Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

- $\dfrac{dy}{dx} = 15x^4 - 2$
- $\dfrac{d^2y}{dx^2} = \mathbf{60x^3}$

2.2

- $f' = (x+1)e^x$, $f'' = (x+2)e^x$
- $f^{(3)} = (x+3)e^x$
- $f^{(n)}(x) = \mathbf{(x+n)e^x}$

2.3

- $\dfrac{d^2y}{dx^2} = -x^{-2}$
- $\dfrac{d^3y}{dx^3} = 2x^{-3}$
- at $x = 2$: $\mathbf{\frac{1}{4}}$

3.1

- $v = 3t^2 - 12t + 9$
- $a = 6t - 12 = 0$ at $t = 2$
- $v(2) = 12 - 24 + 9$
- $\mathbf{-3\ m/s}$

3.2

- $\dfrac{dy}{dx} = 2e^{2x} - 3e^{-3x}$
- $\dfrac{d^2y}{dx^2} = 4e^{2x} + 9e^{-3x}$
- e^{2x} terms: $4 + 2 - 6 = 0$
- e^{-3x} terms: $9 - 3 - 6 = 0$, so the sum is 0

3.3

(a) $\dfrac{dy}{dx} = 2x\ln x + x^2 \cdot \dfrac{1}{x} = 2x\ln x + x$; $\dfrac{d^2y}{dx^2} = 2\ln x + 2 + 1 = 2\ln x + 3$

(b) $\ln x = -\frac{3}{2}$; $\mathbf{x = e^{-3/2}}$; negative for $0 < x < e^{-3/2}$; positive for $x > e^{-3/2}$

(c) quotient rule: $\dfrac{d^2y}{dx^2} = -\dfrac{4y - 4x\frac{dy}{dx}}{16y^2}$; replace $\dfrac{dy}{dx}$: $-\dfrac{4y + x^2/y}{16y^2}$; $= -\dfrac{4y^2 + x^2}{16y^3}$; $= -\dfrac{4}{16y^3} = -\dfrac{1}{4y^3}$

exercises.lol exercises.com.cn

(d) $f' = 2\cos 2x$, $f'' = -4\sin 2x$, $f^{(3)} = -8\cos 2x$, $f^{(4)} = 16\sin 2x$; the pattern repeats every 4 steps, and $10 = 8 + 2$; $f^{(10)}(x) = \mathbf{-1024}\sin\mathbf{2x}$; at $\frac{\pi}{12}$: $-1024\sin\frac{\pi}{6} = \mathbf{-512}$