

6.7 The Fundamental Theorem and Evaluating Integrals

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS net change 净变化 • antiderivative 原函数
 • fundamental theorem of calculus 微积分基本定理 • definite integral 定积分

Objective

Build the skills to answer exam questions on the **Fundamental Theorem of Calculus (evaluation form)** — using an antiderivative to evaluate a definite integral.

You must be able to:

- apply $\int_a^b f(x) dx = F(b) - F(a)$, where $F' = f$
- evaluate integrals of powers, exponentials, and basic trig functions
- interpret $\int_a^b f'(x) dx = f(b) - f(a)$ as a **net change** 净变化

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Evaluating with an antiderivative

$\int_1^3 2x dx$: an antiderivative of $2x$ is x^2 , so

$$\int_1^3 2x dx = [x^2]_1^3 = 3^2 - 1^2 = 8.$$

■ A power and a constant

$\int_0^2 (3x^2 + 1) dx = [x^3 + x]_0^2 = (8 + 2) - 0 = 10$. Antidifferentiate term by term, then subtract the values at the limits.

■ Net change from a rate

If f' is a rate of change, then $\int_a^b f'(x) dx = f(b) - f(a)$ is the **net change** in f . If a tank's level changes at rate $r(t)$ and $\int_0^{10} r dt = -4$, the level **fell** by 4 over those 10

units.

■ Trig and exponential

$$\int_0^\pi \sin x dx = [-\cos x]_0^\pi = -\cos \pi + \cos 0 = 1 + 1 = 2, \text{ and } \int_0^1 e^x dx = [e^x]_0^1 = e - 1.$$

2 Practice

Now apply the methods above.

2.1 Evaluate $\int_0^4 x dx$. [2]

2.2 Evaluate $\int_1^2 (4x^3) dx$. [2]

2.3 Evaluate $\int_0^{\pi/2} \cos x dx$. [2]

3 Exam-style questions

3.1 $\int_a^b f'(x) dx =$ [1]

- **A** $f'(b) - f'(a)$
- **B** $f(b) - f(a)$
- **C** $f(b) + f(a)$
- **D** $\frac{1}{2}(f(a) + f(b))$

3.2 A particle's velocity is $v(t) = 3t^2 - 4$ m/s.

(a) Evaluate $\int_0^2 v(t) \, dt$. [2]

(b) State what this value represents physically. [1]

3.3 Water enters a tank at rate $r(t) = 6t$ L/min. Find the total volume added between $t = 1$ and $t = 4$ minutes. [3]

Solutions

2.1 $[\frac{1}{2}x^2]_0^4 = 8 - 0 = 8$.

2.2 $[x^4]_1^2 = 16 - 1 = 15$.

2.3 $[\sin x]_0^{\pi/2} = 1 - 0 = 1$.

3.1 B — by the FTC, $\int_a^b f' = f(b) - f(a)$.

3.2 (a) $[t^3 - 4t]_0^2 = (8 - 8) - 0 = 0$. (b) The particle's **displacement** over $0 \leq t \leq 2$ is 0 — it returns to its start.

3.3 $\int_1^4 6t \, dt = [3t^2]_1^4 = 48 - 3 = 45$ L.