Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

- $\dfrac{dV}{dt} = 3s^2\dfrac{ds}{dt}$
- $3(16)(0.1) = \mathbf{4.8\ cm^3/s}$

2.2

- $\dfrac{dy}{dt} = (2x+3)\dfrac{dx}{dt}$
- $(2+3)(2)$
- **10**

2.3

- $\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$
- $100 = 4\pi(25)\dfrac{dr}{dt}$
- $\dfrac{dr}{dt} = \frac{1}{\pi} = \mathbf{0.318\ cm/s}$

3.1

- $x\dfrac{dy}{dt} + y\dfrac{dx}{dt} = 0$
- $y = 3$ when $x = 4$
- $4\dfrac{dy}{dt} + 3(3) = 0$
- $\dfrac{dy}{dt} = -\frac{9}{4}$ units per second

3.2

- $z^2 = x^2 + y^2$, $z = 5$
- $z\dfrac{dz}{dt} = x\dfrac{dx}{dt} + y\dfrac{dy}{dt}$
- $5\dfrac{dz}{dt} = 3(2) + 4(-1)$
- $\dfrac{dz}{dt} = \mathbf{0.4\ km/min}$, increasing

3.3

(a) $x^2 + y^2 = 100$; $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

(b) $y = 8$; $6(0.5) + 8\dfrac{dy}{dt} = 0$; $\dfrac{dy}{dt} = -0.375$; the top slides down at $\mathbf{0.375\ m/s}$

(c) $\cos\theta = \dfrac{x}{10}$; $-\sin\theta\dfrac{d\theta}{dt} = \dfrac{1}{10}\dfrac{dx}{dt}$; $\sin\theta = \frac{8}{10}$; $\dfrac{d\theta}{dt} = \mathbf{-0.0625\ rad/s}$

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(d) $A = \frac{1}{2}xy$, $\dfrac{dA}{dt} = \frac{1}{2}\left(\dfrac{dx}{dt}y + x\dfrac{dy}{dt}\right)$; at $x = 6$: $\frac{1}{2}(4 - 2.25) = \mathbf{0.875}$ m^2/s; at $x = 8$: $y = 6$, $\dfrac{dy}{dt} = -\frac{2}{3}$; $\frac{1}{2}(3 - \frac{16}{3}) = \mathbf{-1.167}$ m^2/s; the area grows at first and then shrinks