

8.4 Area Between Curves (Functions of x)

Name: _____ Class: _____ Date: _____

Total: 15 marks

Objective

Build the skills to answer exam questions on the **area between two curves** (integrating in x).

You must be able to:

- find the **intersection points** that give the limits of integration
- integrate **top minus bottom**: $\int_a^b (f_{\text{top}} - f_{\text{bottom}}) dx$
- set up the area correctly even when a curve dips below the other

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Top minus bottom

The area between $y = f(x)$ (upper) and $y = g(x)$ (lower) from a to b is

$$\int_a^b (f(x) - g(x)) dx.$$

■ Finding the limits

Set the curves equal to find where they cross. For $y = x$ and $y = x^2$: $x = x^2 \Rightarrow x = 0, 1$. These are the limits.

■ Which is on top?

Test a point between the crossings. At $x = 0.5$: the line $y = 0.5$ is above the parabola $y = 0.25$, so $y = x$ is the **top**:

$$\int_0^1 (x - x^2) dx = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

■ A worked area

Area between $y = 4 - x^2$ and the x -axis ($y = 0$): crossings at $x = \pm 2$, so $\int_{-2}^2 (4 - x^2) dx = \left[4x - \frac{x^3}{3} \right]_{-2}^2 = \frac{32}{3}$.

2 Practice

Now apply the methods above.

2.1 Find where $y = x^2$ meets $y = 4$. [1]

2.2 Set up (do not evaluate) the integral for the area between $y = x^2$ and $y = 4$. [2]

2.3 Evaluate $\int_0^2 (2x - x^2) dx$. [2]

3 Exam-style questions

3.1 The area between $y = f(x)$ (upper) and $y = g(x)$ (lower) on $[a, b]$ is [1]

- | | |
|---|-----------------------|
| A | $\int_a^b (g - f) dx$ |
| B | $\int_a^b (f - g) dx$ |
| C | $\int_a^b f dx$ |
| D | $\int_a^b (f + g) dx$ |

3.2 Find the area of the region enclosed by $y = x$ and $y = x^2$.

(a) Find the limits of integration.

[1]

(b) Evaluate the area.

[3]

3.3 Find the area of the region enclosed by $y = 6x - x^2$ and $y = x$.

[5]