

5.9 Connecting f, f-prime, and f-double-prime

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS local minimum 极小值 • concave up 上凹 • decreasing 递减 • increasing 递增

Objective

Build the skills to answer exam questions on **connecting a function, its first derivative, and its second derivative**.

You must be able to:

- read f , f' , and f'' together to describe a graph

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Connecting f , f' , f''

- f' tells increasing/decreasing;
- f'' tells concavity;
- a local **min** of $f \Leftrightarrow f' = 0$ and $f'' > 0$;
- an **inflection** point of $f \Leftrightarrow f''$ changes sign.

■ Example

$f(x) = x^3$: $f'(x) = 3x^2$ (zero at 0), $f''(x) = 6x$ (zero at 0).

2 Practice

2.1 State what f'' tells you about f . [1]

2.2 State the condition on f' and f'' at a local minimum. [1]

2.3 For $f(x) = x^3$, state where $f' = 0$ and where $f'' = 0$. [2]

3 Exam-style questions

3.1 f'' gives information about a graph's [1]

- A slope
- B concavity
- C x -intercepts
- D domain

3.2 A local minimum of f satisfies [1]

- A $f' = 0$, $f'' > 0$
- B $f' = 0$, $f'' < 0$
- C $f'' = 0$ only
- D $f = 0$

3.3 $f(x) = x^3$.

(a) Find $f'(x)$. [1]

(b) State where $f' = 0$. [1]

(c) State where $f'' = 0$. [1]

Solutions

2.1 its concavity (concave up where $f'' > 0$, down where $f'' < 0$).

2.2 $f' = 0$ and $f'' > 0$.

2.3 $f' = 0$ at $x = 0$; $f'' = 0$ at $x = 0$.

3.1 B.

3.2 A.

3.3 (a) $3x^2$. (b) $x = 0$. (c) $x = 0$.