6.14 Selecting Techniques for Antidifferentiation

Name: ______________ Class: __________ Date: ____________ **Total: 10 marks**

Objective

Build the skills to answer exam questions on **selecting antidifferentiation techniques** —choosing the right method quickly.

You must be able to:

- recognise when a **basic rule**, a **substitution**, **long division**, or **completing the square** applies
- match an integrand's shape to the fastest method
- carry the chosen method through to a correct antiderivative

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

■ The decision guide

- **Basic rule** —a power, e^x, sin / cos, or $\frac{1}{x}$ on its own.
- **Substitution** —an inside function whose derivative also appears (e.g. $2x$ beside x^2+1).
- **Long division** —a rational function with numerator degree $\geq$ denominator degree.
- **Completing the square** —an irreducible quadratic x^2+bx+c in a denominator (arctangent).

■ Spotting a substitution

$\int x^2 e^{x^3}\,dx$: the x^2 is (up to a constant) the derivative of x^3 —**substitute** $u = x^3$. Answer $\frac{1}{3}e^{x^3} + C$.

■ Spotting a division

$\int \dfrac{x^3}{x^2+1}\,dx$: top-heavy, so **divide** first: $x - \dfrac{x}{x^2+1}$, then integrate (the second piece needs $u = x^2+1$).

■

$\int (2x + \cos x)\,dx$ needs no tricks —just the **basic rules**: $x^2 + \sin x + C$.

2 Practice

Now apply the methods above. For each, name the best method (basic / substitution / division / complete-the-square).

2.1 $\int \frac{1}{x^2+4}\,dx$ [1]

2.2 $\int 5x^4(x^5+2)^7\,dx$ [1]

2.3 $\int \frac{x^2+1}{x}\,dx$ [1]

3 Exam-style questions

3.1 Which integral is best done by **substitution**? [1]

- **A** $\int (x^3 - 2)\,dx$
- **B** $\int \frac{1}{x^2+9}\,dx$
- **C** $\int \cos(x^2) \cdot 2x\,dx$
- **D** $\int \frac{x^2}{x+1}\,dx$

3.2 Find $\int \frac{x^2 + 3}{x}\, dx$ (rewrite the fraction first). [3]

3.3 Find $\int \frac{6x}{x^2 + 1}\, dx$. [3]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **6.14 Selecting Techniques for Antidifferentiation** lesson on the **Learn** page;
- read the **Selecting Techniques for Antidifferentiation** section of the AP Calculus AB handout on the **Know** page.

Solutions

2.1 Complete-the-square / arctangent form (basic arctangent rule).

2.2 Substitution, $u = x^5 + 2$.

2.3 Basic —split into $x + \frac{1}{x}$ first.

3.1 **C** —$2x$ is the derivative of x^2, so substitute $u = x^2$.

3.2 $\dfrac{x^2+3}{x} = x + \dfrac{3}{x}$; $\int \left(x + \frac{3}{x}\right) dx = \frac{x^2}{2} + 3\ln|x| + C$.

3.3 $u = x^2 + 1$, $du = 2x\,dx$, so $6x\,dx = 3\,du$; $\int \frac{3}{u}\,du = 3\ln|u| = 3\ln(x^2+1) + C$.