

# 5.7 The Second Derivative Test

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ Total: 9 marks

**KEY WORDS** local extremum 局部极值 • local minimum 极小值 • critical point 临界点  
• local maximum 极大值 • concave down 下凹

## Objective

Build the skills to answer exam questions on the **Second Derivative Test**.

**You must be able to:**

- use  $f''$  at a critical point to classify a **local extremum** 局部极值

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Second Derivative Test

At a critical point  $c$  where  $f'(c) = 0$ :

- $f''(c) > 0 \rightarrow$  local **minimum** (concave up);
- $f''(c) < 0 \rightarrow$  local **maximum** (concave down);
- $f''(c) = 0 \rightarrow$  **inconclusive**.

### ■ Example

$f(x) = x^2 - 4x$ :  $f''(x) = 2 > 0$ , so the critical point is a local minimum.

## 2 Practice

**2.1** State the Second Derivative Test for a local minimum. [1]

**2.2** For  $f(x) = x^2 - 4x$  with  $f'' = 2$ , classify the critical point. [2]

**2.3** State when the test is inconclusive. [1]

## 3 Exam-style questions

**3.1** If  $f'(c) = 0$  and  $f''(c) < 0$ , then  $c$  is a [1]

- **A** local minimum
- **B** local maximum
- **C** inflection point
- **D** nothing

**3.2** If  $f'(c) = 0$  and  $f''(c) > 0$ , then  $c$  is a [1]

- **A** local maximum
- **B** local minimum
- **C** endpoint
- **D** asymptote

**3.3**  $f(x) = x^2 - 6x$ , with  $f'(3) = 0$ .

(a) Find  $f''(x)$ . [1]

(b) Evaluate  $f''(3)$ . [1]

(c) Classify  $x = 3$ . [1]

## Solutions

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**2.1** if  $f'(c) = 0$  and  $f''(c) > 0$ , then  $c$  is a local minimum.

**2.2**  $f'' = 2 > 0$ , so it is a local minimum.

**2.3** when  $f''(c) = 0$ .

**3.1** B.

**3.2** B.

**3.3** (a) 2. (b)  $2 > 0$ . (c) local minimum.