AP CALCULUS AB • EXERCISE SHEET

6.3 Riemann Sums and Integral Notation

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

Objective

Build the skills to answer exam questions on **Riemann sums and definite-integral notation** —connecting a limit of sums to $\int_a^b f\,dx$.

You must be able to:

- write a Riemann sum with **summation notation** $\sum$
- recognise the **definite integral** 定积分 as the limit of Riemann sums as $n \to \infty$
- read the meaning of the parts of $\displaystyle\int_a^b f(x)\,dx$

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

■ Summation notation for a sum

A right Riemann sum with n equal pieces on $[a, b]$ has width $\Delta x = \dfrac{b-a}{n}$ and sample points $x_k = a + k\,\Delta x$:

$$\sum_{k=1}^{n} f(x_k)\,\Delta x.$$

■ The definite integral as a limit

Letting the rectangles become infinitely thin gives the exact area:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x.$$

The integral **is** this limit —a sum of infinitely many infinitely thin pieces.

■ Reading the notation

In $\displaystyle\int_a^b f(x)\,dx$: a and b are the **limits of integration**, $f(x)$ is the **integrand**, and dx marks the variable and the ”width” of each piece. The result is a **number** (a signed area), not a function.

■ **From a limit back to an integral**

A limit like $\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{1+\frac{3k}{n}}\cdot\frac{3}{n}$ is a right sum with $\Delta x=\frac{3}{n}$ on $[1,4]$, so it equals $\int_1^4 \sqrt{x}\,dx$.

2 Practice

Now apply the methods above.

2.1 For $\int_2^{10} f\,dx$ estimated with $n=4$ equal subintervals, state Δx. [1]

2.2 Write, in summation notation, a right Riemann sum for $\int_0^2 f\,dx$ with n equal subintervals. [2]

2.3 In $\int_1^5 (3x+2)\,dx$, name the integrand and the limits of integration. [2]

3 Exam-style questions

3.1 The definite integral $\int_a^b f\,dx$ is defined as [1]

- **A** an antiderivative of f
- **B** the limit of Riemann sums as $n\to\infty$
- **C** the slope of f
- **D** the average of $f(a)$ and $f(b)$

3.2 Consider $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{4k}{n}\right)^2\cdot\frac{4}{n}$.

(a) State the width Δx and the interval $[a, b]$. [2]

(b) Write the limit as a definite integral. [2]

3.3 A definite integral $\int_0^6 v(t)\,dt$ evaluates to a negative number. Explain what a **negative** value tells you about the signed area, in one sentence. [1]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **6.3 Riemann Sums and Integral Notation** lesson on the **Learn** page;
- read the **Riemann Sums, Summation Notation, and Definite Integral Notation** section of the AP Calculus AB handout on the **Know** page.

Solutions

2.1 $\Delta x = \dfrac{10-2}{4} = 2.$

2.2 $\Delta x = \dfrac{2}{n}$, $x_k = \dfrac{2k}{n}$; sum $= \displaystyle\sum_{k=1}^{n} f\left(\tfrac{2k}{n}\right) \cdot \tfrac{2}{n}.$

2.3 Integrand $3x+2$; limits of integration 1 and 5.

3.1 **B** —the definite integral is the limit of Riemann sums.

3.2 (a) $\Delta x = \dfrac{4}{n}$, so $b-a=4$; the sample $2+\frac{4k}{n}$ starts at 2, so $[a,b]=[2,6]$. (b) $\displaystyle\int_2^6 x^2\,dx.$

3.3 More signed area lies **below** the axis than above it over $[0,6]$.