

# 7.6 Separation of Variables

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 16 marks

**KEY WORDS** separation of variables 分离变量法 • general solution 通解 • separable 可分离  
• differential equation 微分方程 • solution 解

## Objective

Build the skills to answer exam questions on **separation of variables** — the main method for solving a differential equation.

**You must be able to:**

- separate a **separable** 可分离 equation so each variable is on its own side
- integrate both sides and include a single constant  $C$
- solve for  $y$  where the question asks for an explicit solution

## 1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

### ■ Separating the variables

For  $\frac{dy}{dx} = xy$ : put all  $y$  with  $dy$  and all  $x$  with  $dx$ :

$$\frac{1}{y} dy = x dx.$$

### ■ Integrating both sides

$$\int \frac{1}{y} dy = \int x dx \Rightarrow \ln|y| = \frac{x^2}{2} + C.$$

One constant  $C$  is enough (combine the two).

### ■ Solving for $y$

Exponentiate:  $|y| = e^{x^2/2+C} = e^C e^{x^2/2}$ . Writing  $A = \pm e^C$  (a new constant),

$$y = Ae^{x^2/2}.$$

■ A quick separable example

$$\frac{dy}{dx} = \frac{x}{y}: y \, dy = x \, dx, \text{ so } \frac{1}{2}y^2 = \frac{1}{2}x^2 + C, \text{ i.e. } y^2 = x^2 + C_1.$$

## 2 Practice

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Now apply the methods above.

**2.1** Separate the variables in  $\frac{dy}{dx} = 2xy$ . [1]

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**2.2** Solve  $\frac{dy}{dx} = \frac{1}{y}$  (find the general solution). [3]

**2.3** Solve  $\frac{dy}{dx} = 3y$  (general solution in terms of a constant  $A$ ). [3]

## 3 Exam-style questions

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**3.1** After separating  $\frac{dy}{dx} = ky$ , integrating the left side gives [1]

|   |   |
|---|---|
| A | B |
| C | D |

**A**  $y$

**B**  $\ln|y|$

**C**  $\frac{1}{y^2}$

**D**  $e^y$

**3.2** Solve  $\frac{dy}{dx} = xy^2$ .

(a) Separate and integrate both sides. [3]

(b) Solve for  $y$  in terms of  $x$  and a constant. [2]

**3.3** Solve  $\frac{dy}{dx} = \frac{\cos x}{y}$ , giving the general solution. [3]