AP CALCULUS AB • EXERCISE SHEET

6.6 Properties of Definite Integrals

Name: ____________ Class: ________ Date: __________ **Total: 10 marks**

KEY WORDS definite integral 定积分

Objective

Build the skills to answer exam questions on the **properties of definite integrals** — combining and manipulating integral values.

You must be able to:

- use $\int_a^a f = 0$ and $\int_b^a f = -\int_a^b f$
- **split** an integral over adjacent intervals: $\int_a^c f = \int_a^b f + \int_b^c f$
- factor out constants and add integrands: $\int_a^b (cf + g) = c\int_a^b f + \int_a^b g$

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Zero-width and reversing

$\int_a^a f\,dx = 0$ (no width). Reversing the limits flips the sign: $\int_b^a f\,dx = -\int_a^b f\,dx$.

■ Adding adjacent intervals

Integrals join end-to-end: $\int_1^4 f = \int_1^3 f + \int_3^4 f$. If $\int_1^3 f = 5$ and $\int_1^4 f = 9$, then $\int_3^4 f = 9 - 5 = 4$.

■ Linearity

Constants come out and sums split:

$$\int_a^b \big(3f(x) + 2g(x)\big)\,dx = 3\int_a^b f + 2\int_a^b g.$$

If $\int_0^2 f = 4$ and $\int_0^2 g = 1$, then $\int_0^2 (3f + 2g) = 3(4) + 2(1) = 14$.

■ Combining with a reversal

Given $\int_2^5 f = 10$, find $\int_5^2 3f$. Reverse and factor: $\int_5^2 3f = 3\int_5^2 f = 3(-10) = -30$.

2 Practice

Now apply the methods above. You are given $\int_0^3 f = 6$, $\int_3^7 f = -2$, and $\int_0^3 g = 5$.

2.1 Find $\int_0^7 f\,dx$. [1]

2.2 Find $\int_3^0 f\,dx$. [1]

2.3 Find $\int_0^3 (2f + g)\,dx$. [2]

3 Exam-style questions

3.1 $\int_5^5 f(x)\,dx =$ [1]

- **A** $f(5)$
- **B** 1
- **C** 0
- **D** undefined

3.2 You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(a) Find $\int_4^6 h\,dx$. [2]

(b) Find $\int_6^1 h\,dx$. [1]

3.3 Given $\int_0^2 p = 8$ and $\int_0^2 q = -3$, find $\int_0^2 (5p - 4q)\,dx$. [2]

Solutions

2.1 $\int_0^7 f = \int_0^3 f + \int_3^7 f = 6 + (-2) = 4$.

2.2 $\int_3^0 f = -\int_0^3 f = -6$.

2.3 $2(6) + 5 = 17$.

3.1 **C** — a zero-width interval gives 0.

3.2 (a) $\int_4^6 h = \int_1^6 h - \int_1^4 h = 12 - 7 = 5$. (b) $\int_6^1 h = -12$.

3.3 $5(8) - 4(-3) = 40 + 12 = 52$.