6 The area under a curve – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
Riemann sum	黎曼和	

Recall

You can find the area of simple shapes:

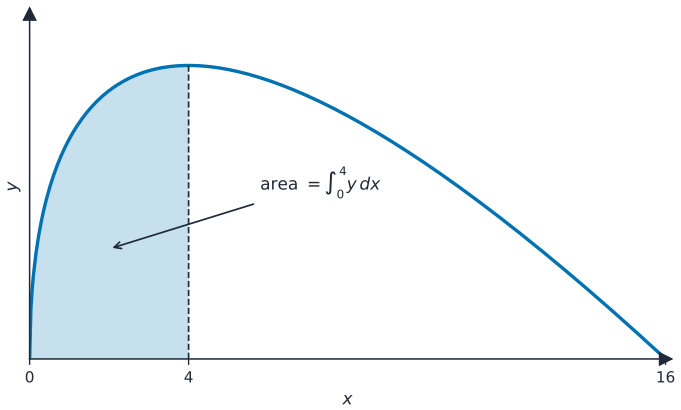
- The area of a rectangle is width × height.
- But how do you find the area under a **curve**, which has no straight edges?

This sheet introduces the idea behind **integration**. Each idea has a worked example.

New ideas

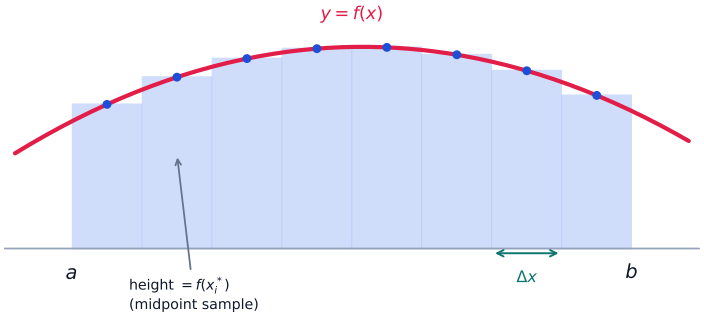
■ 1 Area under a curve

The **area under a curve** (between the curve and the x-axis) is a useful quantity – for example, the area under a velocity–time graph is the distance travelled.



■ 2 Approximating with rectangles

You can estimate that area by covering it with thin **rectangles** and adding their areas – a **Riemann sum** 黎曼和.



$$\sum f(x_i^*)\Delta x \to \int_a^b f \text{ as } n \to \infty$$

■ 3 More rectangles, better estimate

The thinner the rectangles (the more you use), the closer the total gets to the true area.

Worked example. To estimate the area under $y = x$ from 0 to 4 with rectangles: the exact area is a triangle, $\frac{1}{2} \times 4 \times 4 = 8$, and more rectangles get closer and closer to this.

■ 4 The exact area

Making the rectangles infinitely thin gives the **exact** area – this exact value is the **integral**.

Watch out: a Riemann sum with a few rectangles is only an **estimate**. The exact area (the integral) is the limit as the rectangles become infinitely thin and infinitely many.

Practice

Try these in order. The methods are all above.

2.1 State what "the area under a curve" means. [2]

2.2 State how a Riemann sum estimates that area. [2]

2.3 State what happens to the estimate as you use more, thinner rectangles. [1]

2.4 Find the exact area under $y = x$ from 0 to 6 (it is a triangle). [3]

2.5 State what the area under a velocity–time graph represents. [1]