AP CALCULUS AB • EXERCISE SHEET

6.1 Exploring Accumulations of Change

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

Objective

Build the skills to answer exam questions on **accumulation of change** —reading a total change as the area under a rate graph.

You must be able to:

- interpret the **area** under a rate-of-change graph as an **accumulated change** 累积变化
- treat area **above** the axis as positive and area **below** as negative
- attach correct **units** (rate unit × time unit) to an accumulation

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

■ Area under a rate graph is a total

If water flows in at a **rate** $R(t)$ litres per minute, the total water added between $t = 0$ and $t = 5$ is the **area** under the R-vs-t graph over $[0, 5]$. A rate graph that is a rectangle of height 3 L/min over 5 min accumulates $3 \times 5 = 15$ L.

■ Splitting into simple shapes

If the rate graph is made of straight lines, find the area with triangles and rectangles. A rate rising linearly from 0 to 4 L/min over 6 min is a triangle: area $= \frac{1}{2}(6)(4) = 12$ L.

■ Signed area

When a rate goes **negative** (water leaving), that area counts as a **loss**. Over a stretch where the graph is $+8$ area then -3 area, the net accumulated change is $8 - 3 = +5$.

■ Units

The accumulation's unit is (rate unit) × (time unit). A rate in L/min over minutes accumulates **litres**; a velocity in m/s over seconds accumulates **metres**.

2 Practice

Now apply the methods above.

2.1 A tap runs at a constant 2.5 L/min for 8 minutes. How much water flows out? [1]

2.2 A rate graph is a triangle: the rate rises linearly from 0 to 10 units/s over 4 s. Find the accumulated change. [2]

2.3 Over one interval a quantity gains an area of 12 and over the next it loses an area of 5. State the net accumulated change. [1]

3 Exam-style questions

3.1 The area under a velocity-vs-time graph represents [1]

- **A** acceleration
- **B** displacement
- **C** speed
- **D** force

3.2 A machine consumes power at a rate $P(t)$ kilowatts, shown as a graph made of straight segments: a rectangle of height 2 over $0 \le t \le 3$ hours, then a triangle rising from 0 to 4 over $3 \le t \le 5$ hours.

(a) Find the energy used over $0 \le t \le 3$. [1]

(b) Find the energy used over $3 \le t \le 5$. [2]

(c) State the total energy used, with units. [1]

3.3 Sand is added to a pile at a rate that gives an accumulated area of 30 (kg) over the first hour, then removed at a rate giving an area of 8 (kg) over the next hour. State the net change in the pile's mass. [1]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **6.1 Exploring Accumulations of Change** lesson on the **Learn** page;
- read the **Exploring Accumulations of Change** section of the AP Calculus AB handout on the **Know** page.

Solutions

2.1 $2.5 \times 8 = 20$ L.

2.2 Triangle area $= \frac{1}{2}(4)(10) = 20$ units.

2.3 $12 - 5 = 7$.

3.1 B —the area under a velocity–time graph is displacement.

3.2 (a) $2 \times 3 = 6$ kWh. (b) triangle $\frac{1}{2}(2)(4) = 4$ kWh. (c) $6 + 4 = 10$ kWh.

3.3 $30 - 8 = 22$ kg.