

5.6 Concavity and Points of Inflection

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS concave down 下凹 • concave up 上凹 • inflection point 拐点 • concavity 凹凸性

Objective

Build the skills to answer exam questions on **determining concavity**.

You must be able to:

- use the sign of f'' to find **concavity** 凹凸性
- identify an **inflection point** 拐点

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Concavity

- f is **concave up** where $f'' > 0$;
- f is **concave down** where $f'' < 0$.

An **inflection point** is where concavity changes (and f'' changes sign).

■ Example

$f(x) = x^3$: $f''(x) = 6x$, which is > 0 for $x > 0$; the inflection point is at $x = 0$.

2 Practice

2.1 State where f is concave up. [1]

2.2 For $f(x) = x^3$, find f'' and where it is concave up. [2]

2.3 State what an inflection point is. [1]

3 Exam-style questions

3.1 f is concave up where [1]

- | | |
|---|---|
| A | B |
| C | D |
- A $f' > 0$
 - B $f'' > 0$
 - C $f'' < 0$
 - D $f = 0$

3.2 An inflection point is where [1]

- | | |
|---|---|
| A | B |
| C | D |
- A $f' = 0$
 - B concavity changes
 - C $f = 0$
 - D f'' is constant

3.3 $f(x) = x^3$.

- (a) Find $f''(x)$. [1]
- (b) State where $f'' > 0$. [1]
- (c) State the inflection point. [1]