

1.15 Connecting Limits at Infinity and Horizontal Asymptotes

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS horizontal asymptote 水平渐近线 • degrees 次数

• ratio of the leading coefficients 首项系数之比 • limit at infinity 无穷远处极限 • limit 极限

Objective

Build the skills to answer exam questions on **connecting limits at infinity and horizontal asymptotes**.

You must be able to:

- relate a **limit at infinity** 无穷远处极限 to a **horizontal asymptote** 水平渐近线
- find it from the ratio of leading terms

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Limits at infinity

$\lim_{x \rightarrow \infty} f(x) = L$ means $y = L$ is a horizontal asymptote. For a rational function, compare leading terms:

- degree(top) < degree(bottom) $\rightarrow y = 0$;
- degrees equal $\rightarrow y = \frac{\text{leading coeff of top}}{\text{leading coeff of bottom}}$;
- degree(top) > degree(bottom) $\rightarrow$ no horizontal asymptote.

■ Example

$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$, so $y = 0$ is a horizontal asymptote.

2 Practice

2.1 State what $\lim_{x \rightarrow \infty} f(x) = L$ indicates.

[1]

2.2 For $f(x) = \frac{1}{x}$, state $\lim_{x \rightarrow \infty} f(x)$. [1]

2.3 For $f(x) = \frac{2x}{x+1}$, state the horizontal asymptote. [2]

3 Exam-style questions

3.1 $\lim_{x \rightarrow \infty} f(x) = L$ means $y = L$ is a [1]

- | | |
|---|---|
| A | B |
| C | D |
- A** vertical asymptote
B horizontal asymptote
C hole
D root

3.2 $\lim_{x \rightarrow \infty} \frac{1}{x}$ equals [1]

- | | |
|---|---|
| A | B |
| C | D |
- A** 1
B 0
C ∞
D -1

3.3 $f(x) = \frac{3x^2}{x^2 + 1}$.

- (a) Compare the degrees. [1]
(b) Take the ratio of the leading coefficients. [1]
(c) State the horizontal asymptote. [1]