

## 6.14 Selecting Techniques for Antidifferentiation

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 10 marks

KEY WORDS completing the square 配方法 • antiderivative 原函数

### Objective

Build the skills to answer exam questions on **selecting antidifferentiation techniques** — choosing the right method quickly.

You must be able to:

- recognise when a **basic rule**, a **substitution**, **long division**, or **completing the square** applies
- match an integrand's shape to the fastest method
- carry the chosen method through to a correct antiderivative

### 1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

#### ■ The decision guide

- **Basic rule** — a power,  $e^x$ ,  $\sin / \cos$ , or  $\frac{1}{x}$  on its own.
- **Substitution** — an inside function whose derivative also appears (e.g.  $2x$  beside  $x^2 + 1$ ).
- **Long division** — a rational function with numerator degree  $\geq$  denominator degree.
- **Completing the square** — an irreducible quadratic  $x^2 + bx + c$  in a denominator (arctangent).

#### ■ Spotting a substitution

$\int x^2 e^{x^3} dx$ : the  $x^2$  is (up to a constant) the derivative of  $x^3$  — **substitute**  $u = x^3$ .  
Answer  $\frac{1}{3}e^{x^3} + C$ .

#### ■ Spotting a division

$\int \frac{x^3}{x^2 + 1} dx$ : top-heavy, so **divide** first:  $x - \frac{x}{x^2 + 1}$ , then integrate (the second piece needs  $u = x^2 + 1$ ).

#### ■ Spotting a basic rule

$\int (2x + \cos x) dx$  needs no tricks — just the **basic rules**:  $x^2 + \sin x + C$ .

### 2 Practice

Now apply the methods above. For each, name the best method (basic / substitution / division / complete-the-square).

2.1  $\int \frac{1}{x^2 + 4} dx$  [1]

2.2  $\int 5x^4(x^5 + 2)^7 dx$  [1]

2.3  $\int \frac{x^2 + 1}{x} dx$  [1]

### 3 Exam-style questions

3.1 Which integral is best done by **substitution**? [1]

- A  $\int (x^3 - 2) dx$
- B  $\int \frac{1}{x^2 + 9} dx$
- C  $\int \cos(x^2) \cdot 2x dx$
- D  $\int \frac{x^2}{x + 1} dx$

**3.2** Find  $\int \frac{x^2+3}{x} dx$  (rewrite the fraction first). [3]

**3.3** Find  $\int \frac{6x}{x^2+1} dx$ . [3]

## Solutions

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**2.1** Complete-the-square / arctangent form (basic arctangent rule).

**2.2** Substitution,  $u = x^5 + 2$ .

**2.3** Basic — split into  $x + \frac{1}{x}$  first.

**3.1** **C** —  $2x$  is the derivative of  $x^2$ , so substitute  $u = x^2$ .

**3.2**  $\frac{x^2+3}{x} = x + \frac{3}{x}$ ;  $\int (x + \frac{3}{x}) dx = \frac{x^2}{2} + 3 \ln |x| + C$ .

**3.3**  $u = x^2 + 1$ ,  $du = 2x dx$ , so  $6x dx = 3 du$ ;  $\int \frac{3}{u} du = 3 \ln |u| = 3 \ln(x^2 + 1) + C$ .