  is given, integrate to find  $y$ .

*Worked example.*  $\frac{dy}{dx} = 3x^2$  integrates to  $y = x^3 + C$ .

##### ■ 2 A family of curves

The constant  $C$  gives a **family** of solution curves; one extra fact (a known point) picks out the right one.



##### ■ 3 Exponential growth

A very common equation is  $\frac{dy}{dt} = ky$  – “the rate of growth is proportional to how much there is”. Its solution is exponential growth,  $y = Ce^{kt}$ .

##### ■ 4 Where it appears

This models populations, money earning interest, and radioactive decay (with a negative  $k$ ) – anything that grows or shrinks by a fixed proportion.

**Watch out:** exponential growth ( $\frac{dy}{dt} = ky$ ) means the quantity grows **faster and faster** as it gets bigger, because the rate depends on the current amount. It is not steady, straight-line growth.

#### Practice

Try these in order. The methods are all above.

**2.1** State how you solve  $\frac{dy}{dx} = f(x)$  for  $y$ . [1]

**2.2** Solve  $\frac{dy}{dx} = 2x$ . [2]

**2.3** Solve  $\frac{dy}{dx} = 4x^3$ . [2]

**2.4** State the type of growth described by  $\frac{dy}{dt} = ky$ . [1]

**2.5** Give two real situations modelled by exponential growth or decay. [2]