AP CALCULUS AB • HOLIDAY HOMEWORK 假期作业

7 Differential equations and slope fields – Part 1

Name: _____ Class: _____ Date: _____

Recall

You have solved ordinary equations to find a number:

- An equation like $2x + 1 = 7$ has the solution $x = 3$.
- But some equations describe how a quantity **changes** – they involve a derivative.

This sheet introduces differential equations. Each idea has a worked example.

New ideas

1 What a differential equation is

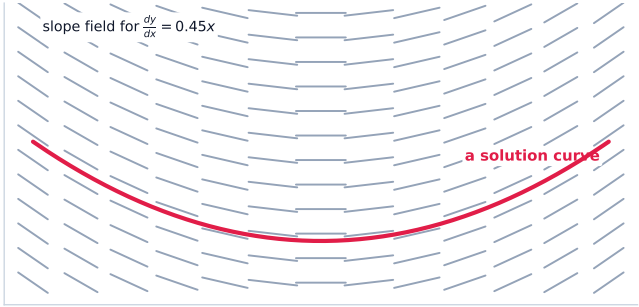
A **differential equation** involves a **derivative** – it tells you the **rate of change**, not the value directly. For example $\frac{dy}{dx} = 2x$ says "the slope at each point is $2x$".

2 Its solution is a function

Solving it means finding the **function** y whose derivative matches. For $\frac{dy}{dx} = 2x$, one solution is $y = x^2$ (its derivative is $2x$).

3 Slope fields

A **slope field** draws a tiny line at many points showing the slope there. It lets you see the shape of the solutions without solving.



each segment is the local slope $\frac{dy}{dx}$; a solution curve is tangent to every segment it meets

4 A family of solutions

There are usually **many** solutions differing by a constant (like $y = x^2$, $y = x^2 + 1$, …) – a whole family that follows the slope field.

Watch out: a differential equation gives the **slope** (dy/dx), not the function itself. You must integrate (or otherwise solve) to recover the actual function y.

Practice

Try these in order. The methods are all above.

2.1 State what makes an equation a "differential equation". [2]

2.2 State what it means to solve a differential equation. [2]

2.3 For $\frac{dy}{dx} = 2x$, verify that $y = x^2$ is a solution. [2]

2.4 State what a slope field shows. [2]

2.5 State why a differential equation usually has many solutions. [1]